



# NEW YORK STATE MIGRANT EDUCATION PROGRAM

## **Title: Visualizing Fractions:**

### **Modeling addition and subtraction using Cuisenaire rods**

- **Part A LIKE Denominators**

**Description:** This module introduces Cuisenaire rods as concrete models of the part-to-whole relationship of fractions with application to fractions with like denominators. Cuisenaire rods allow students to model fractions and develop intrinsic ability to add and subtract fractions through conceptual understanding rather than by rote memorization of algorithms. By engaging students in these modeling activities, the outcome will be to have increased understanding of addition and subtraction of proper fractions with like denominators.

**Developer:** Suzanne K. Fox, Staff Development Specialist, Oswego Center for Instruction, Technology & Innovation (CiTi)

**Series:** This is the second in a series of Professional Development videos about Understanding Fractions through Concrete and Visual designs. The next video, *Visualizing Fractions: Modeling addition and subtraction using Cuisenaire rods \* Part B UNLIKE Denominators*, is the continuation of this video, working with the addition and subtraction of fractions that have different denominators.

## **Facilitator Guide**

**INDIVIDUAL ACCESS/SELF-SERVE:** *(for Individuals (viewing this module independently):* While a robust conversation between colleagues is an enriching way to learn, so is self-reflection. Read and use this Guide as the Facilitator of your own learning. To get the most out of the activities and questions, make sure you have the recommended handouts and supplies listed below, before beginning.

### **CTLE CREDIT**

**Group Workshop:** If you are facilitating this workshop for your METS, you will have to decide which process you will use for granting CTLE credit. You can use your local LEA process, or the M-TASC process:

- a. Contact M-TASC in advance of the workshop to confirm date and module.
- b. Use the M-TASC Participant Sign-In Sheet and submit.

- c. Submit Workshop Evaluations via link or hard copy. If you use the Evaluation link, M-TASC will forward the compiled evaluations once you have informed the office that all evaluations are complete.

**Individual Access/Self-Serve:** For those who would like to request Continuing Teacher Leader Education (CTLE) credit for On-Demand professional development, please complete the CTLE Credit Request for each module. Find the link for this process on the NYS-MEP website: <https://www.nysmigrant.org/resources/pd>

## OBJECTIVES/LEARNING TARGET(S)

- I can use visual models to show the relationship of fractions to the whole.
- I can transition from concrete to symbolic representations when modeling addition and subtraction of fractions.
- I can use modeling to add and subtract proper fractions with like denominators using grade level material.

## WORKSHOP/MODULE DESIGN

Brief Overview: One of the major emphasis domains in grades 3-5 mathematics is an understanding of and the ability to work with fractions. Students having difficulty with the traditional teaching using algorithms will benefit from modeling fractions concretely using Cuisenaire rods. Building on the relationship between fractions and the whole allows students conceptually understand the relationship of the part to the whole when adding and subtracting fractions with both like and unlike denominators, as well as the importance of having common denominators. The target audience for this workshop is educators working in grades 3-6 mathematics where the emphasis is on conceptual understanding of fractions.

## CONNECTION TO THE NYS MEP THEORY OF ACTION

### Subject Content and Instruction Subject

- Focus on assuring that in-school students the foundational skills and strategies to succeed in the classroom and on state and other assessments.

### Advocacy to Self-Advocacy

- Learner independence integrates key (meta) cognitive strategies and subject content knowledge with a focus on creating thinkers; problem solvers; and self-regulated, life-long learners.

## SUPPLIES AND MATERIALS

- Video: *Visualizing Fractions: Modeling addition and subtraction using unlike denominators \* Part A like denominators*
  - This video is for NYS MEP use only.
  - Use the video link on the NYS migrant website in the Professional Development section for, “*Visualizing Fractions: Modeling addition and subtraction \* Part A like denominators*”

- Participant Handouts
  1. Power Point Notes
  2. Paper Cuisenaire with Extra Rods
  3. Moving from Concrete to Symbolic Using the Rods
  4. Fractions on a Number Line
- Other Supplies
  - paper and pencil for notes
- PD Documentation
  - Workshop Sign-in Sheet
  - Workshop Exit Survey/Evaluation

**Facilitator Note:** For the best results, print the paper Cuisenaire rods in color on white cardstock. Participants only need one copy of the Cuisenaire rods handout for the workshop, but may need a second copy for use with students.

## PREPARE AHEAD OF TIME

The Cuisenaire rods need to be cut out ahead of time, so participants can use them in the workshop. Add five minutes to the beginning of the workshop time for participants to cut their own rods apart.

**Note:** Participants can leave (and use) the white squares in strips of 2, 3, 4, or 5, until more singles are needed.

## GETTING STARTED

- Disseminate handouts and supplies.
- 5 minutes for participants to cut the paper Cuisenaire rods
- Start the Video Presentation: *Visualizing Fractions – Modeling addition and subtraction using Cuisenaire rods \* Part A like denominators*

Video Presentation: 32 minutes

Plan for an additional 5 to 10 minutes if participants need extra time to complete the independent practice.

## Key Points

- Cuisenaire rods provide concrete modeling to show the relationship of unit fractions to the whole.
- Cuisenaire rods provide concrete modeling to show the relationship of unit fractions to each other.
- Addition and subtraction of fractions are based on the unit fraction and its relationship to the whole.

- Cuisenaire rods show physical modeling of addition and subtraction which form deeper understanding of why denominators stay the same in equivalent forms of answers (i.e.  $\frac{7}{6} = 1\frac{1}{6}$ )
- Cuisenaire rods show physical modeling of addition and subtraction, helping students form a deeper understanding as to why we add and subtract the numerators but not the common denominators.

**Facilitator Note:** The video will play background music while participants work (sometimes in pairs) to complete the activities within the presentation. If the music stops before the group is ready, please feel free to pause the video.

If participants struggle with an activity, know that the music plays for a maximum of 1 minute so that explanations can be modeled by Suzanne in a timely manner.

## RELATIONSHIP OF PART TO WHOLE USING THE RODS

### Key Points

- The “whole” can be different sizes. The relationship of the part to the whole is important.
- “1/2” can be different sizes. The relationship of the part to the whole is important.

### Activity 1: Participants are given the relationship of the “part” and need to find the “whole”

1. If the RED rod is “1/2,” what color Cuisenaire rod is the “whole” that goes with it?
2. If the YELLOW rod is “1/2,” what color Cuisenaire rod is the “whole” that goes with it?
3. If the DARK GREEN rod is “2/3,” what color Cuisenaire rod is the “whole” that goes with it?

### Activity 2: Handout: Moving from Concrete to Symbolic Using the Rods – Box #1

Use the models from the first activity to answer the questions in the first box.

## RELATIONAL UNDERSTANDING OF FRACTIONS USING THE RODS

### Key Points

- Students are solid in knowing that “eight” is greater than “four,” even written as 1/8.
- When comparing fractions, students can understand that the rod for 1/8 (one of eight parts to the whole) is smaller than the rod that is 1/4 (one of four parts to the same whole).

### Activity 3: Show Equivalence

Using the BROWN rod for the “whole,” participants find the colors to model Equivalent fractions in halves, fourths, and eighths.

### Activity 4: Comparing and Ordering

Using the DARK GREEN rod for the whole, participants find the colors to model  $\frac{1}{2}$ ,  $\frac{1}{3}$ , and  $\frac{1}{6}$ .

### Activity 5: Handout: Moving from Concrete to Symbolic Using the Rods – Box #2

Participants use the models from Equivalence and Comparing/Ordering to answer the questions and apply the math to the second box.

## MODELING SITUATIONS USING MATHEMATICS AND THE RODS

Suzanne models and asks participants to copy her with their own Cuisenaire rods.

**Facilitator Note:** To get the most out of the activities, encourage participants to copy Suzanne’s examples with their own Cuisenaire rods.

The “Fractions on a Number Line” handout is designed to use with the real Cuisenaire rods. The paper version is a different size.

## ADDITION OF FRACTIONS WITH LIKE DENOMINATORS

### Key Point

- Modeling with Cuisenaire rods and writing the math symbols ( $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$ ) reinforces the understanding that you don’t add the denominators when adding the numerators.

**For each problem,** Suzanne provides the color of the rod participants should use for the “whole.” Participants need to determine which colors to use for the parts and how to model the addition.

Use the DARK GREEN rod as the “whole” for  $\frac{5}{6} + \frac{2}{6}$ .

Use the BLUE rod as the “whole” for  $\frac{4}{3} + \frac{2}{3}$ .

Use the ORANGE rod as the “whole” for  $\frac{3}{5} + \frac{4}{5}$ .

## SUBTRACTION OF FRACTIONS WITH LIKE DENONMINATORS

### Key Point

- Modeling with Cuisenaire rods and writing the math symbols ( $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$ ) reinforces the understanding that you don’t change the common denominators when subtracting the numerators.

**For each problem**, Suzanne provides the color of the rod participants should use for the “whole.” Participants need to determine which colors to use for the parts and how to model the addition.

Use the BROWN rod as the “whole” for  $5/4 - 1/4$ .

Use the BROWN rod as the “whole” for  $4/8 - 1/8$ .

Use the DARK GREEN rod as the “whole” for  $5/6 - 3/6$ .

**Facilitator Note:** The ANNENBERG LEARNER website is suggested at the end of the video and Power Point Notes, to use for a virtual lesson using the Cuisenaire rods. This website is free.

**Click on this link:** <http://www.learner.org/courses/learningmath/number/session8/>

to go to Mathematics Session 8: Rational Numbers and Proportional Reasoning on the ANNENBERG LEARNING website.

You will see the three lesson choices shown on the video and power point handouts: Part A, Part B and Part C.

**Select Part B:** Fractions with Cuisenaire Rods.

This session has three pages of examples for modeling fractions with Cuisenaire rods that Educators can read and use with students.

The fourth page has Cuisenaire rods to click and drag so students can model problems on the computer.

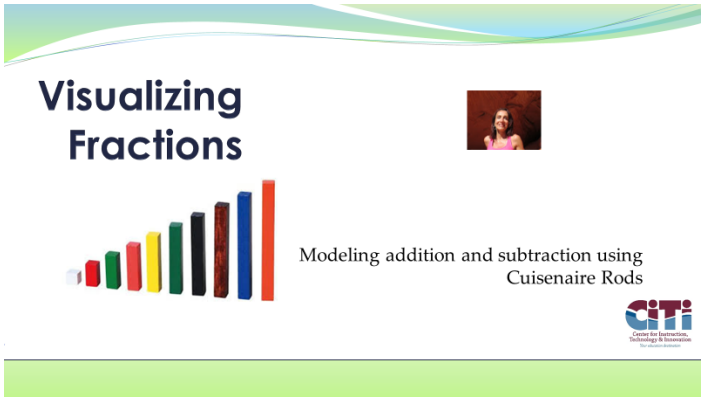
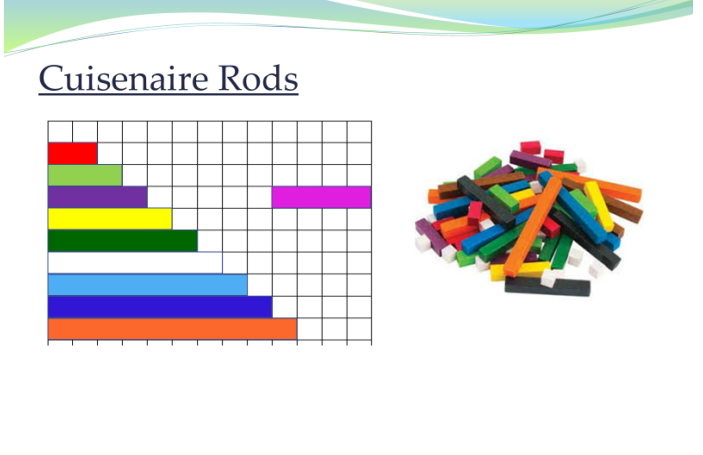

### Closure for Group Workshops

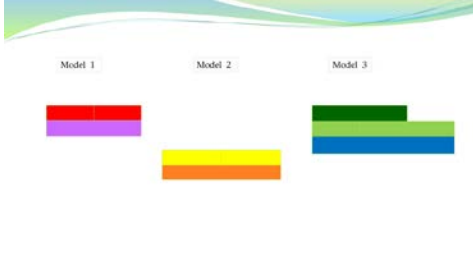
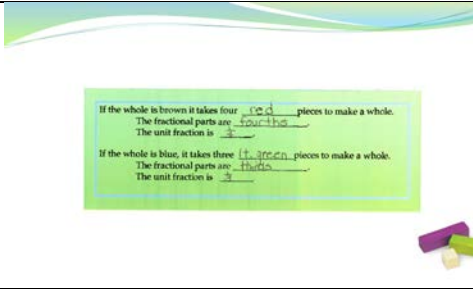
- Facilitators are welcome to use the Workshop Evaluation provided on the website or you can use your own version.
- *Continuing Teacher Leader Education (CTLE)* – Follow the CTLE process at your METS program center for staff who are tracking credit.

### Closure for Individual Access/Self-Serve



- Complete the Workshop Evaluation and give it to your Director.
- *Continuing Teacher Leader Education (CTLE)* – If you would like to request credit for this module, please follow the CTLE Credit Request process. Find the link for this process on the NYS-MEP website: <https://www.nysmigrant.org/resources/pd>

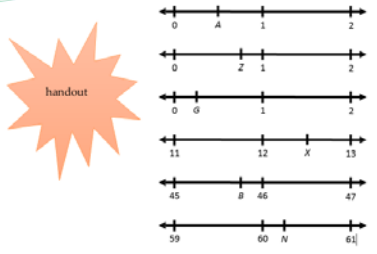

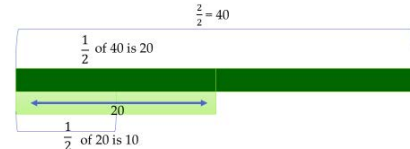
**Facilitator Note:** The following Appendix contains the workshop “Talking Points” used by Developer, Suzanne K. Fox, to support your facilitation when participants need something repeated.






 <p><b>Visualizing Fractions</b></p> <p>Modeling addition and subtraction using Cuisenaire Rods</p>	<p>Today’s web learning session will allow you to bring fractions to a concrete modeling version. By engaging your students in these modeling activities, students will understand the relationship of fractions to each other, the relationship of fractions to the whole, as well as understand the why behind addition and subtraction of proper fractions. This will be accomplished by using Cuisenaire rods. These rods allow students to manipulate fractions in a concrete manner. To help you in this understanding, you will need your cut outs of the Cuisenaire rods and the handouts titled “Moving from the symbolic to the Concrete”. It would also be helpful if you had paper and pencil for taking notes.</p>
 <p><u>Cuisenaire Rods</u></p>	<p>Let’s begin by playing with the Cuisenaire rods. Whenever I use manipulatives for the first time in class I always give students time to play and get a feel for what they might be used for. Take a moment and look at the relationship between the rods.</p> <p><b>MUSIC</b></p> <p>We will now start to use these rods to conceptually understand fractions.</p>
 <p><u>Relationships of Part to Whole Using the Rods</u></p>	<p>Using your rods let’s start to show relationships between fractions and one whole. This is the start of fractions and begins in grade 3. CLICK In the first, take a red rod. If it represents one half, what would you need to make one whole? PAUSE Now what color rod would represent that 1 whole? PAUSE.</p>






	<p>Leave this model constructed and try the second. PAUSE 15 seconds. Leave this model constructed and try the third. PAUSE for 15 seconds.</p>
	<p>Check your models with the three here. How did your models stack up? In the third model, if the dark green is two thirds, then we need to find three thirds before we find the whole.</p>
<p><u>Moving From Concrete to Symbolic Using the Rods</u></p> <div style="border: 1px solid green; padding: 5px; margin: 10px auto; width: fit-content;"> <p>If the whole is brown it takes four _____ pieces to make a whole. The fractional parts are _____ The unit fraction is _____</p> <p>If the whole is blue, it takes three _____ pieces to make a whole. The fractional parts are _____ The unit fraction is _____</p> </div>	<p>Here is one way you can help your student move from the concrete to the symbolic in fractions. Take a moment to complete these sentences on your handout that matches the slide. <b>MUSIC</b></p>
	<p>How did you do? Allowing your student to explore and realize that one whole can be different sizes and how one whole can be partitioned in different ways is understanding of fractions as part of the whole.</p>
<p><u>Relational Understanding of Fractions Using the Rods</u></p> <ul style="list-style-type: none"> <li>• Showing equivalence</li> <li>• Comparing and ordering</li> </ul> <div style="display: flex; justify-content: space-around; margin-top: 10px;"> <div style="width: 100px; height: 100px; background-color: #90EE90;"></div> <div style="width: 100px; height: 100px; background-color: #90EE90;"></div> </div>	<p>Another conceptual understanding in fractions is the ability to show equivalence and comparison in size and value. First, let's model an example that shows equivalence. CLICK Using your brown rod to represent one whole, model each of the rods that represent one whole using the unit fractions one half, one fourth and one eighth. <b>MUSIC</b> Leave this model and now construct a second model to be used for comparing fractions. CLICK Let's use these concrete models to think about the math.</p>



<p><u>Moving From Concrete to Symbolic Using the Rods</u></p> <p>If the whole is brown:</p> <p>The purple represents _____ + _____ = 1</p> <p>The red represents _____ + _____ = 1</p> <p>The white represents _____ + _____ = 1</p> <p><math>\frac{1}{2} = \frac{2}{4}</math>    <math>\frac{1}{3} = \frac{2}{6}</math>    <math>\frac{1}{4} = \frac{2}{8}</math></p> <p><math>\frac{1}{2} &gt; \frac{1}{3}</math>    <math>\frac{1}{3} &gt; \frac{1}{4}</math>    <math>\frac{1}{4} &lt; \frac{1}{5}</math>    <math>\frac{1}{5} &lt; \frac{1}{6}</math></p>	<p>Find the handout that looks like this slide. Using your Cuisenaire rod models, take a few moments to complete the sentences. Of course you may work alone or with a partner.</p> <p><b>MUSIC</b></p>
<p>If the whole is brown:</p> <p>The purple represents <math>\frac{1}{2} = \frac{2}{4} + \frac{2}{4} = \frac{4}{4} = 1</math></p> <p>The red represents <math>\frac{1}{3} = \frac{2}{6} + \frac{2}{6} + \frac{2}{6} = \frac{6}{6} = 1</math></p> <p>The white represents <math>\frac{1}{4} = \frac{2}{8} + \frac{2}{8} + \frac{2}{8} = \frac{6}{8} = 1</math></p> <p><math>\frac{1}{2} = \frac{2}{4}</math>    <math>\frac{1}{3} = \frac{2}{6}</math>    <math>\frac{1}{4} = \frac{2}{8}</math> (patterns)</p> <p><math>\frac{1}{2} &gt; \frac{1}{3}</math>    <math>\frac{1}{3} &gt; \frac{1}{4}</math>    <math>\frac{1}{4} &lt; \frac{1}{5}</math>    <math>\frac{1}{5} &lt; \frac{1}{6}</math></p> 	<p>How do your responses compare with the ones you see here? The top section lends itself to discussion on unit fractions, addition of unit fractions, and equivalence.</p> <p>The bottom section can start a rich dialogue on how equivalent fractions are written, the mathematical patterning involved, and the concept of greater than or less than – which is really hard for students to grasp if just using symbolic. They come into fractions knowing 8 is greater than 2 and then are told that in fractions it is the reverse! Yikes! Can you see how the rods actually show one eighth is a smaller amount than one half? You are creating the “aha” moment!</p>
<p><u>Moving From Concrete to Symbolic Using the Rods</u></p> <p>If the whole is dark green:</p> <p>The light green represents _____ + _____ = 1</p> <p>The red represents _____ + _____ = 1</p> <p>The white represents _____ + _____ = 1</p> <p>What would you come up with as examples of comparing fractions with equivalence and comparison?</p>	<p>Still staying with the handout, go through the three questions presented on the slide. See if you can create your own model, using a color as 1 whole and partitioning it into fractional pieces. Remember to model the questions using the Cuisenaire rods first.</p> <p><b>MUSIC</b></p>
<p>If the whole is dark green:</p> <p>The light green represents <math>\frac{1}{2} = \frac{3}{6} + \frac{3}{6} = \frac{6}{6} = 1</math></p> <p>The red represents <math>\frac{1}{3} = \frac{2}{6} + \frac{2}{6} + \frac{2}{6} = \frac{6}{6} = 1</math></p> <p>The white represents <math>\frac{1}{4} = \frac{3}{12} + \frac{3}{12} + \frac{3}{12} = \frac{9}{12} = 1</math></p> <p>What would you come up with as examples of comparing fractions with equivalence and comparison?</p> 	<p>Again, modeling with the rods gives the concrete reasoning behind equivalence and comparisons in fractions. You could ask students to circle equivalent fractions, list fractions in order from least to greatest, or even put these fractions on a number line.</p>

	<p>Here is an activity for the rods using a number line. You have the hand out for this as well. It is scaled for the rods. The second three number lines show that fractions occur beyond just 0 and 1. We may think this is a no-brainer, your student will need your guidance in seeing this partitioning between many different numbers.</p>
<p><u>Modeling Situations Using Mathematics and the Rods</u></p> 	<p>Cuisenaire rods can model relationships between fractional parts of the whole. Would you please model with me for this next example. PAUSE Take 2 dark green rods to represent the whole. CLICK Notice that we are describing one whole with two halves. CLICK Now place two light green rods CLICK under one of the halves. We have now identified one fourth of the whole. Now if we look at this one part of the whole CLICK can we also see that one fourth can also be described as one half CLICK of one half? Leave your model and let's take this to the next level.</p>
 <p>There were 40 kids on a school bus. Half of the kids brought their own lunch. Half of the kids who brought a lunch also brought a snack. How many kids brought a lunch <u>and</u> a snack?</p>	<p>What if we said that the whole is equal to 40. CLICK Then our one half would be CLICK twenty. Looking at our two light CLICK green rods we would know that the two together CLICK are twenty. So what would be the value of one rod? CLICK Half of 20 is 10! CLICK</p> <p>Take a moment to read the application problem. This whole situation can be modeled using the rods! There were no complex algorithms to know – we just described what was happening using mathematical modeling!</p>

<p><u>Addition of Fractions with Like Denominators</u> <u>Using the Rods</u></p> $\frac{5}{6} + \frac{2}{6}$  	<p>Now we come to some really nifty math modeling using the rods. Adding and subtracting. In order to do this, we rely on the foundational concept of the relation of the parts to the whole. This video will focus on adding and subtracting with like denominators. The Adding and subtracting Part B video works with unlike denominators. You will see what I mean in this first example. In order to do this segment justice, I will be doing the majority of the rest of the video using the document camera. For each of the modeling pages, I ask that you use your rods to model as I do. Then for each of the practice slides, you may work with a partner or solo, and I will go through every practice problem using the document camera.</p> <p>Let's start with this first example. Notice I give us the starting color rod to use. This rod will represent the whole.</p> <p><b>MOVE TO DOCUMENT CAMERA</b></p>
<p><u>Addition of Fractions with Like Denominators</u> <u>Using the Rods</u></p> <p>Independent Practice</p> $\frac{4}{3} + \frac{2}{3}$  $\frac{3}{5} + \frac{4}{5}$ 	<p>Here are two practice problems for you to model using the rods. I have given you the color rod to use in representing the whole in each.</p> <p><b>MUSIC</b> <b>THEN MOVE TO DOCUMENT CAMERA</b></p>
<p><u>Subtraction of Fractions with Like Denominators</u> <u>Using the Rods</u></p> $\frac{5}{4} - \frac{1}{4}$ 	<p>Subtraction with the rods is easy to model after going through addition. Let's model the first one together. As we move to the document camera, you will need your brown rod to represent the whole.</p> <p><b>MOVE TO DOCUMENT CAMERA</b></p>

<p>Subtraction of Fractions with Like Denominators          Using the Rods</p>  <p>Independent Practice</p>	<p>Here are two practice problems for you to model using the rods. I have given you the color rod to use in representing the whole in each.</p> <p><b>MUSIC</b>  <b>THEN MOVE TO DOCUMENT CAMERA</b></p>
<p>A virtual lesson using the rods</p> 	<p>Once again, the Annenberg Learner site does a really nice job in providing lessons and virtual modeling using the rods. Please explore this site for additional planning purposes.</p>
<p>Visualizing Fractions</p> <ul style="list-style-type: none"> <li>Were you able to “see” adding and subtracting happen without using traditional rules and algorithms?</li> </ul>   	<p>Working with fractions often has students stumbling as they understand how to work with whole numbers and then have to change or grow their mindset to now include numbers in between. Cuisenaire rods are meant to assist students in their conceptual understanding of working with fractions. Having this confidence in knowledge will enable them to work further with adding and subtracting fractions with unlike denominators. Using the rods for this work can be accessed through the part B video of this series. Thank you for allowing me to be a part of your math journey.</p>