Lesson 2 – Linear and Exponential Functions

Think about your own family. Everyone in your family has things about them that make them unique. But now think about what makes your family similar. Maybe it's the way you speak, or dress, or behave that makes the people in your family similar.

Just like you, certain functions belong to a family. In Algebra 1, we focus on four main families: linear, exponential, quadratic, and absolute value.

This lesson will aim to deepen your understanding of the characteristics of linear and exponential function "families." We will also learn a technique, called **regression**, which will allow us to write a function rule using a set of coordinates.

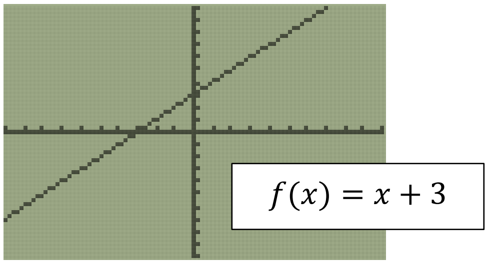
Linear Functions

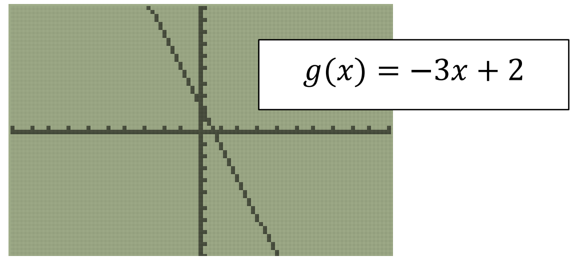
The following four situations are examples of linear functions.

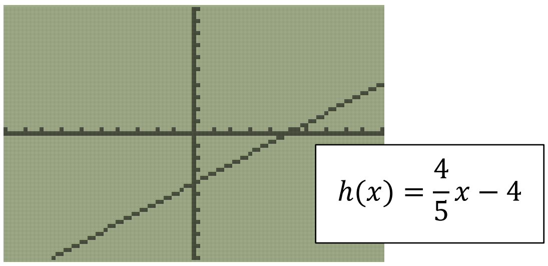
- Pedro earns $12 per hour at his job.
- A plant's height increases 3 inches per week.
- As water empties from a bathtub, its volume decreases at a rate of 2 gallons per minute.
- A car's distance traveled is changing at a constant speed of 55 miles per hour.

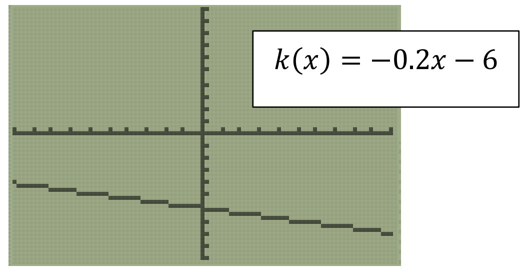
Write all the ways these situations are similar and how they are different.

The following graphs and rules are linear functions.









What is similar about the graphs of linear functions? What are some differences between the graphs?

How are the linear function rules similar to one another? How are the linear function rules different?

Use your calculator the fill in tables for *f*, *g*, *h*, and *k* from $x = -5$ to $x = 5$.

x	$f(x) = x + 3$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

x	$g(x) = -3x + 2$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

x	$h(x) = \frac{4}{5}x - 4$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

x	$f(x) = -0.2x - 6$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

What similarities and differences do you notice about the tables of linear function rules?

Every linear function rule can be written in the form

$$\boldsymbol{y = mx + b}$$

The ***m*** and ***b*** represent numbers. Here are important ideas about them that you may have noticed from the similarities and differences work you just did.

m

- Also called the rate of change
- It is the slope of the graph
- How much the y-values increases or decreases by when the x-value increases by 1
- When m is positive, the graph increases as x increases from left to right
- When m is negative, the graph decreases as x increases from left to right

b

- The value of y when x = 0
- It is where the graph intersects the y-axis
- The starting amount for linear situations

Exponential Functions

Exponential functions are a type of nonlinear function, because their patterns do not result in straight lines. The following situations are examples of exponential functions.

A. 500 bacteria double every hour.
B. A car bought for $25000 is worth half of its value each year.
C. A savings account has $1000 initially and gains 5% interest annually.
D. A school with 900 students is decreasing its enrollment by 2% each year.

What do each of these situations have in common? How are they different?

Here are rules for each of the situations A through D.

$$A(x) = 500(2)^x$$

$$B(x) = 25000\left(\frac{1}{2}\right)^x$$

$$C(x) = 1000(1.05)^x$$

$$D(x) = 900(0.98)^x$$

How are these exponential rules similar? How are they different?

What connections do you see between the situation described and its function rule?

Here are tables for each of the rules.

$A(x) = 500(2)^x$

X	Y1
0	500
1	1000
2	2000
3	4000
4	8000
5	16000
6	32000

$B(x) = 25000\left(\frac{1}{2}\right)^x$

X	Y1
0	25000
1	12500
2	6250
3	3125
4	1562.5
5	781.25
6	390.63

$C(x) = 1000(1.05)^x$

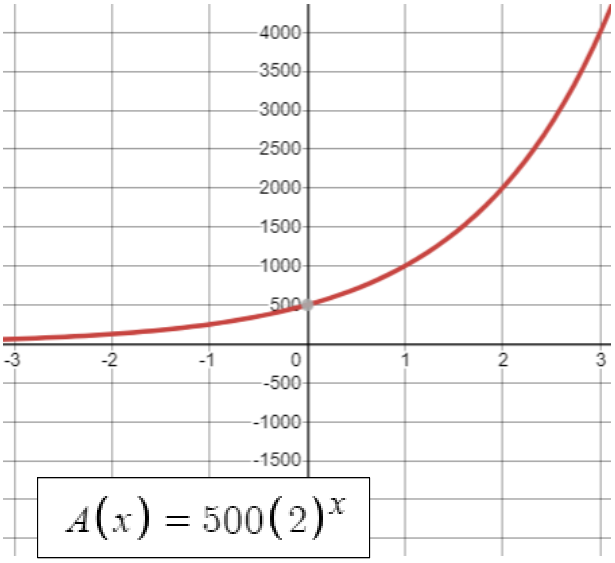
X	Y1
0	1000
1	1050
2	1102.5
3	1157.6
4	1215.5
5	1276.3
6	1340.1

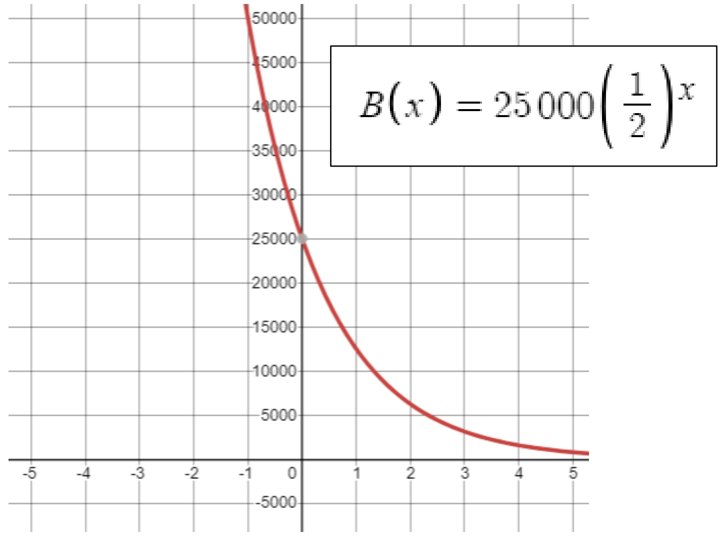
$D(x) = 900(0.98)^x$

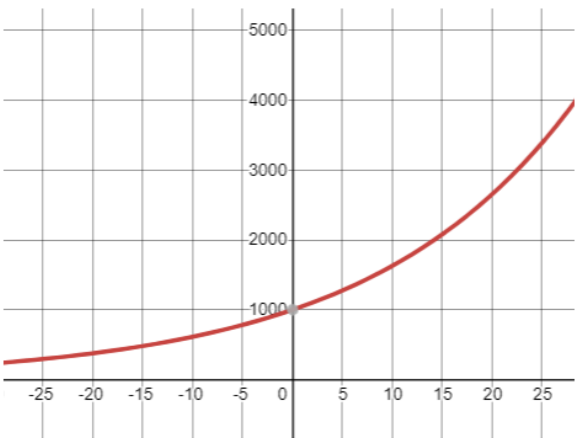
X	Y1
0	900
1	882
2	864.36
3	847.07
4	830.13
5	813.53
6	797.26

What similarities and differences do you notice between the exponential patterns in the tables? What connections do you notice about the function rule and the table of values?

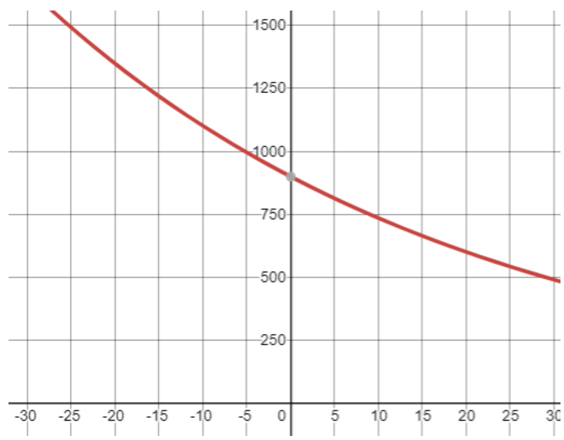
Here are graphs of each exponential function.







$C(x) = 1\,000(1.05)^x$



$D(x) = 900(0.98)^x$

What are some similarities and differences you notice about how each graph is shaped?

How do the exponential graphs compare to the linear graphs?

What are some observations you can make about the how the exponential function rules and their graphs are related?

Every exponential function rule can be written in the form

$$\boldsymbol{y} = \boldsymbol{a}(\boldsymbol{b})^{\boldsymbol{x}}$$

Where **a** and **b** are numbers. Here are some important ideas about how these numbers affect the behavior of tables and graphs of exponential function rules.

a

- This is the initial (starting) value of the function
- On a table, this is the y-value when x = 0
- On a graph, this is where the graph will intersect the y-axis (vertical axis)

b

- This is the multiplier of the function
  If the function is growing or shrinking by a percent, b will be equal to $1 + \frac{\%}{100}$ or $1 - \frac{\%}{100}$
- To get the next y-value in the table, multiply the current y-value by b
- If b is greater than 1, the graph will be increasing from left to right
- If b is between 0 and 1, the graph will be decreasing from left to right

Decide whether the rule belongs to the linear function family, the exponential function family, or neither. Use evidence from the calculator or your knowledge of mathematics to justify your answer.

A. $y = 4 + 2x$

B. $y = 2(x+1)^2 - 5$

C. $f(x) = 3^x$

D. $g(x) = |x+1| - 3$

E. $k(x) = 0.5x - 6 + x$

F. $p(t) = 5000(0.87)^x$

Decide which situations below are best modeled by linear functions, exponential functions, or neither. Explain your reasoning.

i. A bamboo plant grows at a constant rate of 3 inches per day.

ii. An amusement park allows 50 people to enter every 30 minutes

iii. The value of a cell phone decreciates by 3.5% each year.

iv. A baseball tournament eliminates half of the teams after each round.

v. A football is kicked into the air from an initial height of 4 feet. It reaches a maximum height of 82 feet before it returns to the ground.

Linear Regression

This technique will transform a set of data into a line of best fit.

For example, in January 2019, the Regents exam gave data on the height of a certain breed of dog based upon its mass (weight).

First, make a prediction. What do you think is the relationship between the height of a dog and its weight? Would a linear function make sense to model this relationship?

Here is the question:

34 The data given in the table below show some of the results of a study comparing the height of a certain breed of dog, based upon its mass.

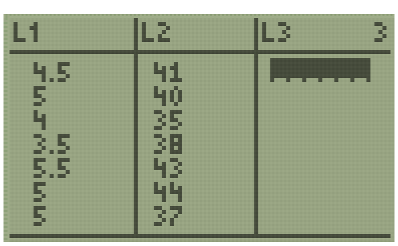
Mass (kg)	4.5	5	4	3.5	5.5	5	5	4	4	6	3.5	5.5
Height (cm)	41	40	35	38	43	44	37	39	42	44	31	30

Write the linear regression equation for these data, where x is the mass and y is the height. Round all values to the *nearest tenth*.

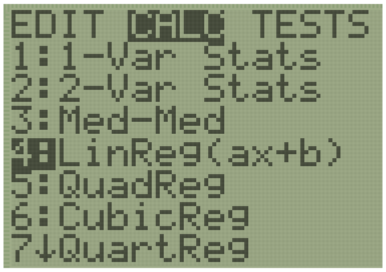
State the value of the correlation coefficient to the *nearest tenth*, and explain what it indicates.

The solution starts here. Follow along on your calculator as well.

Press **STAT**, **ENTER**



Type the x-values in **L1** and the y-values in **L2**

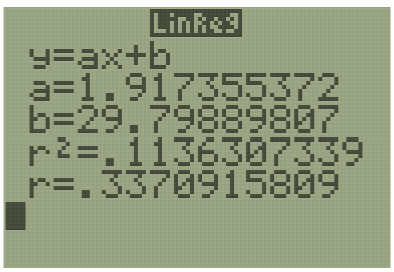


Press **STAT**, →

Since this problem asks for a <u>linear</u> regression, choose **4**

(<u>Exponential</u> regression is choice **0**)

Highlight "Calculate" and press **ENTER**



The first line, **y = ax + b**, is the function rule.

The next lines tell you the numbers to write in place of **a** and **b**. **x** and **y** will remain letters.

Ignore r^2

"r" is called the **correlation coefficient**, and it is always between 0 and 1. The closer **r** is to 1, the better fit this data is to the type of function rule you chose.

Notice that this question asked us to round the values of the function rule to the nearest tenth.

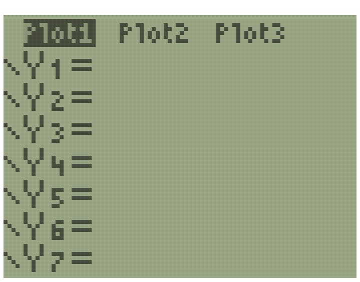
The linear regression equation is $\boldsymbol{y = 1.9x + 29.8}$

The correlation coefficient, to the nearest tenth, is **0.3**

Since 0.3 is not close to 1, this is a <u>weak</u> correlation.

Notice how the *r*-value 0.3 was closer to zero than one. This tells us that there is a weak relationship between the breed's height and weight.

To better see this, do the following:

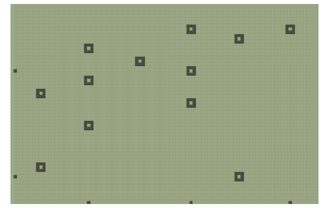


Step 1: When you press **Y=** be sure that **"Plot1"** is highlighted. If it isn't, use the **arrow keys** to highlight it and press **ENTER**

All **Y=** rules should be blank.

Step 2: Press **ZOOM**, and choose **"9: ZoomStat"** and then press **ENTER**

Describe the overall pattern of the data points. How do they compare to a linear function?



Next, to see how well our function fits the data, type $\boldsymbol{y = 1.9x + 29.8}$ into **Y1** by pressing **Y=**. This is the line that best fits our data. You can see that the line traces the pattern in the data, but that the points are relatively spread out. This makes for a weak correlation.

When working with real world measurements, or data, it is rare that the measurements make perfectly linear patterns. However, data is often close to linear, which is why the correlation coefficient is important. It tells us how well our data fits a line.

The correlation coefficient, **r**, is either strong or weak depending on whether it is closer to zero or one.

Complete the following problems on your own.

36 The percentage of students scoring 85 or better on a mathematics final exam and an English final exam during a recent school year for seven schools is shown in the table below.

Percentage of Students Scoring 85 or Better	
Mathematics, x	**English, y**
27	46
12	28
13	45
10	34
30	56
45	67
20	42

Write the linear regression equation for these data, rounding all values to the *nearest hundredth*.

State the correlation coefficient of the linear regression equation, to the *nearest hundredth*. Explain the meaning of this value in the context of these data.

35 The table below shows the number of hours ten students spent studying for a test and their scores.

Hours Spent Studying (x)	0	1	2	4	4	4	6	6	7	8
Test Scores (y)	35	40	46	65	67	70	82	88	82	95

Write the linear regression equation for this data set. Round all values to the *nearest hundredth*.

State the correlation coefficient of this line, to the *nearest hundredth*.