NEW YORK STATE MIGRANT EDUCATION PROGRAM

Title: Visualizing Fractions:
Modeling addition and subtraction using Cuisenaire rods
- **Part B *UNLIKE* denominators**

Description: A 45-minute presentation on bringing fractions to a concrete model using Cuisenaire rods. Cuisenaire rods allow students to model fractions and develop intrinsic ability to add and subtract fractions through conceptual understanding rather than by rote memorization of algorithms. By engaging students in these modeling activities, the outcome will be to have increased understanding of addition and subtraction of proper fractions with unlike denominators.

Developer: Suzanne K. Fox, Staff Development Specialist, Oswego Center for Instruction, Technology & Innovation (CiTi)

Series: This is the third in a series of Professional Development videos about Understanding Fractions through Concrete and Visual designs. It is recommended to participate in *Visualizing Fractions: Modeling addition and subtraction using Cuisenaire rods * **Part A** **like** denominators,* prior to this video, for the introduction to Cuisenaire rods and practice modeling simpler fractions.

Facilitator Guide

INDIVIDUAL ACCESS/SELF-SERVE *(for Individuals viewing this module independently):* While a robust conversation between colleagues is an enriching way to learn, so is self-reflection. Read and use this Guide as the Facilitator of your own learning. To get the most out of the activities and questions, make sure you have the recommended handouts and supplies listed below, before beginning.

CTLE CREDIT

Group Workshop: If you are facilitating this workshop for your METS, you will have to decide which process you will use for granting CTLE credit. You can use your local LEA process, or the M-TASC process:

a. Contact M-TASC in advance of the workshop to confirm date and module.
b. Use the M-TASC Participant Sign-In Sheet and submit.

c. Submit Workshop Evaluations via link or hard copy. If you use the Evaluation link, M-TASC will forward the compiled evaluations once you have informed the office that all evaluations are complete.

Individual Access/Self-Serve: For those who would like to request Continuing Teacher Leader Education (CTLE) credit for On-Demand professional development, please complete the CTLE Credit Request for each module. Find the link for this process on the NYS-MEP website: https://www.nysmigrant.org/resources/pd

WORKSHOP/MODULE DESIGN

Brief Overview: One of the major emphasis domains in grades 3-5 mathematics is an understanding of and the ability to work with fractions. Students having difficulty with the traditional teaching using algorithms will benefit from modeling fractions concretely using Cuisenaire rods. Building on the relationship between fractions and the whole allows students to conceptually understand the relationship of the part to the whole when adding and subtracting fractions with both like and unlike denominators, as well as the importance of having common denominators. The target audience for this workshop is educators working in grades 3-6 mathematics where the emphasis is on conceptual understanding of fractions.

OBJECTIVES/LEARNING TARGET(S)

- To use visual models to show the relationship of fractions to the whole.
- To use modeling to add and subtract proper fractions with like denominators using grade level material.
- To use modeling to add and subtract proper fractions with unlike denominators using grade level material.
- To transition from concrete to pictorial representations when modeling addition and subtraction of fractions.

CONNECTION TO THE NYS MEP THEORY OF ACTION

- **Subject Content and Instruction Subject:** Focus on assuring that in-school students the foundational skills and strategies to succeed in the classroom and on state and other assessments.
- **Advocacy to Self-Advocacy:** Learner independence integrates key (meta) cognitive strategies and subject content knowledge with a focus on creating thinkers; problem solvers; and self-regulated, life-long learners.

SUPPLIES AND MATERIALS

- Video: *Visualizing Fractions: Modeling addition and subtraction using unlike denominators* **** Part B*** **unlike** *denominators*
    - This video is for NYS MEP use only.

- Use the video link on the NYS migrant website in the Professional Development section for "*Visualizing Fractions: Modeling addition and subtraction * **Part B** **unlike** denominators".*

- Participant Handouts
  1. Power Point Notes
  2. Paper Cuisenaire rods – Extra #2

  Facilitator Note: For the best results, print the paper Cuisenaire rods in color on white cardstock.

  - Print two copies per person.
  - Cut apart the colored rods.
  - Cut five (5) of the smallest rod (white). Then cut the rest of the white rods into strips of 2, 3 or 4 for easier traveling.

  3. ¼ Inch graph paper
- Other Supplies
  - paper and pencil for notes
- PD Documentation
  - M-TASC Sign-in Sheet
  - M-TASC Evaluation

Getting Started

- Disseminate handouts and supplies.
- Start the Video Presentation: *Visualizing Fractions – Modeling addition and subtraction using Cuisenaire rods *** Part B** **unlike** denominators.*

(Video Presentation: 45 minutes)

Plan for an additional 10 to 15 minutes if stopping video for extra time for participants to complete the independent practice.

Key Points

- Cuisenaire rods provide concrete modeling to show the relationship of unit fractions to the whole.
- Cuisenaire rods provide concrete modeling to show the relationship of unit fractions to each other.
- Addition and subtraction of fractions are based on the unit fraction and its relationship to the whole.
  - Cuisenaire rods show physical modeling of addition and subtraction which form deeper understanding of why denominators stay the same in equivalent forms of answers (i.e. $\frac{7}{6} = 1\frac{1}{6}$)
  - Cuisenaire rods show physical modeling of addition and subtraction which form deeper understanding of why common denominators are needed prior to the operation.

- Students can transfer concrete representations of the rods to pictorial using graph paper when working with more advanced fractions.

SHORT REVIEW

Symbolic ↔ Concrete, addition and subtraction practice using Cuisenaire rods with ***like*** denominators
(10 minutes)

- Activities One through Three constitute a brief warm-up
- As well as time for participants to re-connect with what they learned about modeling fractions with like denominators in during *Visualizing Fractions: Modeling addition and subtraction using Cuisenaire rods* ***** **Part A *like*** *denominators*

Facilitator Note: Participants need the Cuisenaire rods for Activities One through Five and will use the graph paper for Activity Six. To save time, the video tells participants which color Cuisenaire rods to use for modeling the solutions.

Activity 1: Let's play with the Cuisenaire rods! (rods needed)

Facilitator Note: The video will play background music while participants work (sometimes in pairs) to complete the activities within the presentation. If the music stops before the group is ready, feel free to pause the video. Conversely, if participants struggle, know that the music plays for a maximum of 1 minute so that explanations can be modeled in a timely manner.

Activity 2: Modeling addition with like denominators

- one example as a group
- one example done independently and checked as a group

Activity 3: Modeling subtraction with like denominators

- one example as a group
- one example done independently and checked as a group

Facilitator Note: To get the most out of the activities, encourage participants to copy Suzanne's examples with their own Cuisenaire rods.

Symbolic ↔ Concrete, addition and subtraction practice using Cuisenaire rods with *unlike* denominators

(Presentation: 35 minutes)

Plan an additional time (10 to 15 minutes) to stop the video when participants want more time with the independent practice.

Activity 4: Modeling addition with unlike denominators

- one example as a group
  - Find the whole for each fraction.
  - If the breaks in the rods do not line up, you need to find a new rod, denominator, that is COMMON to both fractions.
- two examples done independently and checked as a group
  - Model the whole for each fraction with Cuisenaire rods.
  - Model the common whole for both fractions with Cuisenaire rods.

Facilitator Note – The video tells participants which colors to use to model the independent practice problems, but participants need to figure out which color goes with which fraction.

The clue is that the larger unit fraction uses the larger Cuisenaire rod. In the problem 3/5 +1/4, the (unit) fraction 1/4 is larger than the (unit) fraction 1/5, so the fourths are modeled with the larger Cuisenaire rod.

Activity 5: Modeling subtraction with unlike denominators

- two examples as a group
  - When the "breaks" in the rods line up with each other, then the common rod, common denominator, is already one of the fractions in the model.
- two examples done independently and checked as a group
  - Model the whole for each fraction with Cuisenaire rods.
  - Model the common whole for both fractions with Cuisenaire rods.

Activity 6: Modeling a grade level example problem using graph paper

- one example as a group
- need graph paper to work along with Suzanne
- transitioning from Cuisenaire rods to Tape Diagrams

Facilitator Note: Website suggested as a good starter for Migrant Educators to use:
PBS Learning Media – Modeling Fractions with Cuisenaire Rods
https://ny.pbslearningmedia.org/resource/rttt12.math.cuisenaire/modeling-fractions-with-cuisenaire-rods/#.WrUbeOjwaUk
The website asks participants to sign up, but you can use the website without signing in.

The Visualizing Fractions series concludes with a video presentation about using Cuisenaire rods and graph paper to:

- model addition and subtraction of fractions in application problems;
- provide foundational understanding of fraction to decimal equivalence.

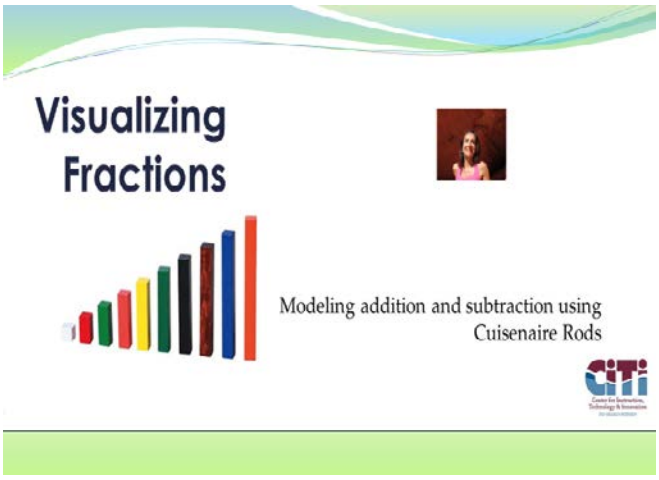
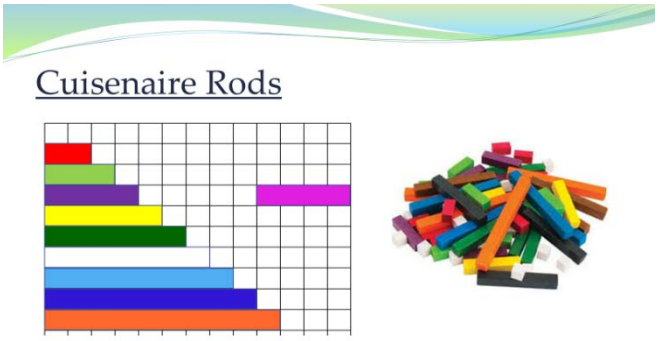
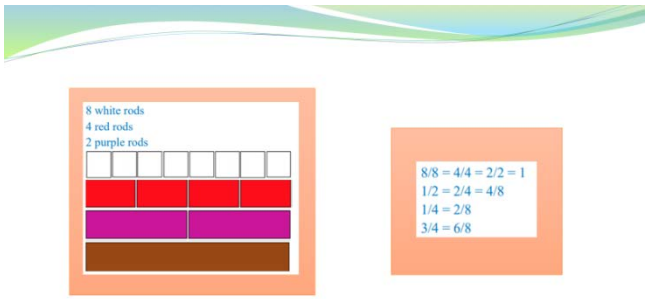
Closure for Group Workshops

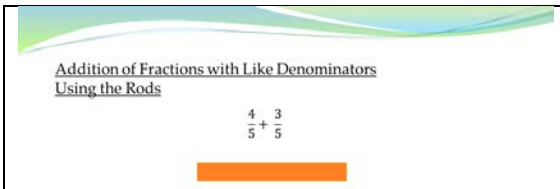
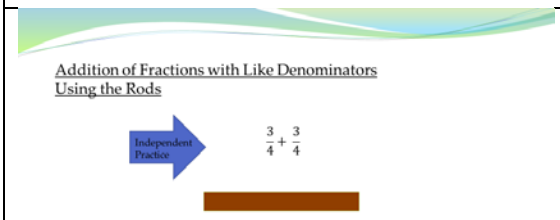
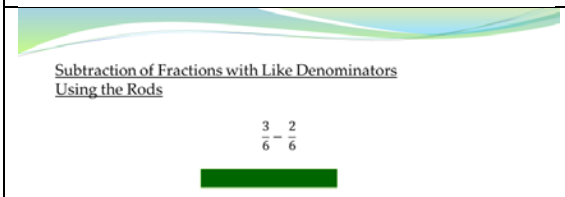
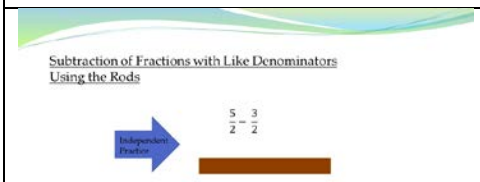
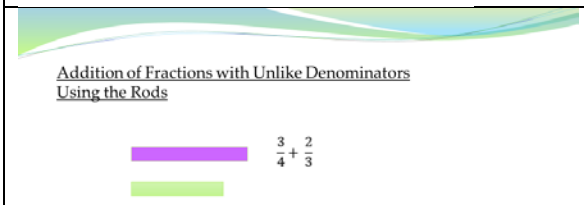
- Facilitators are welcome to use the Workshop Evaluation provided on the website or you can use your own version.
- *Continuing Teacher Leader Education (CTLE)* – Follow the CTLE process at your METS program center for staff who are tracking credit.

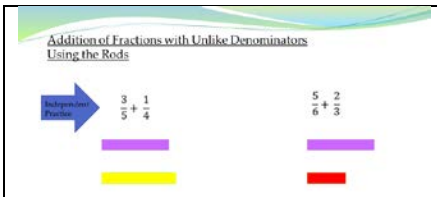
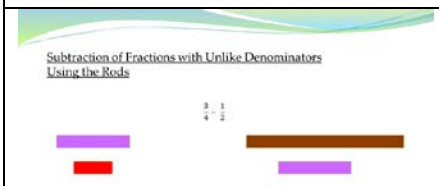
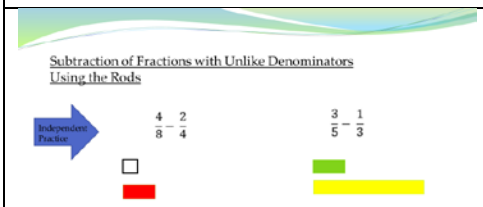
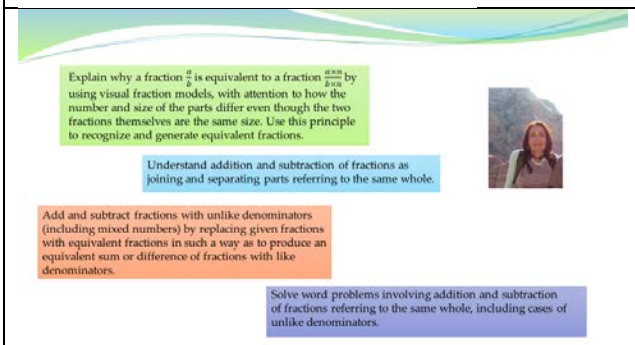
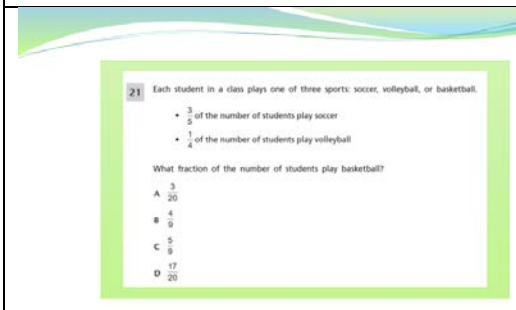
Closure for Individual Access/Self-Serve

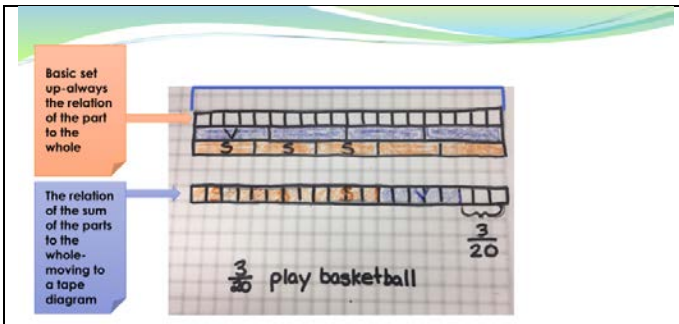
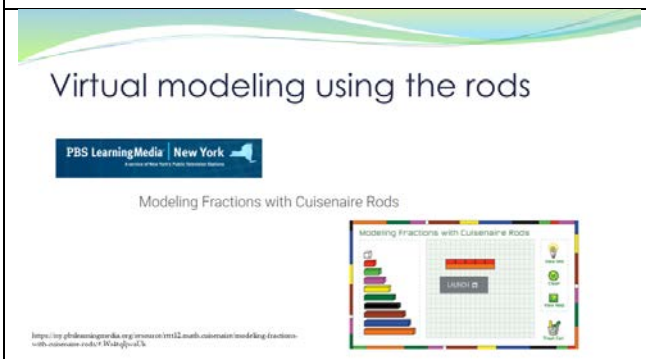
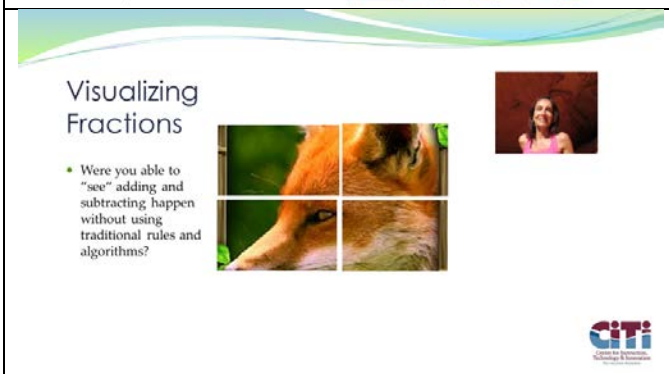
- Complete the Workshop Evaluation and give it to your Director.
- *Continuing Teacher Leader Education (CTLE)* – If you would like to request credit for this module, please follow the CTLE Credit Request process. Find the link for this process on the NYS-MEP website: https://www.nysmigrant.org/resources/pd

Facilitator Note: The Presenter's "Talking Points" are in the appendix for reference.

	Today's web learning session will allow you to bring fractions to a concrete modeling version. By engaging your students in these modeling activities, students will understand the relationship of fractions to each other, the relationship of fractions to the whole, as well as understand the why behind addition and subtraction of proper fractions with unlike denominators. This will be accomplished by using Cuisenaire rods. These rods allow students to manipulate fractions in a concrete manner. To help you in this understanding, you will need your cut outs of the Cuisenaire rods and 2 sheets of quarter inch graph paper. It would also be helpful if you had paper and pencil for taking notes.
	Let's begin by playing with the Cuisenaire rods. I always give students time to develop their own "I notice and I wonder" about the rods and what they could be used for. Take a moment and look at the relationship between the rods. MUSIC Let's take a few moments to reconnect our understanding of fractions and Cuisenaire rods from the first video.
	The foundation of fraction understanding is the relationship of the part to the whole. In this model, the rods explain this concretely using halves, fourths, and eighths. This model can also be used to find equivalent forms of fractions. One way of telling if an example is "good" is to see how many connections can be made to other concepts. Here we have 10 connections in one model!

	Let's model this example together using the document camera. MOVE TO DOCUMENT CAMERA
	Now try this one on your own. Then we will go through it together. The brown rod will represent the whole. MUSIC THEN MOVE TO DOCUMENT CAMERA
	Now let's model a subtraction with like denominators together. MOVE TO DOCUMENT CAMERA
	Here is a practice problem for you to model using the rods. representing the whole in each. MUSIC THEN MOVE TO DOCUMENT CAMERA
	Addition of fractions with unlike denominators often has students adding both the numerators and denominators when solely using the symbolic as a teaching method. Modeling with the rods almost forces your student to find the common rod aka the common denominator and really sees the reason for doing so. As I move to the document camera, you will need to start out with your purple and light green rods. MOVE TO DOCUMENT CAMERA

	Here are two practice problems for you to model using the rods. I have given you the color rods to use in representing the whole in each. MUSIC THEN MOVE TO DOCUMENT CAMERA
	Once again, by using the rods, subtraction is a natural follow through after doing addition. Here we can explore two ways of modeling three fourths minus one half. I will model both methods. We can start out with the purple and red model. MOVE TO THE DOCUMENT CAMERA
	Here are two practice problems for you to model using the rods. I have given you the color rods to use in representing the whole in each. MUSIC THEN MOVE TO DOCUMENT CAMERA
	These learning standards from grades 3 through 5 can be modeled beautifully using Cuisenaire rods. Visualizing brings conceptual understanding which in turn organically brings in the development of procedural understanding. The combination of the two is crucial for the application of fractions to situations.
	Our final example is both addition of fractions with unlike denominators and comparing the part to the whole. Let's see how modeling can make sense of the situation for everyone.

	Because we would need 20 white rods, it may make more sense to transition to graph paper. Here I have used quarter inch. Please try to model this along with me using your graph paper. Using a pencil, block of groups of 5 and underneath groups of 4. I recommend a pencil because you may not get the correct number of units in each on the first try. Eventually students will realize that common denominators can be found by finding a common multiple. Once you have five fifths and four fourths drawn, block off your common denominator of 20. PAUSE. Now redraw the array of 20 and fill in your soccer and volleyball. You can just use S and V as representations. Right away you should see that the remaining squares represent the number who play basketball. This modeling is an excellent transition to working with tape diagrams. Tape diagrams are used all the way through grade 7.
	A great way for students to use technology with the rods is using the practice from PBS kids. You can search directly from their home page. The website is also listed here.
	Having the ability to model fractions is such a positive and interactive way to conceptually understand the why behind all the algorithms taught. Being able to *figure things out* and developing a mathematical mindset will help them understand the more complex and challenging concepts in later grades. Thank you, as always, for making me part of your mathematical journey.