Topic 2 – Analyzing Functions Using Tables and Graphs

Function Definition and Representations

A **function** is one of the most important concepts in Algebra. Consider the following example. At East High School, students are assigned to a guidance counselor based on the first letter of their last name.

First letter of student's last name	**Guidance Counselor**
A through K	Ms. Burnell
L through O	Mr. Matos
P through Z	Mrs. Gilbert

Something is a **function** if every *x*-value (or input) in the domain is assigned to only one *y*-value (or output) in the range.

In this example, the letters of students' names, A through Z, are the domain. These students are assigned to a guidance counselor in the range. Notice how each student only has one guidance counselor, or in other words, no student has two guidance counselors.

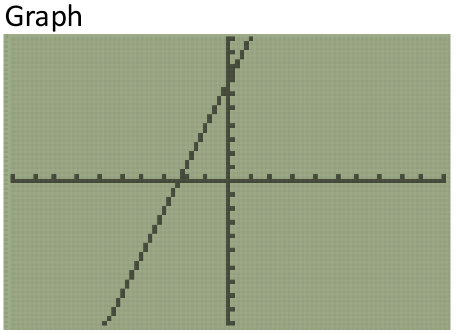
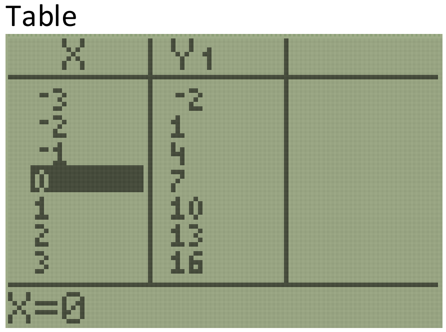
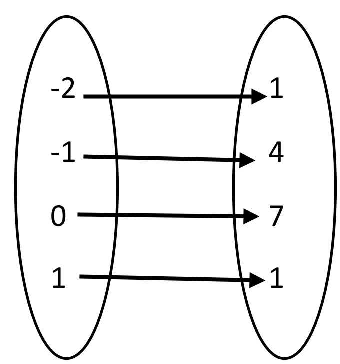
A function is usually some mathematical rule that tells you what to do with *x* in order to get *y*.

Consider the linear function

$$y = 3x + 7$$

This rule tells you to multiply each *x*-value by 3 then add 7 to get the *y*-value.

Here are other ways to represent $y = 3x + 7$.

Graph	Table	Arrow Diagram (mapping)
		

Every x-value has exactly one y-value.

Solve this next Regents question.

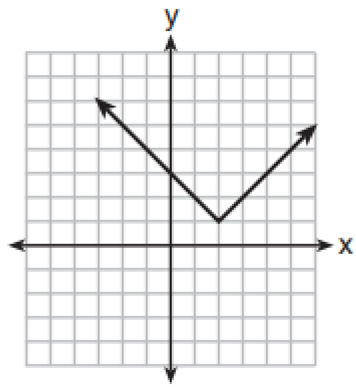
7 Which relation does *not* represent a function?

x	1	2	3	4	5	6
y	3.2	4	5.1	6	7.4	8.8

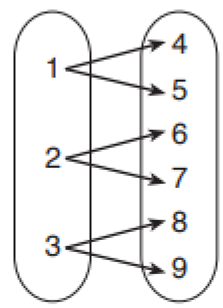
(1)

$$y = 3\sqrt{x+1} - 2$$

(3)



(2)



(4)

The answer is choice (4). Every value of *x* goes to two values of *y*. Choice (4) is not a function.

An equivalent way to write this linear function is with function notation: $f(x) = 3x + 7$.

You say "f of x" when you see $f(x)$ and it is another way of writing the variable y.

Function notation is used to input numbers for *x*.

For instance, consider $f(8)$. This means, "Find the *y*-value when *x* equals 8." For a simple linear function such as $f(x) = 3x + 7$, you may be able to find y in your head, or with the home screen of your calculator.

$$f(8) = 3 \cdot 8 + 7$$

$$f(8) = 24 + 7$$

$$f(8) = 31$$

Determine the value of $f(3)$, $f(25)$, and of $f(-6)$.

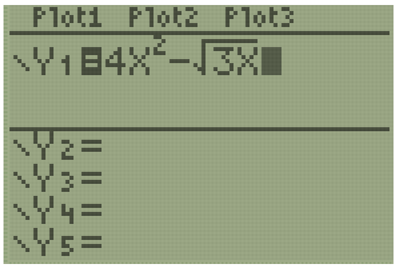
Did you get it right? Did you make a mistake? How long did that take you?

Remember, linear functions like $f(x) = 3x + 7$ are the easiest ones to work with.

Using the TABLE

Consider $g(x) = 4x^2 - \sqrt{3x}$. Notice that this function has been named $g(x)$ "*g* of *x*". We can use any letter to name a function, which is helpful when a problem involves more than one function. Let's explore $g(x)$ with the calculator.

Every function can be represented with a table or a graph.



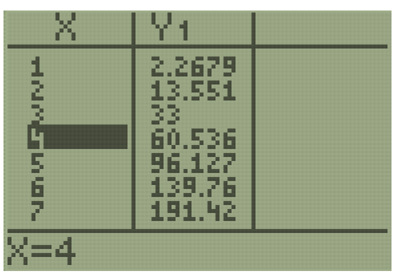
All function rules can be put into **Y=**.

The 2 on top of the x is an exponent. To get an exponent, press the **^** key then the value of the exponent. Since 2 is a very common exponent, you can also press the x^2 key. Notice that anything you type will stay in the exponent until you press the right arrow button →.

Notice that this function involves the square root of 3x. This symbol $\sqrt{}$ is called a radical. Find the radical symbol on your calculator by pressing **2nd**, $\boldsymbol{x^2}$.

From the Y= menu, we can find specific values of $g(x)$ by using tables or graphs.

Alejandro wants to find $g(4)$ using a table.



Press **2nd, GRAPH**

Use the up and down arrows until you get to *x* = 4.

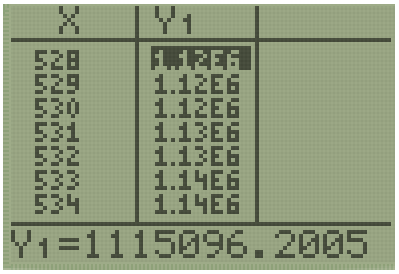
$$g(4) = 60.536$$

Next, Alejandro wants to find $g(528)$ using a table. He definitely doesn't want to press the down arrow that many times.

He presses **2nd, WINDOW** and changes TblStart to 528

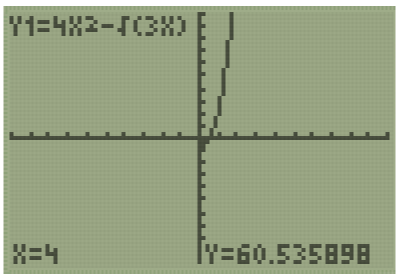
$\Delta Tbl = 1$ means that the *x*-values will count by 1.

Don't worry about the rest.



When Alejandro presses **2nd GRAPH**, his table starts with x = 528. The y-value is too large to display properly unless he highlights it using the arrow keys. This sometimes happens when x is very large.

Using the GRAPH



Tatiana is working with the same function, $g(x) = 4x^2 - \sqrt{3x}$

Tatiana wants to find $g(4)$ using a graph.

Press **GRAPH**. If your viewing window is different, press **ZOOM, 6** to get a standard window.

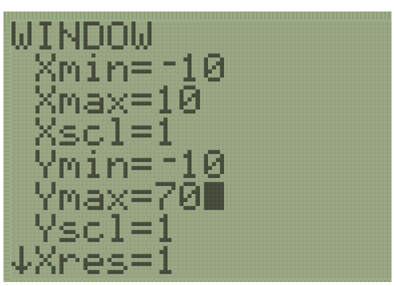
Press **TRACE**, **4**, **ENTER**

Tatiana notices that when x = 4, y = 60.535898.

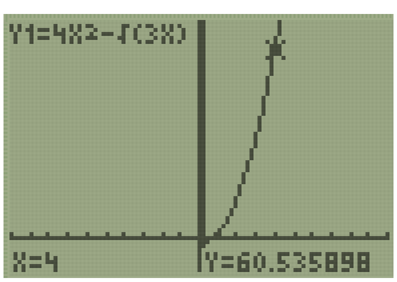
Alejandro's and Tatiana's answers for $g(4)$ are slightly different, but that's okay.

Tatiana notices that her calculator tells her that $g(4) = 60.535898$ at the bottom of her screen, but that she can't see the y-value 60.535898 on the graph of the function. She counts along the y-axis and notices that it only goes up to 10.

She decides to make her graph show more y-values.



She presses **WINDOW** and sets her maximum y-value to 70.



When she presses **GRAPH**, the y-axis goes up to 70.

When she presses **TRACE**, **4**, **ENTER**, she can see the location of $g(4)$ on the graph.

In general, make the Xmax and Ymax higher than the number you want and make the Xmin and Ymin lower than the number you want. You never need to change the Xscl or Yscl, but if you do, they tell the calculator how far to space the marks on the x-axis and y-axis.

Try the following Regents questions. Try to use tables and graphs, and what you know about functions to answer them.

1.

The function $g(x)$ is defined as $g(x) = -2x^2 + 3x$. The value of $g(-3)$ is

(1) -27 (3) 27

(2) -9 (4) 45

2.

If $k(x) = 2x^2 - 3\sqrt{x}$, then $k(9)$ is

(1) 315 (3) 159

(2) 307 (4) 153

3.

Marc bought a new laptop for $1250. He kept track of the value of the laptop over the next three years, as shown in the table below.

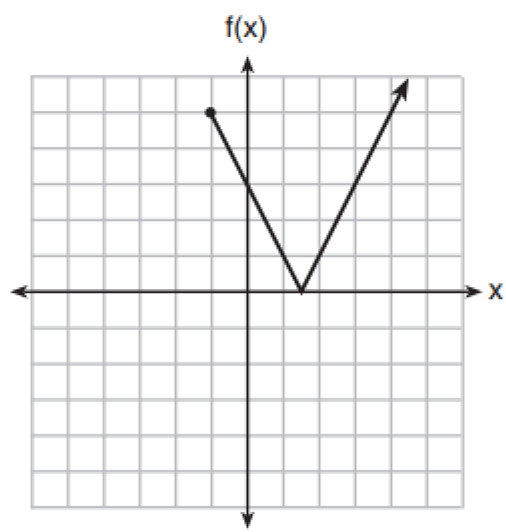
Years After Purchase	Value in Dollars
1	1000
2	800
3	640

Which function can be used to determine the value of the laptop for x years after the purchase?

(1) $f(x) = 1000(1.2)^x$
(2) $f(x) = 1000(0.8)^x$
(3) $f(x) = 1250(1.2)^x$
(4) $f(x) = 1250(0.8)^x$

4.

The function $f(x)$ is graphed below.

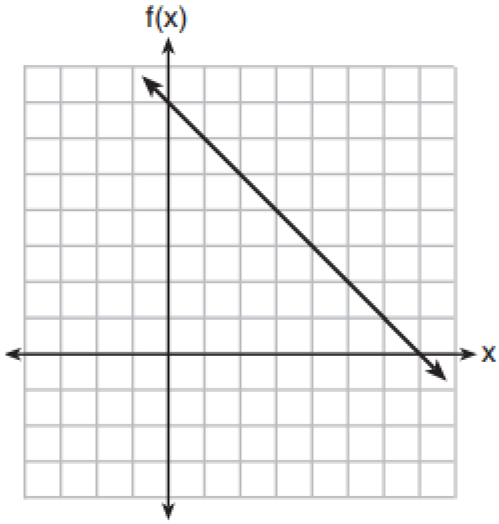


The domain of this function is

(1) all positive real numbers
(2) all positive integers
(3) $x \geq 0$
(4) $x \geq -1$

5.

The functions $f(x)$, $q(x)$, and $p(x)$ are shown below.



$$q(x) = (x - 1)^2 - 6$$

x	p(x)
2	5
3	4
4	3
5	4
6	5

When the input is 4, which functions have the same output value?

(1) $f(x)$ and $q(x)$, only
(2) $f(x)$ and $p(x)$, only
(3) $q(x)$ and $p(x)$, only
(4) $f(x)$, $q(x)$, and $p(x)$

6.

Materials A and B decay over time. The function for the amount of material A is $A(t) = 1000(0.5)^{2t}$ and for the amount of material B is $B(t) = 1000(0.25)^t$, where t represents time in days. On which day will the amounts of material be equal?

(1) initial day, only
(2) day 2, only
(3) day 5, only
(4) every day

In this lesson you learned some very important ideas:

- A function assigns every *x* to exactly one *y*-value.
- Functions can be represented by algebraic rules, tables, and graphs.
- How to view function tables in the calculator, how to adjust both the start value, and how much *x* changes by.
- How to view graphs in the calculator, change the window to get a better picture, and trace an input *x* to determine an output *y*.