Math Fluency

For the NYS Migrant Education Program

Tina Bonfiglio
Math Instructional Specialist

Your Educational Partner of Choice
BOCES 2
Department for Exceptional Children

Essential Questions

What is math fluency?

Why is math fluency important?

How can we help students develop math fluency?

What is math fluency?

FLUENCY **IS**

applying conceptual math thinking (flexible)

focused on efficiency

FLUENCY **IS NOT**

memorizing facts

using algorithms

focused on speed (timed drills)

What is math fluency?

Procedural fluency is the ability to…

- **apply procedures efficiently, flexibly, and accurately.**
- **transfer procedures to different problems and contexts.**
- **build or modify procedures from other procedures.**
- **recognize when one strategy or procedure is more appropriate to apply than another.**

(NCTM 2014, 2020; National Research Council 2001, 2005, 2012; Star 2005)

Four Declarations from NCTM

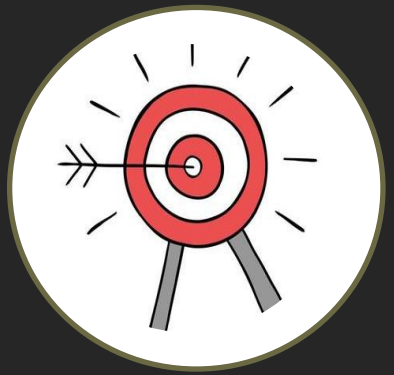
Conceptual understanding must precede and coincide with instruction on procedures.

Procedural fluency requires having a repertoire of strategies.

Basic facts should be taught using number relationships and reasoning strategies, not memorization.

Assessing must attend to fluency components and the learner. Assessments often assess accuracy, neglecting efficiency and flexibility.

Our Goal



Accurate

We want students to be precise in their problem solving.

Efficient

We want students to solve problems quickly and without excessive steps, but speed should not be a focus.

Flexible

We want students to be able to use multiple strategies appropriately.

Why is math fluency important?

Research shows it is linked with progress in later grades.

- It helps students advance when learning multi-digit arithmetic, fractions, and long division.
- It frees up brainpower or working memory to do more complex mathematical work.

What Can We Do?

- Help students develop relationships between facts within the same operation.
- Then relate addition to subtraction and multiplication to division.

General Strategies

Make it Visual (number lines, arrays, charts)

Use Manipulatives (counters, ten frames, place value blocks)

Skip Counting (2, 4, 6, 8...)

Fact Families (4+5=9 so 9-5=4, 3x6=18 so 18÷6=3)

Commutative Property (2+3=3+2, 7x8=8x7)

Identify Patterns (while calculating, on addition and multiplication charts)

Make Real-World Connections (time, money, cooking)

Make it Fun (play games, use songs)

Practice Regularly

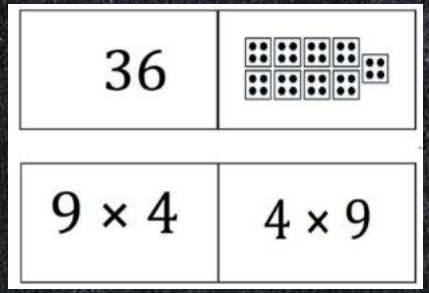
Flash Cards

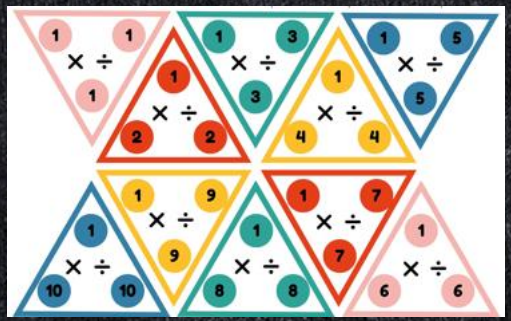
DO NOT use only traditional flash cards and only to develop speed or for rote memorization.

Flash Cards

Use flash cards to:

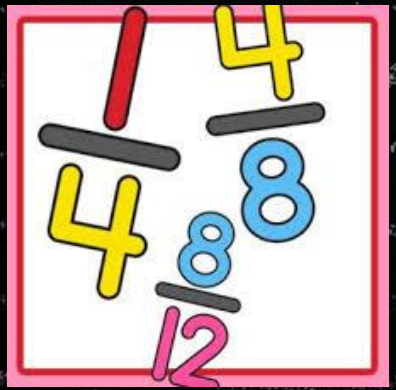
- match digits to number names or images (groups of objects, ten frames, base ten blocks, etc.)
- match a number to an expression or a pictorial model
- build fact families





How Do We Do It?

- ❑ Develop Addition Fluency
- ❑ Develop Multiplication Fluency
- ❑ Develop Fraction Sense



Developing Addition Fluency

Methods (in progression)

Method 1: Counting All

Method 2: Counting On (Counting Up)

Method 3: Adding a Single Digit to 10

Method 4: Making a Ten

Method 5: Doubles Plus a Number

Method 6: Decomposing to Make Tens and Ones

Method 1: Counting All

6 + 4 =

Young students first learn to add by **counting every number up to the sum.**

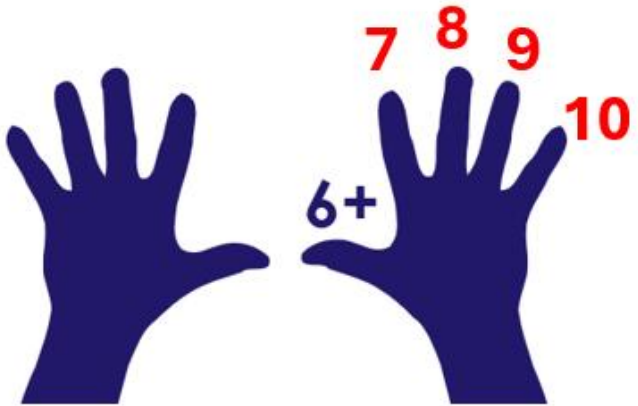
1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

Method 2: Counting On (Up)

4 + 6 =

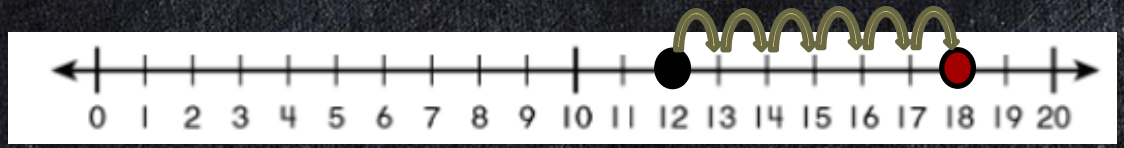
Students can use their fingers, a number line, or other manipulatives for tactile or visual assistance.

Students realize that the sum is **six more than 4** in the counting sequence, and they can simply **begin counting from 4.**
Eventually, they automatically choose the larger number or "addend" and count from there.



Method 2: Counting On (Up)

12+6=?



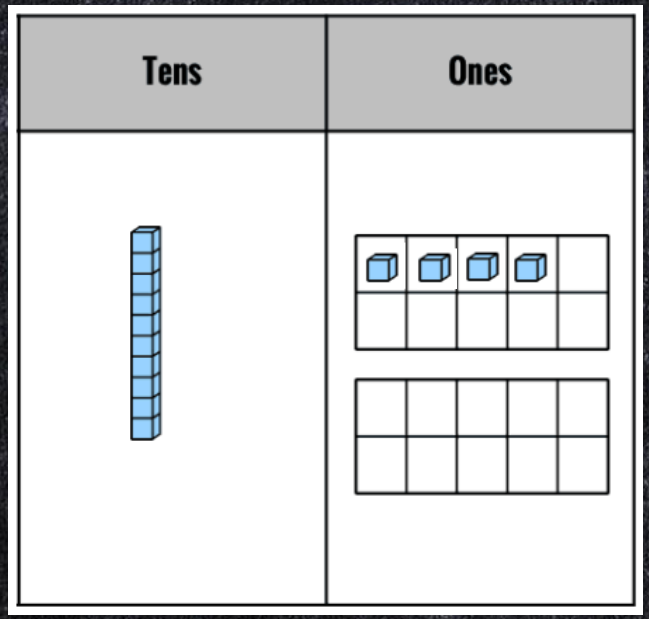
12+6=18

Method 3: Adding a Single Digit to 10

10 + 4 = ?

The number 10 has no ones so when you add 4 ones to it, it simply becomes the ones place.

14



Method 4: Making a 10

8 + 9 = ?

8+(2+7) = ?

10 + 7= 17

Students need to know ten facts for this strategy.

Decomposing a number means breaking it into smaller parts.

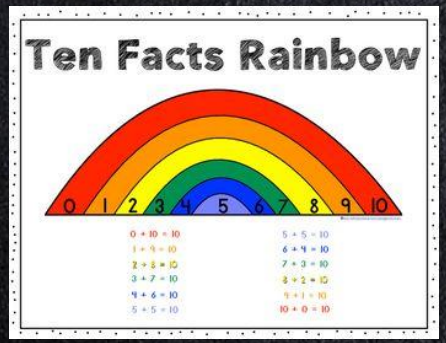
8 + 9 = 17

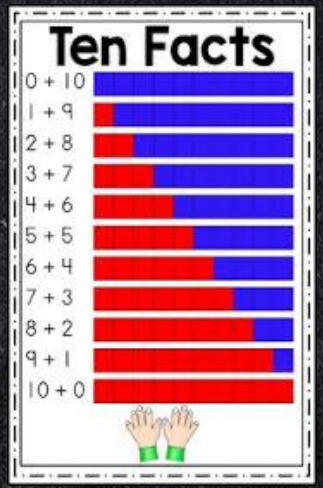
Grade 1

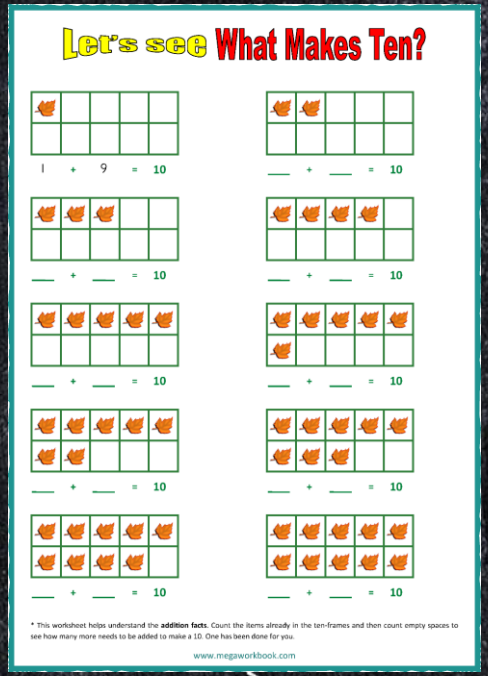
Method 4: Making a 10

Try it!

FIRST, practice Ten Facts







Method 4: Making a 10

Try it!

$7 + 9 = ?$

$(6+1) + 9 = ?$

$6 + 10 = 16$

$7 + 9 = 16$

Method 4: Making a 10

Try it!

8 + 6= ?

8+(2+4) = ?

10 + 4 = 14

8 + 6 = 14

Method 5: Doubles Plus a Number

8 + 9 = ?

8+(8+1) = ?

16 + 1= 17

Students need to know doubles facts for this strategy.

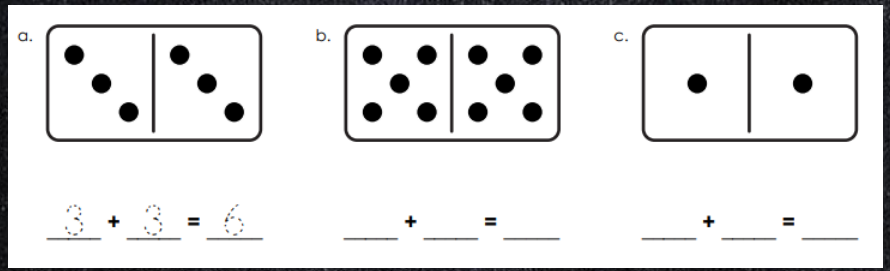
8 + 9 = 17

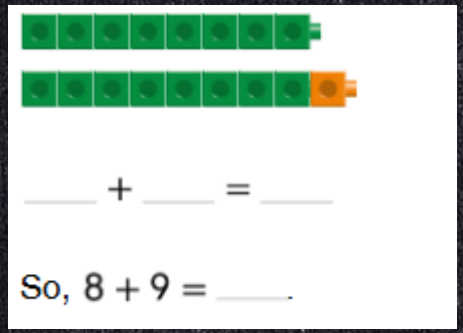
Grade 1

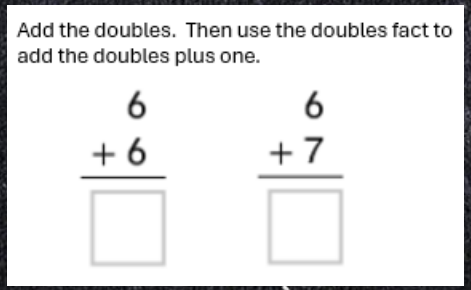
Method 5: Doubles Plus a Number

Try it!

FIRST, practice Doubles







Method 5: Doubles Plus a Number

Try it!

$7 + 9 = ?$

$7+(7+2) = ?$

$14 + 2 = 16$

$7 + 9 = 16$

Method 5: Doubles Plus a Number

Try it!

$8 + 6 = ?$

$(2+6) + 6 = ?$

$2 + 12 = 14$

$8 + 6 = 14$

Grade 2+

Method 6: Decomposing to Make Tens and Ones

34 + 42=?

(30+4) + (40+2) = ?

Add tens: 30 + 40 = 70

Add ones: 4 + 2 = 6

Add sums: 70 + 6 = 76

Students need to understand place value for this strategy.

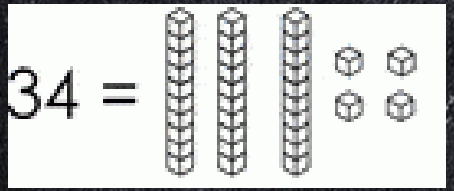
34 + 42= 76

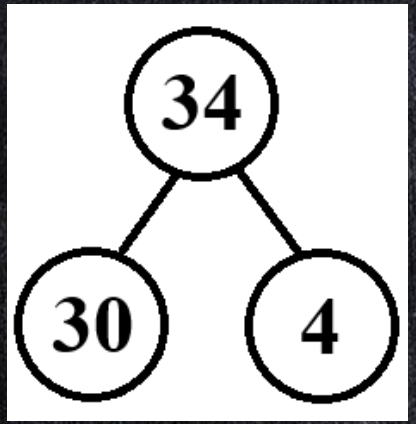
Grade 2+

Method 6: Decomposing to Make Tens and Ones

Try it!

FIRST, practice decomposing





Grade 2+

Method 6: Decomposing to Make Tens and Ones

Try it!

47 + 21= ?

(40+7)+(20+1) = ?

Add tens: 40 + 20 = 60

Add ones: 7 + 1 = 8

Add sums: 60 + 8 = 68

47 + 21= 68

Grade 2+

Method 6: Decomposing to Make Tens and Ones

Try it!

53 + 16= ?

(50+3)+(10+6) = ?

Add tens: 50 + 10 = 60

Add ones: 3 + 6 = 9

Add sums: 60 + 9 = 69

53 + 16= 69

Grade 2+

Method 6: Decomposing to Make Tens and Ones

(decompose second number only)

74 + 13= ?

74 + (10+3)=?

74 + 10 = 84

84 + 3 = 87

74 + 13= 87

Grade 2+

Method 6: Decomposing to Make Tens and Ones

(decompose second number only)

Try it!

25 + 43= ?

25 + (40+3) = ?

25 + 40 = 65

65 + 3 = 68

25 + 43= 68

Grade 2+

Method 6: Decomposing to Make Tens and Ones

(decompose second number only)

Try it!

What happens when the ones add to more than 9???

51 + 22= ?

51 + (20+2) = ?

51 + 20 = 71

71 + 2 = 73

51 + 22= 73

Grade 2+

Method 6: Decomposing to Make Tens and Ones

(with regrouping)

34 + 57= ?

(30+4) + (50+7) = ?

Add tens: 30 + 50 = 80

Add ones: 4 + 7 = 11

Add sums: 80 + 11 =

Add sums: 80 + 11 =

Decompose: 80+(10+1)=

Add tens: 90 + 1=

Add: 91

34 + 57= 91

Grade 2+

Method 6: Decomposing to Make Tens and Ones

(with regrouping)

Try it!

18 + 34= ?

(10+8)+(30+4) = ?

Add tens: 10 + 30 = 40

Add ones: 8 + 4 = 12

Add sums: 40 + 12 =

Add sums: 40 + 12 =

Decompose: 40+(10+2)=

Add tens: 50 + 2=

Add: 52

18 + 34= 52

Grade 2+

Method 6: Decomposing to Make Tens and Ones

(with regrouping)

Try it!

67 + 19= ?

(60+7)+(10+9) = ?

Add tens: 60 + 10 = 70

Add ones: 7 + 9 = 16

Add sums: 70 + 16 =

Add sums: 70 + 16 =

Decompose: 70+(10+6)=

Add tens: 80 + 6=

Add: 86

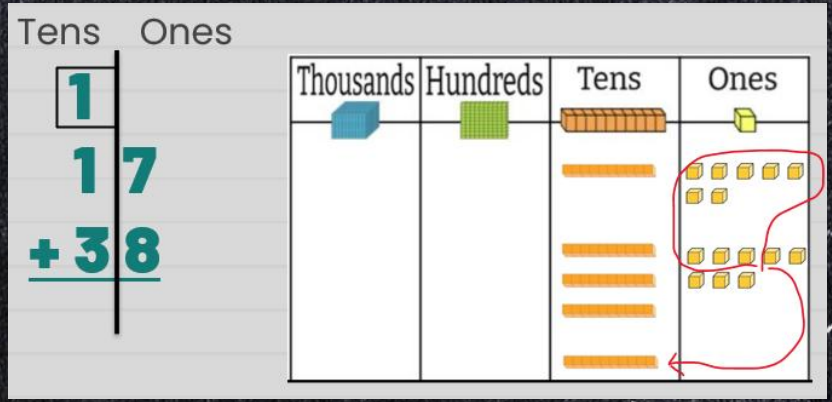
67 + 19= 86

Developing Addition Fluency

Next Steps

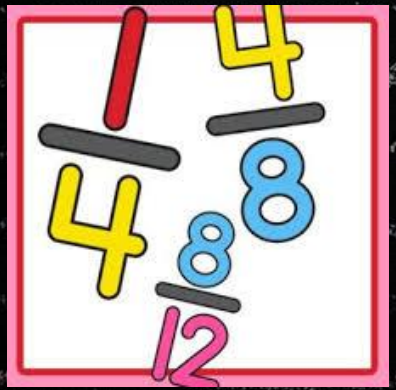
Multi-Digit Addition

- Using place value blocks and mats
- Without regrouping
- With regrouping



How Do We Do It?

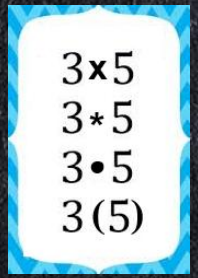
- ✓ Develop Addition Fluency
- ❑ Develop Multiplication Fluency
- ❑ Develop Fraction Sense



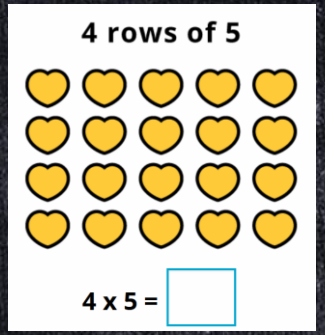
Multiplication Vocabulary

Multiply
Multiplied by
Times

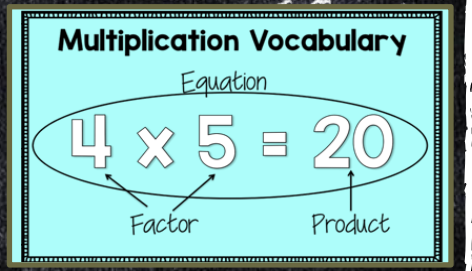
Symbols:



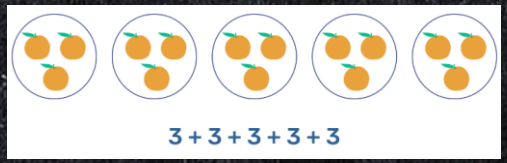
Arrays



Equation
Factor
Product



Equal Groups/Groups of
Repeated Addition



Developing Multiplication Fluency

Conceptual understanding is the priority!

Show multiplication as:

- ❑ repeated addition
- ❑ equal groups
- ❑ an array

Developing Multiplication Fluency

Repeated Addition

4 x 3

“Add 4 three times.”

4 + 4 + 4 = 12

so…

4 x 3 = 12

Developing Multiplication Fluency

Repeated Addition

Try it!

6 x 4

“Add 6 four times.”

6 + 6 + 6 + 6 = 24

so…

6 x 4 = 24

Developing Multiplication Fluency

Show multiplication as:

- ☑ repeated addition
- ☐ equal groups
- ☐ an array

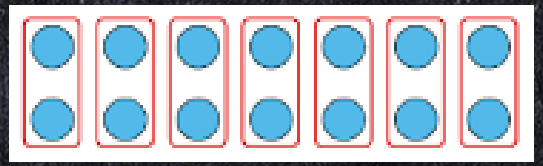
Developing Multiplication Fluency

Equal Groups

7 x 2

“7 groups of 2.”

Draw it.



Then skip count or use repeated addition.
2, 4, 6, 8, 10, 12, 14

so... 7 x 2 = 14

Developing Multiplication Fluency

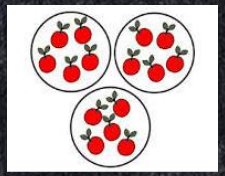
Equal Groups

Try it!

3 x 5

"3 groups of 5."

Draw it.



Then skip count or use repeated addition.

5, 10, 15 or 5+5+5=15

so... 3 x 5 = 15

Developing Multiplication Fluency

Show multiplication as:

- ☑ repeated addition
- ☑ equal groups
- ☐ an array

Developing Multiplication Fluency

Arrays

3 x 4

“3 rows of 4.”

Draw it:

Then skip count or use repeated addition.

3, 6, 9, 12 or 3+3+3+3=12

so... 3 x 4 = 12

Developing Multiplication Fluency

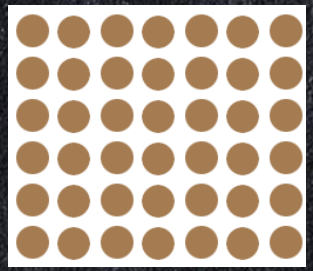
Arrays

Try it!

6 x 7

"6 rows of 7."

Draw it:



so... 6 x 7 = 42

Then skip count or use repeated addition.

6, 12, 18, 24, 30, 36, 42 or 6+6+6+6+6+6+6=42

Developing Multiplication Fluency

Show multiplication as:

- ☑ repeated addition
- ☑ equal groups
- ☑ an array

Developing Multiplication Fluency

Recommended order to teach multiplication facts

2: doubles, skip count
10: shift number to the left one place, skip count
5: half of multiplying by 10, skip count
1: use conceptual understanding of multiplication
0: don't just memorize
11: multiply by 10 then add one more group
3: double then add one more group
4: double the double
6: multiply by 5 then add one more group
9: multiply by 10 then subtract one group
8: multiply by 4 then double or double, double, double
7: multiply by 5 then add a double
12: multiply by 10 then add a double

Developing Multiplication Fluency

Multiplying by 2

6 x 2

Relate it to addition. Use doubles facts.

The double of 6 is 12.

So…6 x 2 = 12

Developing Multiplication Fluency

Multiplying by 10

5 x 10

Don't teach "Add a zero". Teach place value.

Multiplying by 10 "shifts the number one place to the left".

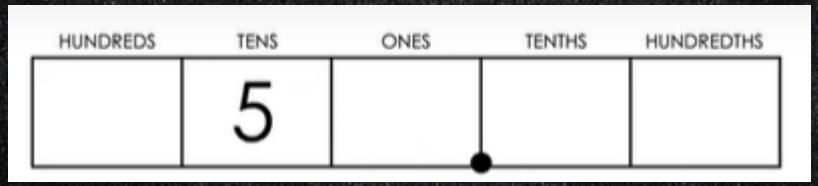
Developing Multiplication Fluency

Multiplying by 10

5 x 10

Don’t teach “Add a zero”. Teach place value.

Multiplying by 10 “shifts the number one place to the left”.



Developing Multiplication Fluency

Multiplying by 10

5 x 10

Don't teach "Add a zero". Teach place value.

Multiplying by 10 "shifts the number one place to the left".

HUNDREDS	TENS	ONES	TENTHS	HUNDREDTHS
	5	0		

Add 0 as a place holder in the empty space.

So...5 x 10 = 50

Developing Multiplication Fluency

Multiplying by 10

Now let's try
50 x 10

Developing Multiplication Fluency

Multiplying by 10

50 x 10

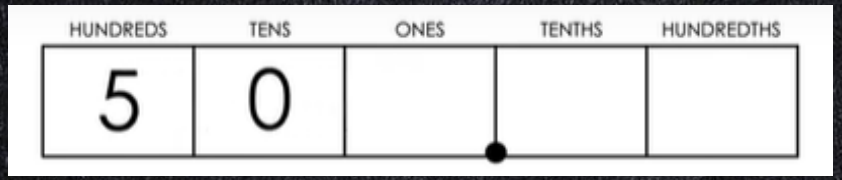
Shift the number one place to the left.

Developing Multiplication Fluency

Multiplying by 10

50 x 10

Shift the number one place to the left.



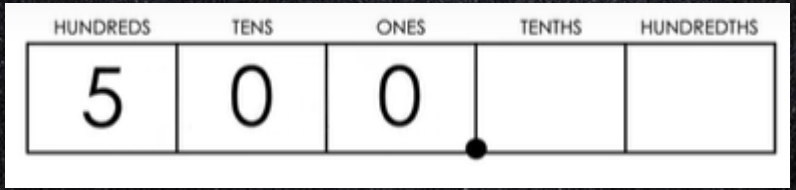
Developing Multiplication Fluency

Multiplying by 10

50 x 10

Shift the number one place to the left.

Add 0 as a place holder in the empty space.



So...50 x 10 = 500

Developing Multiplication Fluency

Does this work with decimals?

Developing Multiplication Fluency

Multiplying by 10

8.6 x 10

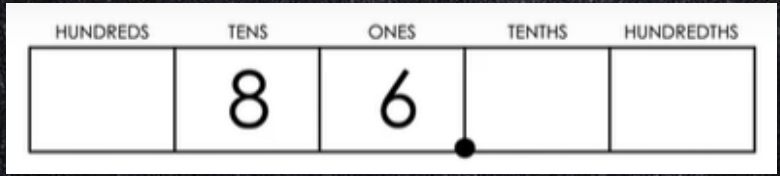
Shift the number one place to the left.

Developing Multiplication Fluency

Multiplying by 10

8.6 x 10

Shift the number one place to the left.



There is no need for a place holder.

So...8.6 x 10 = 86

Developing Multiplication Fluency

Multiplying by 10

Note: "Add a zero" does not work with decimals!

8.6 x 10
does not equal
8.60

8.6 x 10 = 86

Developing Multiplication Fluency

Multiplying by 5

6 x 5

Multiply 6 by 10, then take half.

6 x 10 = 60 and half of 60 is 30.

So...6 x 5 = 30

Developing Multiplication Fluency

Multiplying by 1

6 x 1

Don't teach "any number times 1 is that number".

Teach "any number times 1 means one group of that number".

1 group of 6 is 6. (or 6 groups of 1 is 6.)

So...6 x 1 = 6

Developing Multiplication Fluency

Multiplying by 0

6 x 0

Don't teach "any number times 0 is 0".

Teach "any number times 0 means zero groups of that number".

0 groups of 6 is 0. (or 6 groups of 0 is 0.)

So...6 x 0 = 0

Developing Multiplication Fluency

Multiplying by 11

6 x 11

Think of this as 11 groups of 6. We know 10 groups is 60.
One more group would mean 60 + 6, so 66.

IN OTHER WORDS...

Multiply by 10. (6 x 10 = 60)

Then add one more group. (60 + 6 = 66)

So...6 x 11 = 66

Developing Multiplication Fluency

Multiplying by 11

Try it!

9 x 11

Think of this as 11 groups of 9. We know 10 groups is 90.
One more group would mean 90 + 9, so 99.

IN OTHER WORDS...

Multiply by 10. (9 x 10 = 90)

Then add one more group. (90 + 9 = 99)

So...9 x 11 = 99

Developing Multiplication Fluency

Multiplying by 3

6 x 3

Think of this as 3 groups of 6. Double 6 then add one more group.

The double of 6 is 12.

12 plus one more group of 6 is 12 + 6 which is 18.

So...6 x 3 = 18

Developing Multiplication Fluency

Multiplying by 3

Try it!

7 x 3

Think of this as 3 groups of 7. Double 7 then add one more group.

The double of 7 is 14.

14 plus one more group of 7 is 14 + 7 which is 21.

So...7 x 3 = 21

Developing Multiplication Fluency

Multiplying by 4

6 x 4

Think of this as doubling the double.

The double of 6 is 12.

The double of 12 is 24.

So...6 x 4 = 24

Developing Multiplication Fluency

Multiplying by 4

Try it!

3 x 4

Think of this as doubling the double.

The double of 3 is 6.

The double of 6 is 12.

So...3 x 4 = 12

Developing Multiplication Fluency

Multiplying by 6

9 x 6

Think of this as 6 groups of 9. Multiply 9 by 5 then add one more group of 9.

9 x 5 = 45

45 plus one more group of 9 is 45 + 9 which is 54.

So...9 x 6 = 54

Developing Multiplication Fluency

Multiplying by 6

Try it!

7 x 6

Think of this as 6 groups of 7. Multiply 7 by 5 then add one more group of 7.

7 x 5 = 35

35 plus one more group of 7 is 35 + 7 which is 42.

So…7 x 6 = 42

Developing Multiplication Fluency

Multiplying by 9

7 x 9

Think of this as 9 groups of 7. Multiply 7 by 10 then subtract one group of 7.

7 x 10 = 70

70 minus one group of 7 is 70 - 7 which is 63.

So...7 x 9 = 63

Developing Multiplication Fluency

Multiplying by 9

Try it!

8 x 9

Think of this as 9 groups of 8. Multiply 8 by 10 then subtract one group of 8.

8 x 10 = 80

80 minus one group of 8 is 80 - 8 which is 72.

So...8 x 9 = 72

Developing Multiplication Fluency

Multiplying by 8

2 x 8

OR

Multiply 2 by 4. Then double.

2 x 4 = 8

The double of 8 is 16.

So...2 x 8 = 16

Double, double, double

The double of 2 is 4.

The double of 4 is 8.

The double of 8 is 16.

So...2 x 8 = 16

Developing Multiplication Fluency

Multiplying by 8

Try it!

6 x 8

Multiply 6 by 4. Then double.

6 x 4 = 24

The double of 24 is 48.

So...6 x 8 = 48

OR

Double, double, double

The double of 6 is 12.

The double of 12 is 24.

The double of 24 is 48.

So...6 x 8 = 48

Developing Multiplication Fluency

Multiplying by 7

8 x 7

Multiply 8 by 5. Then add a double.

8 x 5 = 40

The double of 8 is 16.

40 + 16 = 56

So...8 x 7 = 56

Developing Multiplication Fluency

Multiplying by 7

Try it!

6 x 7

Multiply 6 by 5. Then add a double.

6 x 5 = 30 **The double of 6 is 12.**

30 + 12 = 42

So...6 x 7 = 42

Developing Multiplication Fluency

Multiplying by 12

4 x 12

Multiply 4 by 10. Then add a double.

4 x 10 = 40

The double of 4 is 8.

40 + 8 = 48

So...4 x 12 = 48

Developing Multiplication Fluency

Multiplying by 12

Try it!

6 x 12

Multiply 6 by 10. Then add a double.

6 x 10 = 60 **The double of 6 is 12.**

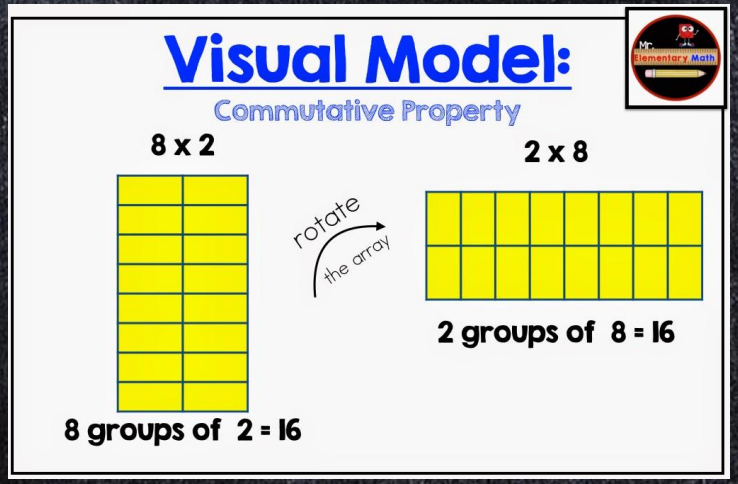
60 + 12 = 72

So...6 x 12 = 72

Developing Multiplication Fluency

Multiplication is commutative!

Like addition, you can multiply numbers in any order and get the same result.



Developing Multiplication Fluency

Recommended order to teach multiplication facts

2: doubles, skip count
10: shift number to the left one place, skip count
5: half of multiplying by 10, skip count
1: use conceptual understanding of multiplication
0: don't just memorize
11: multiply by 10 then add one more group
3: double then add one more group
4: double the double
6: multiply by 5 then add one more group
9: multiply by 10 then subtract one group
8: multiply by 4 then double or double, double, double
7: multiply by 5 then add a double
12: multiply by 10 then add a double

Developing Multiplication Fluency

Multiplication Table

x	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Products in the 5s column alternate between ending in 5 and 0.

Products in the diagonals repeat in reverse order. (Commutative Property!)

Any number multiplied by an even number is even.

Encourage students to look for patterns in the multiplication table.

The products in the 9s column decrease by 1 in the ones place and increase by 1 in the tens place.

Developing Multiplication Fluency

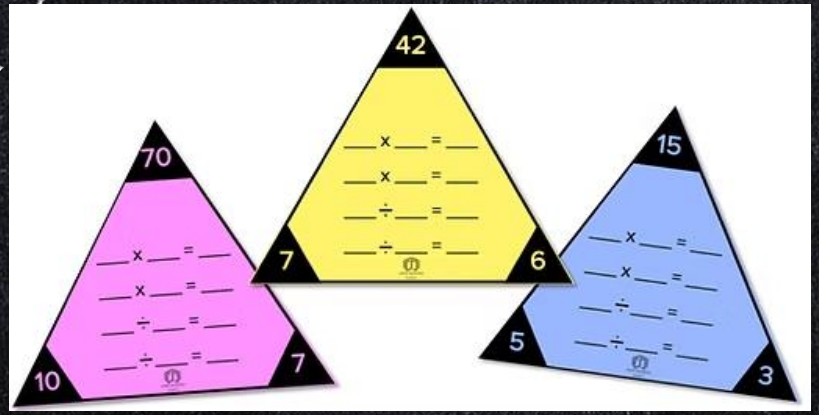
x	1	2	3	4	5	6	7	8	9	10	11	12
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												
12												

Give students a blank table to see how much they can complete using strategies and patterns.

Developing Multiplication Fluency

Resources

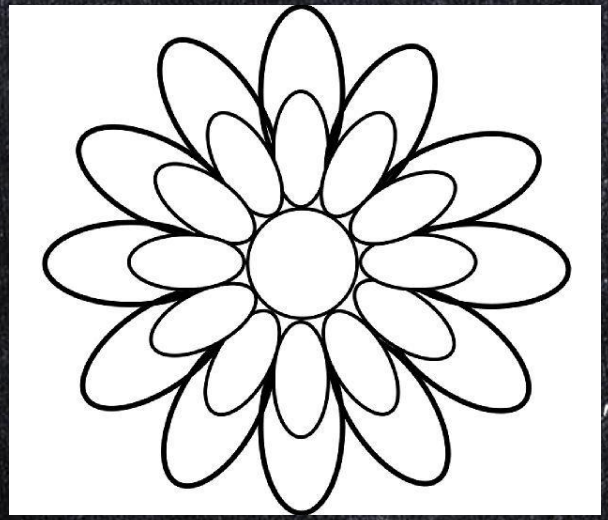
Fact Family Triangles



Developing Multiplication Fluency

Resources

Multiplication Flowers



Developing Multiplication Fluency

Resources

Multiplication Flowers

Repeated Addition

Equal Groups

Arrays

Commutative Property

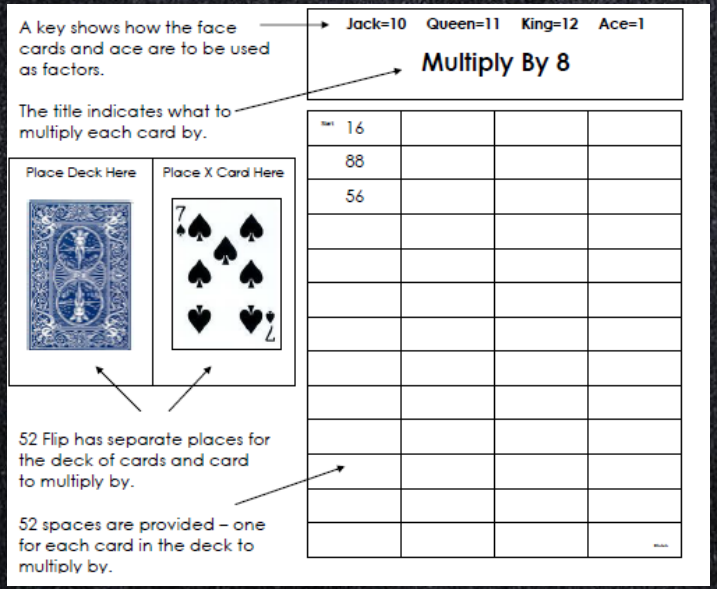
Fact Families

Factor Pairs

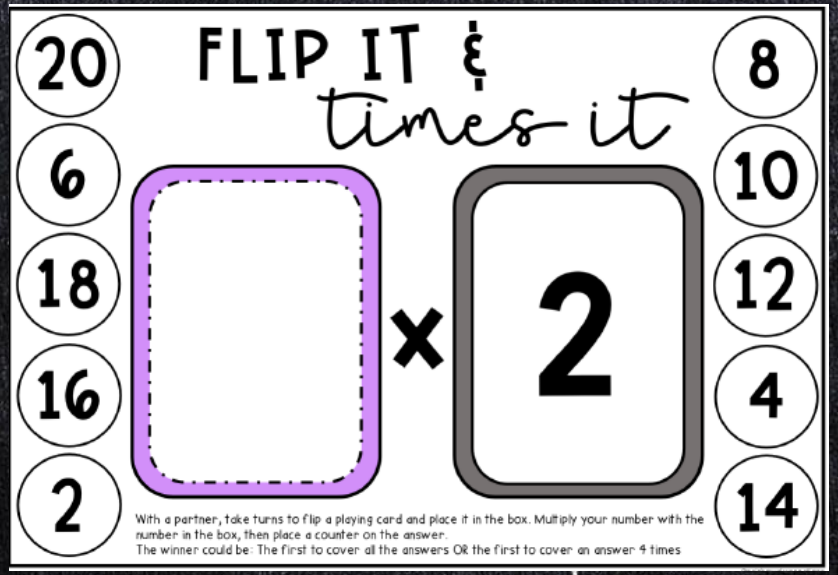
Developing Multiplication Fluency

Resources

52 Flip



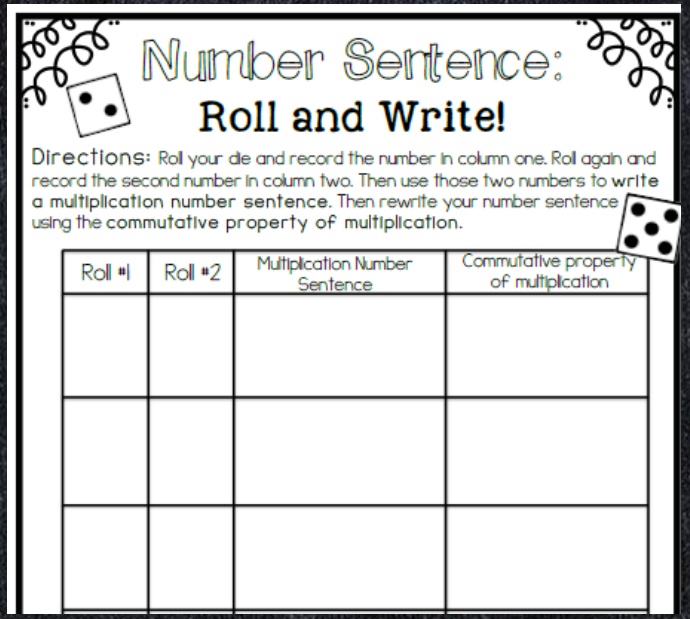
Flip it & Times it



Developing Multiplication Fluency

Resources

Roll and Write



Number Sentence:

Roll and Write!

Directions: Roll your die and record the number in column one. Roll again and record the second number in column two. Then use those two numbers to write a multiplication number sentence. Then rewrite your number sentence using the commutative property of multiplication.

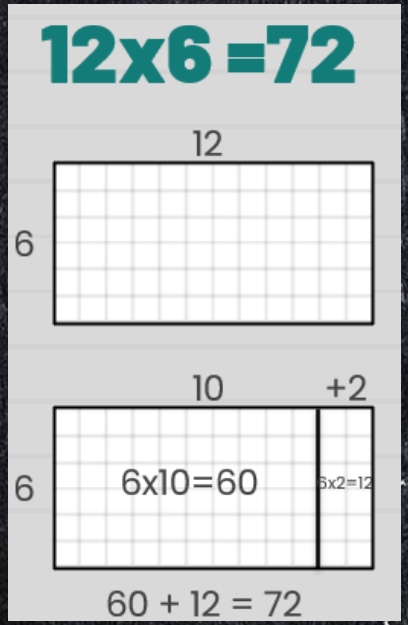
Roll #1	Roll #2	Multiplication Number Sentence	Commutative property of multiplication

Developing Multiplication Fluency

Next Steps

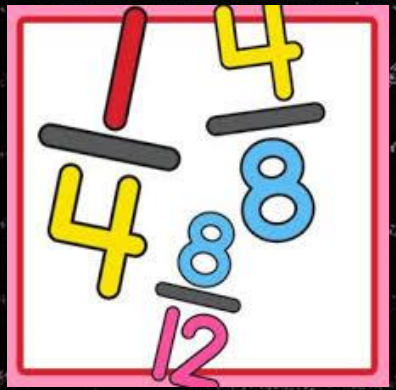
Multi-Digit Multiplication

- Area Models
- Partial Products

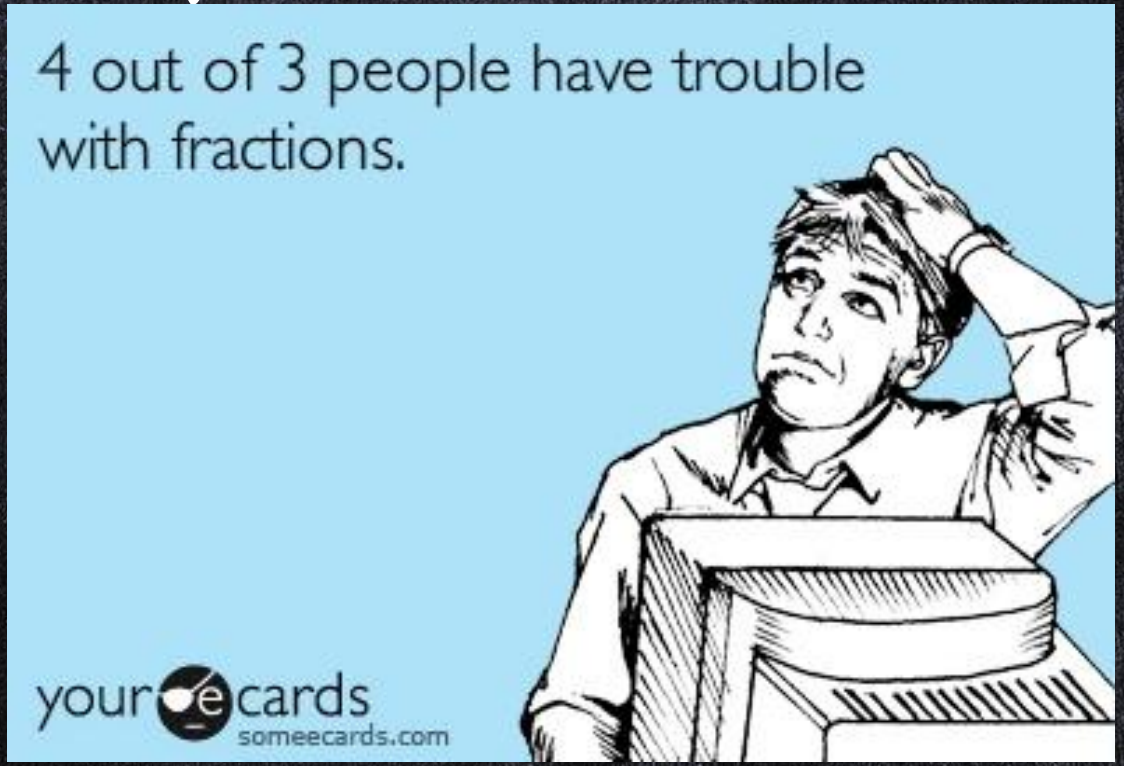


How Do We Do It?

- ✓ Develop Addition Fluency
- ✓ Develop Multiplication Fluency
- ☐ Develop Fraction Sense



Developing Fraction Sense

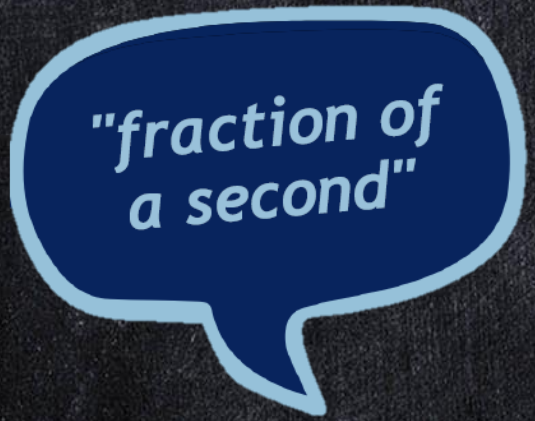


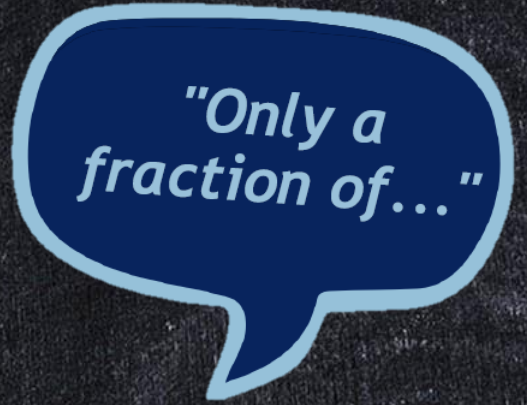
Developing Fraction Sense

"What is a Fraction?"

Why are these common responses?
What are the implications of this?

Developing Fraction Sense





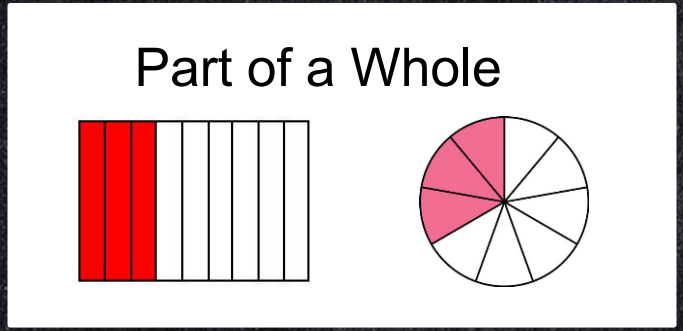
Developing Fraction Sense

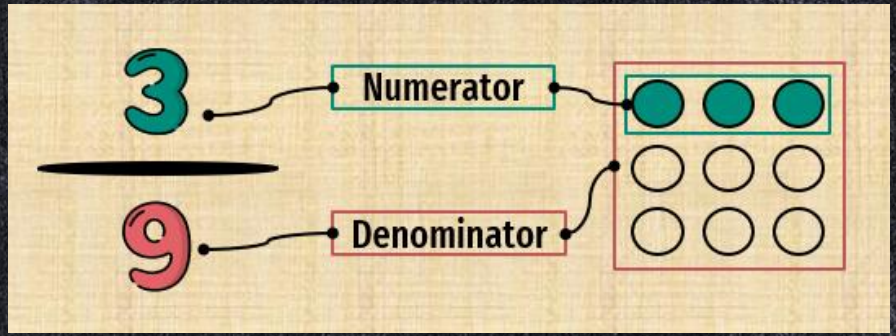
Introducing Fractions

Don't Say...	Instead Say...
$\frac{5}{7}$	$\frac{5}{7}$
"The numbers in this fraction..."	"This number..."
"top number", "bottom number"	"numerator", "denominator"
"5 over 7"	"five-sevenths"

Developing Fraction Sense

Introducing Fractions

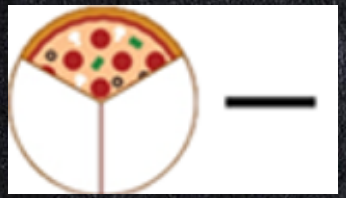


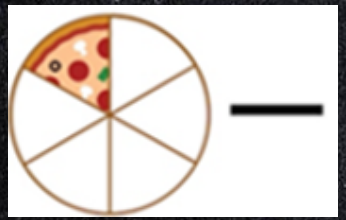


Developing Fraction Sense

Introducing Fractions

Write what fraction of the pizza is shown.





These don't teach Fraction Magnitude!

Color or lightly shade to indicate the fraction given.

$\frac{1}{7} =$ 

$\frac{4}{5} =$ 

Developing Fraction Sense

Teacher/Student Conversation

Teacher: Which fraction is smaller...5/8 or 5/4?

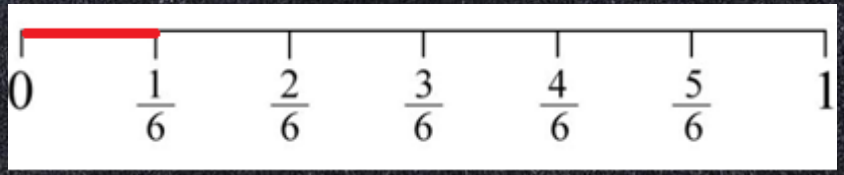
Student: Well, isn't 5/4 sort of impossible?

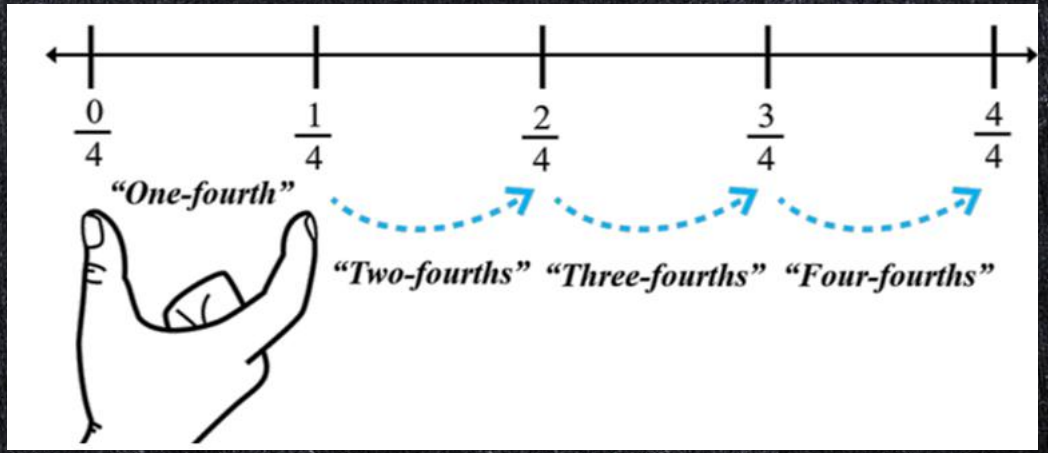
Teacher: Why?

Student: Because...if we have 5 pieces of pizza out of 4...
But how could you get 5 out of 4? How could you get 5/4?

Developing Fraction Sense

Fraction Magnitude





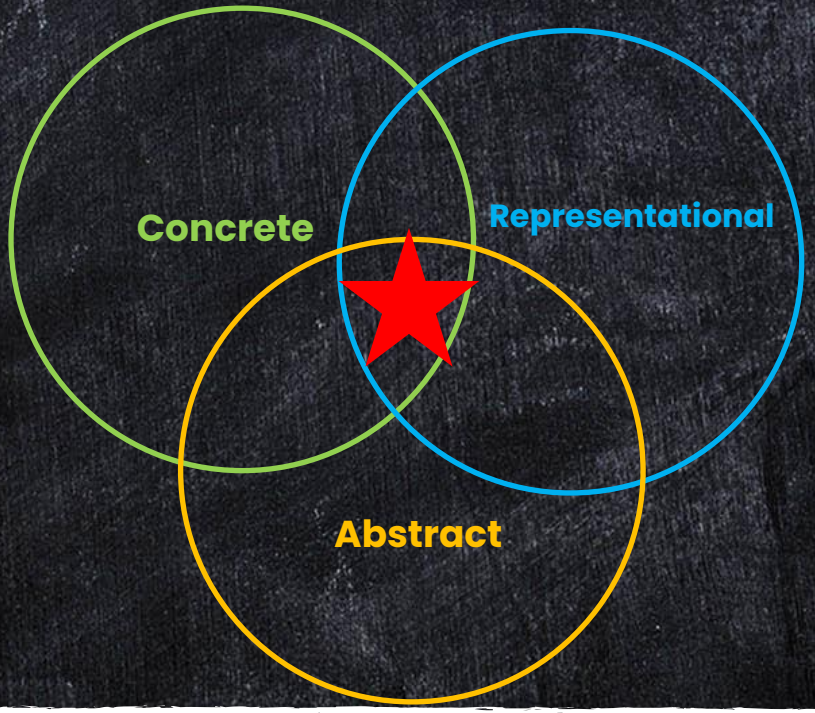
Developing Fraction Sense

CRA Approach

Concrete- make it hands on, use manipulatives

Representational- use pictorial drawings and models

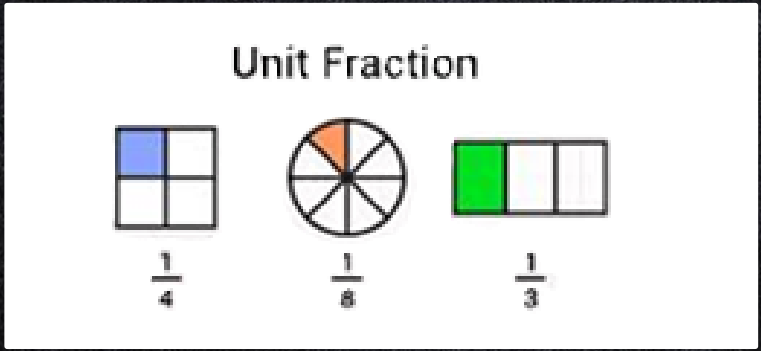
Abstract- use numbers, equations, and algorithms

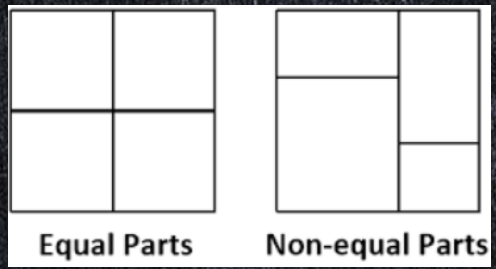


Developing Fraction Sense

Unit Fraction

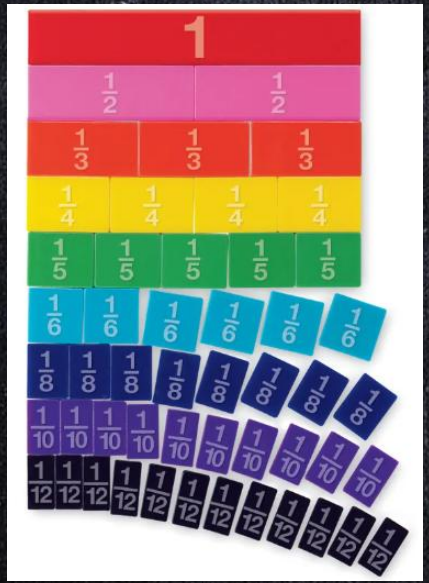
- A unit fraction is one part of all the **equal parts** of a whole
- The numerator is always 1 and the denominator is the number corresponding to the total number of equal parts.

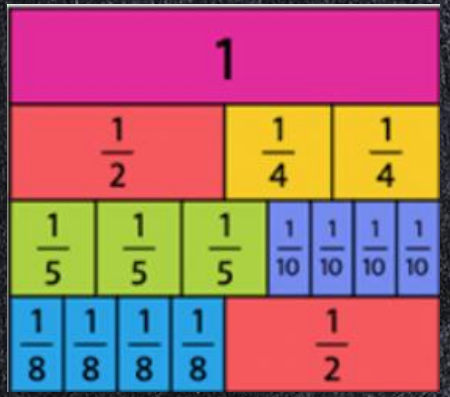




Developing Fraction Sense

Concrete: Fraction Tiles

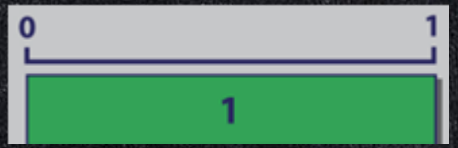


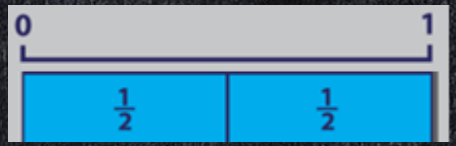


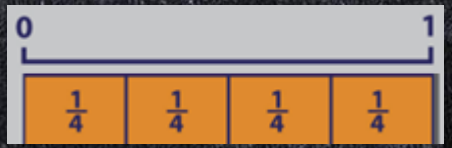
virtual fraction tiles at www.toytheather.com

Developing Fraction Sense

Representational: Fraction Tiles on a Number Line



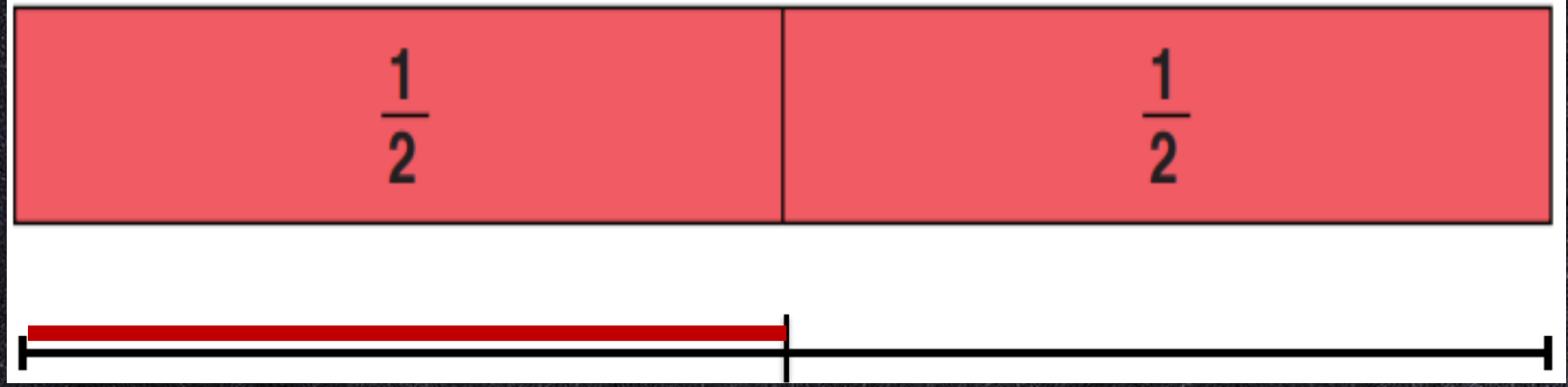




Developing Fraction Sense

Representational: Fraction Tiles on a Number Line

What is the unit fraction? $\frac{1}{2}$ How many unit fractions make up the whole? 2



Developing Fraction Sense

Representational: Fraction Tiles on a Number Line

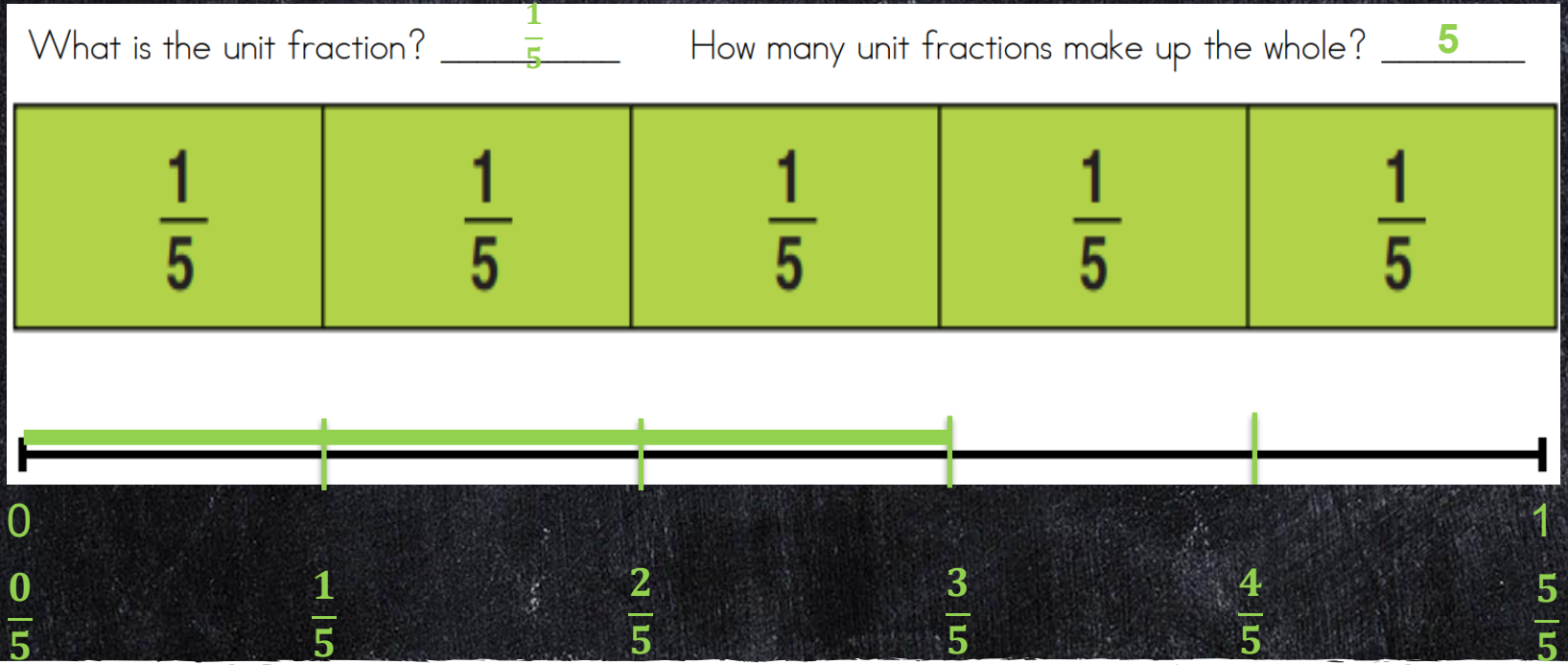
What is the unit fraction? $\frac{1}{3}$ How many unit fractions make up the whole? 3

0 1

$\frac{0}{3}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{3}{3}$

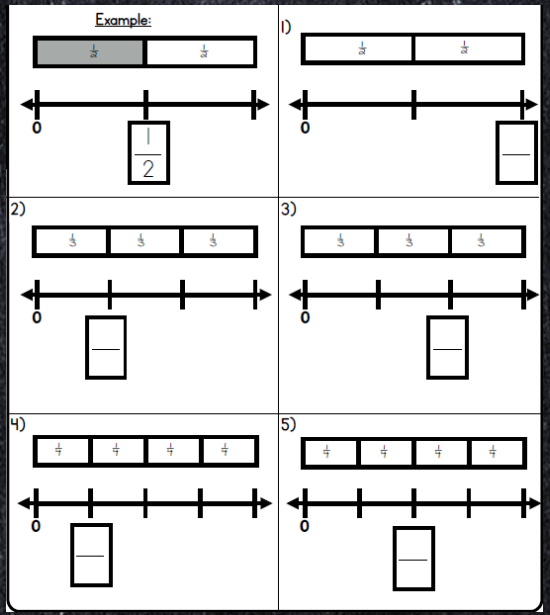
Developing Fraction Sense

Representational: Fraction Tiles on a Number Line



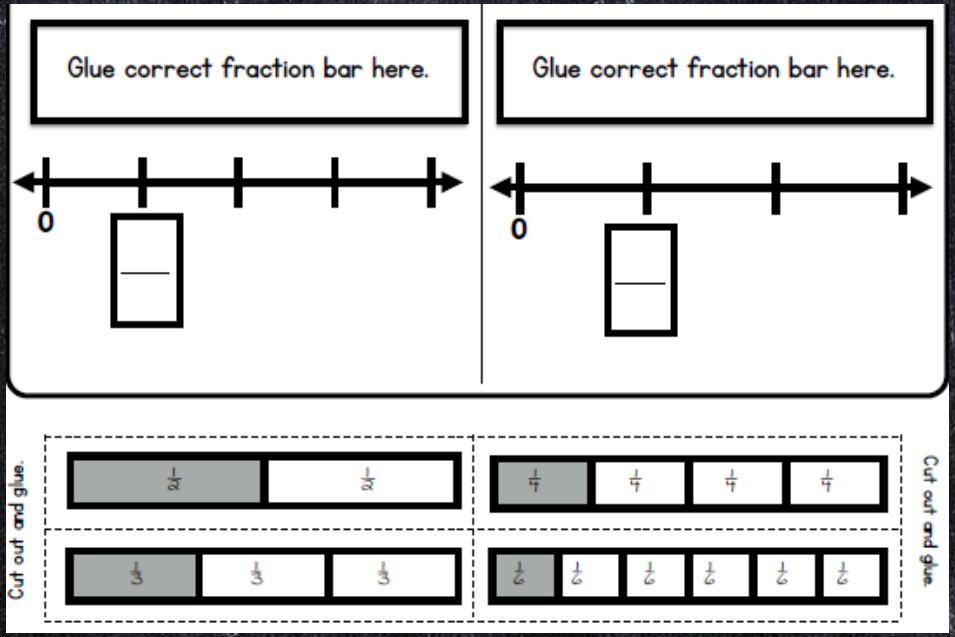
Developing Fraction Sense

Representational: Fraction Tiles on a Number Line



Developing Fraction Sense

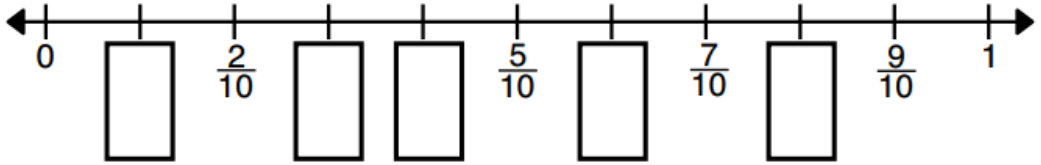
Representational: Fraction Tiles on a Number Line

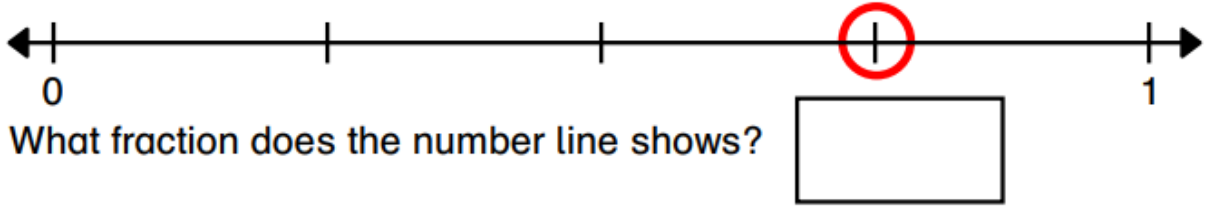


Developing Fraction Sense

Abstract: Fractions on a Number Line

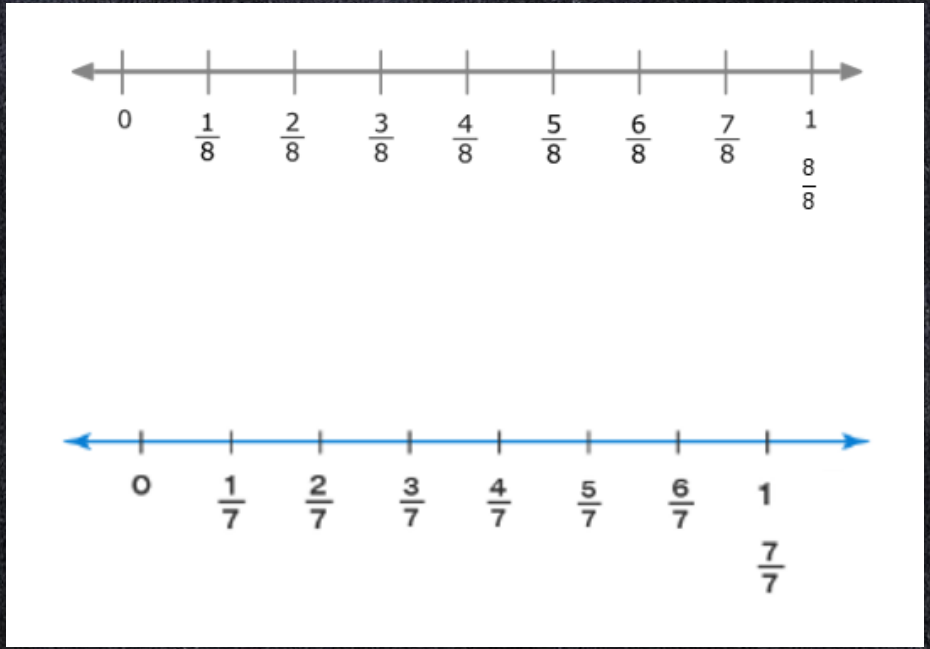
Find missing fractions on a number line





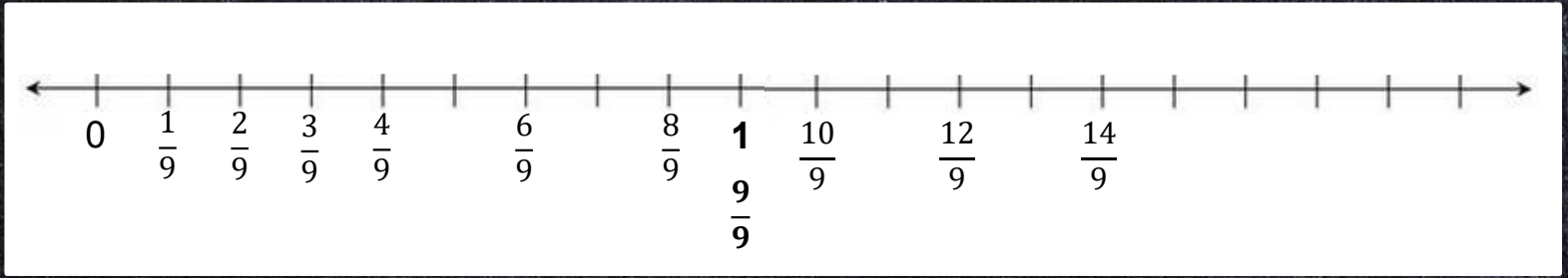
Developing Fraction Sense

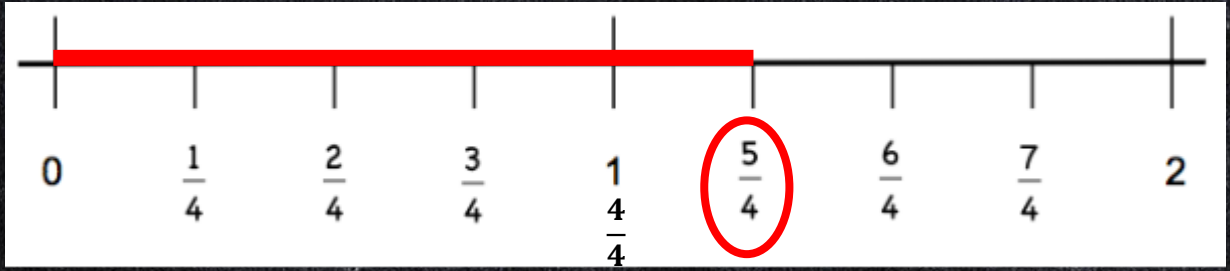
Counting by Fractions



Developing Fraction Sense

Counting by Fractions





Developing Fraction Sense

Fractions Greater than 1

...are fractions whose numerators are greater than their denominators.

Examples: $\frac{5}{4}$ $\frac{8}{3}$

Developing Fraction Sense

Fractions Greater than 1

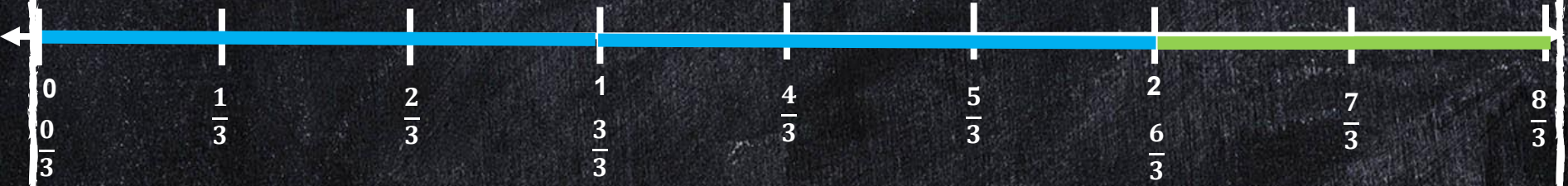
$\frac{5}{4}$

$$\frac{5}{4} = 1\frac{1}{4}$$

Developing Fraction Sense

Fractions Greater than 1

$\frac{8}{3}$



$$\frac{8}{3} = 2\frac{2}{3}$$

Developing Fraction Sense

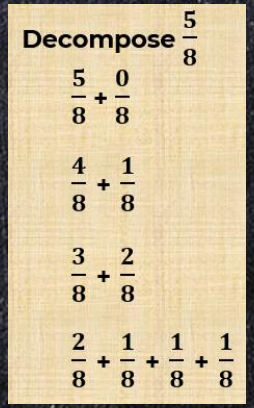
Next Steps

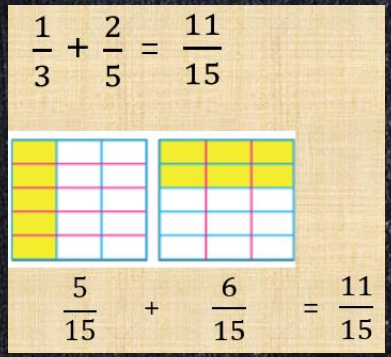
Comparing Fractions

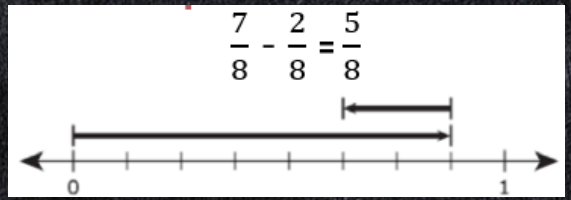
Decomposing Fractions

Adding Fractions

Subtracting Fractions

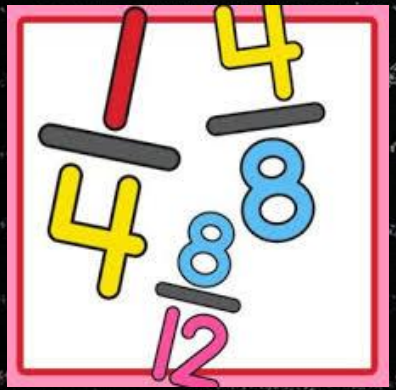




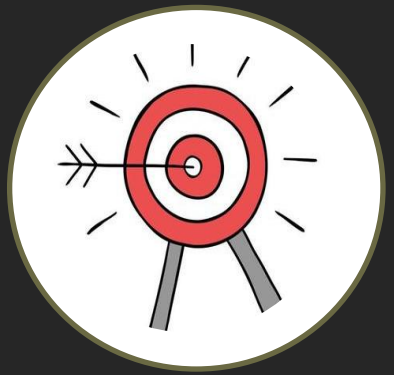


How Do We Do It?

- [x] Develop Addition Fluency
- [x] Develop Multiplication Fluency
- [x] Develop Fraction Sense



Our Goal



Accurate

We want students to be precise in their problem solving.

Efficient

We want students to solve problems quickly and without excessive steps, but speed should not be a focus.

Flexible

We want students to be able to use multiple strategies appropriately.

Developing Math Fluency

Essential Questions

What is math fluency?

Why is math fluency important?

How can we help students develop math fluency?

Any Questions for Me?