Topic 3 – Function Families

Congratulations! You are nearing the end of this four-topic series. This lesson will aim to deepen your understanding of functions by grouping them into three main "families."

Think about your own family. Everyone in your family has things about them that make them unique. But now think about what makes your family similar. Maybe it's the way you speak, or dress, or behave that makes your family similar.

Just like you, certain functions belong to a family. In Algebra I, we focus on three main families: linear, exponential, and quadratic.

Linear Functions

The following four situations are examples of linear functions.

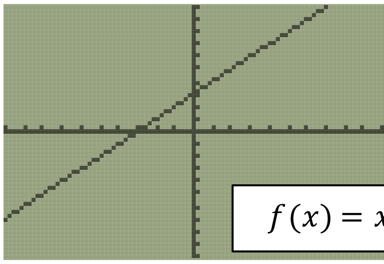
- Pedro earns $12 per hour at his job.
- A plant's height increases 3 inches per week.
- Water is emptying out of a bathtub such that its volume is decreasing at a rate of 2 gallons per minute.
- A car's distance traveled is changing at a constant speed of 55 miles per hour.

Write all the ways these situations are similar and how they are different.

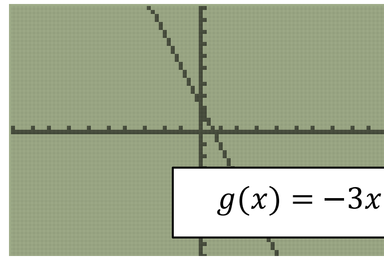
Similar	Different

Note: Linear functions involve adding or subtracting a constant amount to a starting value to get the next amount. The speed example is tricky. Try to get students to verbalize that each hour the car will add 55 miles to its total distance.

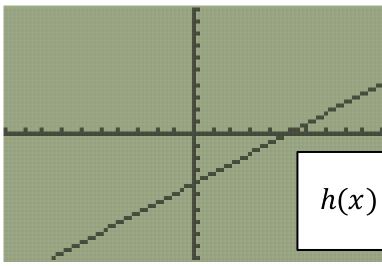
The following graphs and rules are linear functions.



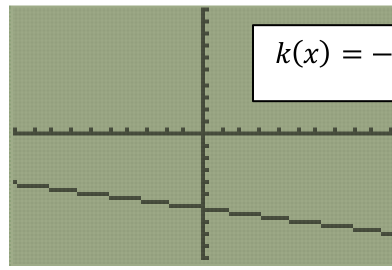
$f(x) = x + 3$



$g(x) = -3x + 2$



$h(x) = \frac{4}{5}x - 4$



$k(x) = -0.2x - 6$

What is similar about the graphs of linear functions? What are some differences between the graphs?

Answers: Linear function graphs all look like perfectly straight lines. The lines will either slope up and increase from left to right or slope down and decrease from left to right (as x increases).

Linear function graphs will also intersect the x- and y-axes in different locations.

How are the linear function rules similar to one another? How are the linear function rules different?

Answers: Linear function rules are usually in the form $y = ax + b$. The number in place of a is the amount that is being constantly added or subtracted. When a is positive, the graph will increase from left to right as x increases. When a is negative, the graph will decrease as x increases.

The number for b is the starting amount in a linear situation and the y-intercept on the graph when x = 0.

Use your calculator the fill in tables for *f*, *g*, *h*, and *k* from $x = -5$ to $x = 5$.

x	$f(x) = x + 3$
-5	-2
-4	-1
-3	0
-2	1
-1	2
0	3
1	4
2	5
3	6
4	7
5	8

x	$g(x) = -3x + 2$
-5	17
-4	14
-3	11
-2	8
-1	5
0	2
1	-1
2	-4
3	-7
4	-10
5	-13

x	$h(x) = \frac{4}{5}x - 4$
-5	-8
-4	-7.2
-3	-6.4
-2	-5.6
-1	-4.8
0	-4
1	-3.2
2	-2.4
3	-1.6
4	-0.8
5	0

⍰	$f(x) = -0.2x - 6$
-5	-5.9
-4	-5.92
-3	-5.94
-2	-5.96
-1	-5.98
0	-6
1	-6.02
2	-6.04
3	-6.06
4	-6.08
5	-6.1

What similarities and differences do you notice about the tables of linear function rules?

Answers: Each table adds or subtracts a constant amount when x increases by 1. The amount that adds or subtracts is the same as the number that is attached to x in the function rule.

Notice that when x = 0, the y-value matches the number at the end of each function rule.

Every linear function rule can be written in the form

$$y = mx + b$$

The *m* and *b* represent numbers. Here are important ideas about them that you may have noticed from the similarities and differences work you just did.

m

- Also called the rate of change.
- It is the slope of the graph.

b

- The value of *y* when *x* = 0.
- It is where the graph intersects the *y*-axis.
- The starting amount for linear situations.

- How much the *y*-value increases or decreases by when the *x*-value increases by 1
- When *m* is positive, the graph increases as *x* increases from left to right.
- When *m* is negative, the graph decreases as *x* increases from left to right.

Exponential Functions

Exponential functions are a type of nonlinear function, because their patterns do not result in straight lines. The following situations are examples of exponential functions.

A. 500 bacteria double every hour.

B. A car bought for $25,000 is worth half of its value each year.

C. A savings account has $1000 initially and gains 5% interest annually.

D. A school with 900 students is decreasing its enrollment by 2% each year.

What do each of these situations have in common? How are they different?

Answers: Exponential function rules always involve multiplying (or dividing) by a constant amount to get the next amount. It is more useful to think of exponential functions as multiplying functions. Growing or shrinking by a percent also involved multiplication. Students generally think of halving a number (function B) as dividing repeatedly by 2.

Note: Discuss that you can multiply repeatedly by $\frac{1}{2}$ or 0.5 to produce the same amount. You may also wish to expand on growing or shrinking by a percent. When we multiply something by 1 it is unchanged. To grow or shrink by a percent, multiply by $1 + \frac{\%}{100}$ or by $1 - \frac{\%}{100}$.

Here are rules for each of the situations A through D.

$$A(x) = 500(2)^x$$

$$B(x) = 25000\left(\frac{1}{2}\right)^x$$

$$C(x) = 1000(1.05)^x$$

$$D(x) = 900(0.98)^x$$

How are these exponential rules similar? How are they different?

Answers: Each exponential function rule involves a number multiplied by another number that is raised to an exponent. In every case, the exponent contains x. The starting number and the amount in the parentheses vary.

What connections do you see between the situation described and its function rule?

Answers: The starting number in the function rule is the initial amount in each situation. The number in the parentheses is the multiplier for each situation.

Note: You may wish to point out that growing by 5% is the same as taking 100% of what you start with and adding 5%. 105% = 1.05. Shrinking by 2% is the same as taking 100% of what you start with and subtracting 2%. 100% - 2% = 98%, or 0.98.

Here are tables for each of the rules.

$A(x) = 500(2)^x$

X	Y1
0	500
1	1000
2	2000
3	4000
4	8000
5	16000
6	32000

$B(x) = 25000\left(\frac{1}{2}\right)^x$

X	Y1
0	25000
1	12500
2	6250
3	3125
4	1562.5
5	781.25
6	390.63

$C(x) = 1000(1.05)^x$

X	Y1
0	1000
1	1050
2	1102.5
3	1157.6
4	1215.5
5	1276.3
6	1340.1

$D(x) = 900(0.98)^x$

X	Y1
0	900
1	882
2	864.36
3	847.07
4	830.13
5	813.53
6	797.26

What similarities and differences do you notice between the exponential patterns in the tables?

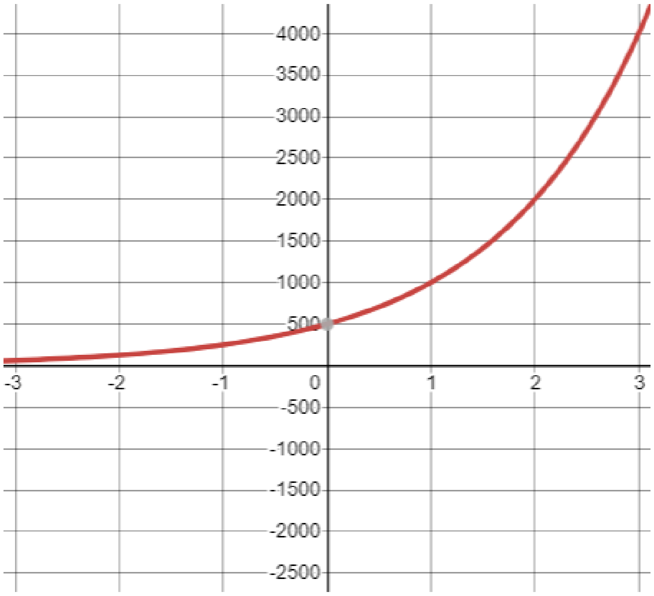
What connections do you notice about the function rule and the table of values?

Answers: Each table multiplies by a constant amount to get the next value. The pattern in tables A and B are more obvious than C and D. Help students see the connection between the number in parentheses in the function rule and how it is the multiplier to produce the next value in the table.

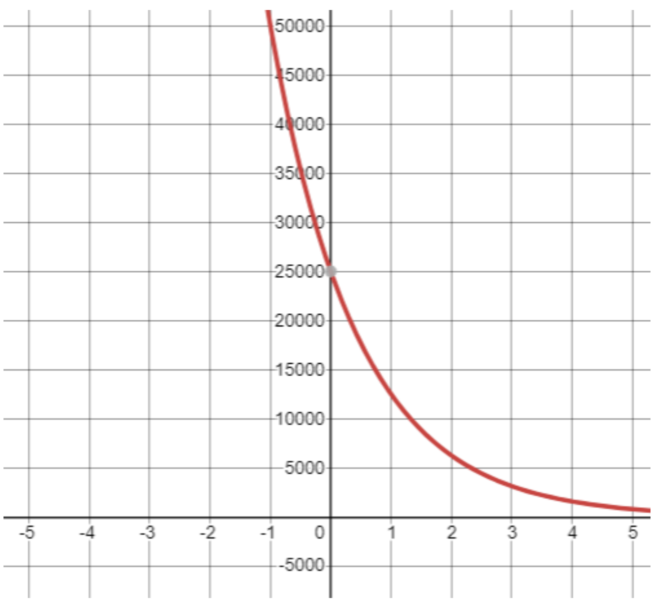
Have students verify that this will always work for each table.

Note: Students should also notice that the starting number in the function rule is the y-value when x = 0.

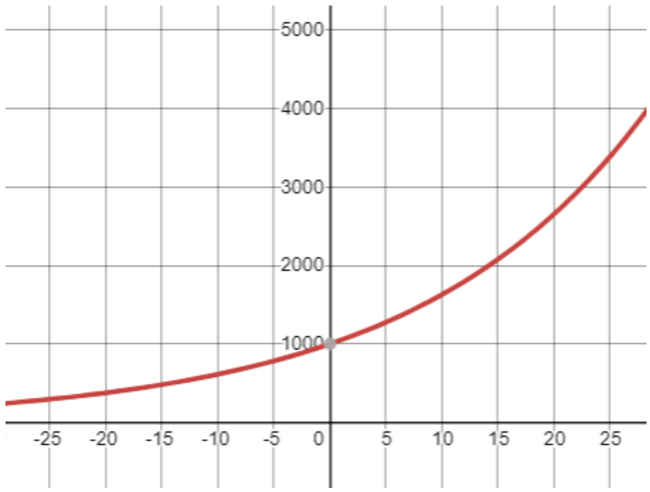
Here are graphs of each exponential function.



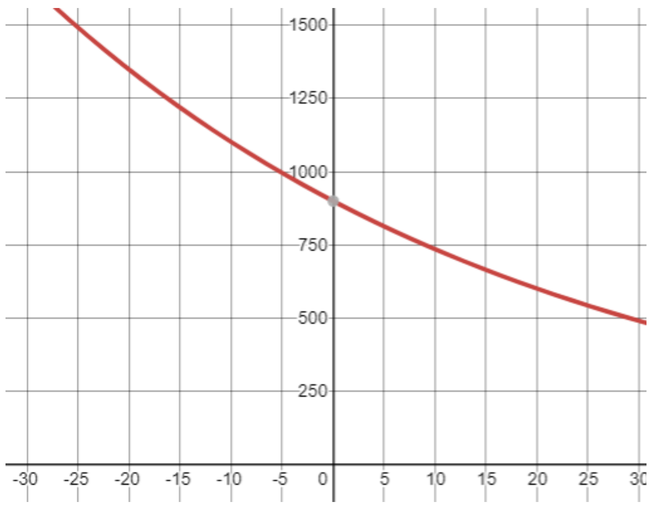
$$A(x) = 500(2)^x$$



$$B(x) = 25\,000\left(\frac{1}{2}\right)^x$$



$$C(x) = 1000(1.05)^x$$



$$D(x) = 900(0.98)^x$$

What are some similarities and differences you notice about how each graph is shaped?
How do the exponential graphs compare to the linear graphs?

Answers: Similar to linear function graphs, each exponential graph is always increasing or always decreasing. Unlike linear functions, these are not straight lines. Exponential function graphs are always above the x-axis and the y-values are positive. Exponential functions will never equal zero, and do not have x-intercepts or zeros.

Note: Don't feel compelled to discuss all of these ideas, but they are nice to know.

What are some observations you can make about the how the exponential function rules and their graphs are related?

Answers: The outer number of the function rule is equal to the y-intercept of the exponential function graph. The graph will be increasing when the number in parentheses is greater than one. The graph will be decreasing when the number in parentheses is between 0 and 1.

Every exponential function rule can be written in the form

$$y = a(b)^x$$

Where a and b are numbers. Here are some important ideas about how these numbers affect the behavior of tables and graphs of exponential function rules.

a

- This is the initial (starting) value of the function.
- On a table, this is the *y*-value when *x* = 0.
- On a graph, this is where the graph will intersect the y-axis (vertical axis).

b

- This is the multiplier of the function.
  If the function is growing or shrinking by a percent, *b* will be equal to $1 + \frac{\%}{100}$ or $1 - \frac{\%}{100}$
- To get the next *y*-value in the table, multiply the current *y*-value by *b.*
- If *b* is greater than 1, the graph will be increasing from left to right.
- If *b* is between 0 and 1, the graph will be decreasing from left to right.

Quadratic Functions

The third major function family, quadratic functions, are also nonlinear. Situations that are modeled with quadratic function rules typically involve gravity or motion. For example:

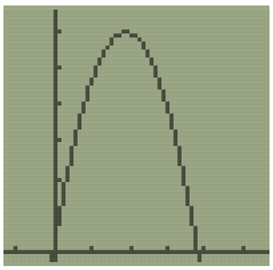
- Jonathan kicks a football into the air. The height in feet at any given time (*t)* can be modeled by the function rule $h(t) = -16t^2 + 60t + 3$.
- A skydiver jumps from a height of 3,500 meters. The total distance fallen at any given time (*t)* can be modeled by the function rule $d(t) = 3500 - 9.8t^2$.
- A concert venue's revenue depends on the ticket price, *x*. The concert revenue can be modeled by the quadratic function rule $R(x) = x(200 - 20x)$.
- The path of a comet as it slingshots around planet Earth can be modeled by the quadratic function $p(x) = (x - 3)^2 + 4$.

What similarities and differences do you notice about these function rules?

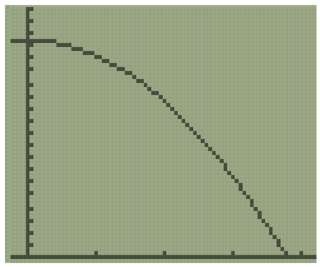
Answers: Three of the rules involve an exponent of 2. Function h(t) and d(t) have the exponent of 2 directly attached to the variable, while p(x) is squaring the parentheses with the variable inside. R(x) is a different form of a quadratic called factored form. It is a product of two linear functions.

Note: Depending on the background knowledge of your students, you may wish to point out that multiplying two factors of x will produce x^2. Also, two functions use h as the independent variable, and two of the functions use x, although this difference is not significant.

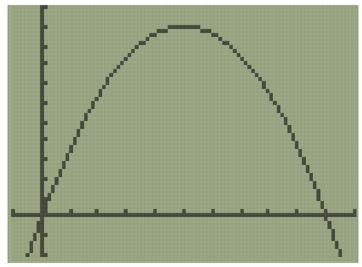
Quadratic function rules appear in three major forms. We are not going to focus on how to convert between these forms, but it makes it more difficult to recognize what exactly makes function rules quadratic. Did you notice how three rules involved an exponent of 2? Quadratic function rules do have x^2 in them. It is less obvious that $R(x) = x(200 - 20x)$ is quadratic, but it is. Let's take a look at each of their graphs.



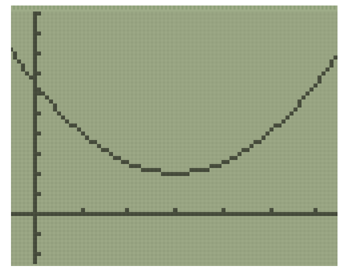
$h(t)$



$d(t)$



$R(x)$



$p(x)$

Which graphs are similar? How are they similar? Which graph is different? How is it different?

Answers: This question is intentionally open ended to generate discussion.

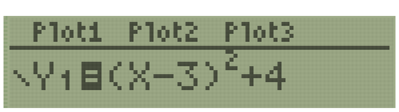
Three of the functions, *h*, *R*, and *p*, show graphs that change directions and clearly have a turning point (vertex). These graphs increase and decrease. Functions *h*, *d*, and *R* open downward while *p* opens upward. Functions *h* and *R* both appear to share an x-intercept at x = 0 (although *h* actually does not).

Students may identify function *p* as being different because it opens up or because it doesn't have any x-intercepts. Some students may identify function *d* as being different because it only decreases in this viewing window.

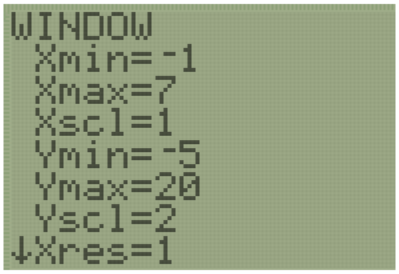
Looking at a graph will always reveal whether a rule is quadratic. These U-shaped graphs are called **parabolas**. Notice that it is not obvious that $d(t)$ is a parabola. Next lesson, we will discuss how to adjust the calculator WINDOW in greater detail.

Notice that linear and exponential functions either always increase or always decrease from left to right. Parabolas change directions. This turning point is called the **vertex**. If the quadratic graph opens up (shaped like a cup), the vertex is the minimum of the graph. If the parabola opens down (shaped like a frown), the vertex is a maximum point.

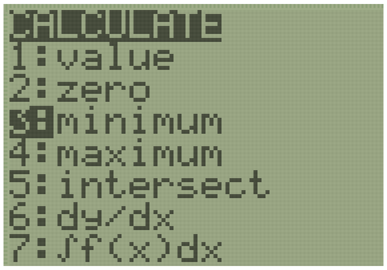
Suppose you want to find the minimum point of $p(x)$.



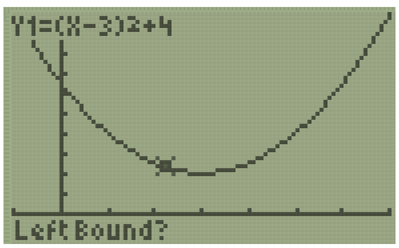
Press Y= and enter the function rule for Y1.

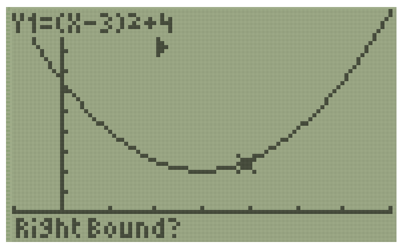


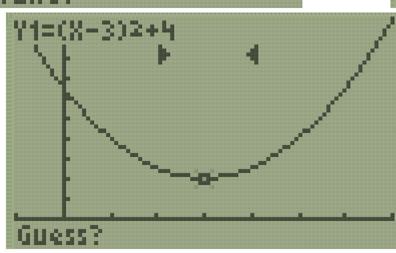
To follow along with this example, press WINDOW and enter the following settings.



Press **2nd**, **TRACE**, and choose choice **3** for the minimum finder.



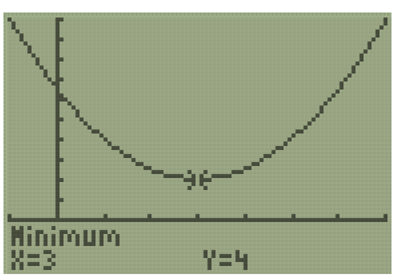




Move the cursor using the arrow keys until it is on the left side of the minimum, then press **ENTER**.

Move the cursor to the right side of the minimum and press **ENTER.**

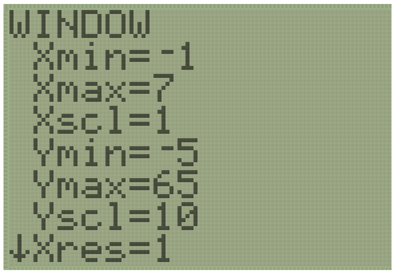
Finally, move the cursor to somewhere close to the minimum point an press **ENTER.**



The coordinates of the minimum point of $p(x)$ are $(3,4)$.

This process works nearly the same for finding a maximum.

Try to find the coordinates of the maximum point on $h(t) = -16t^2 + 60t + 3$. Note that when you put the rule into **Y=** you always use *x* as your variable.



These settings produce a nice viewing window for *h*. Again, we will discuss strategies for how to do this next lesson.

Use these Window settings and write the coordinates of the maximum point of $h(t) = -16t^2 + 60t + 3$.

Answer: (1.87, 59.25)

The Regents exam also loves to ask about the **zeros** of quadratic functions. **Zeros** are *x*-values for which *y* = 0. Sometimes we can find zeros on a table.

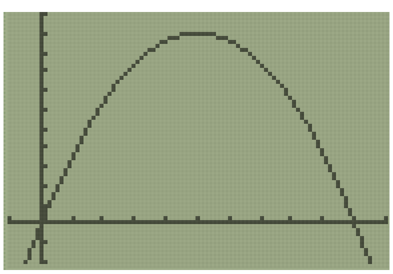
Locate the zeros of $R(x) = x(200 - 20x)$ using your table.

Note: The answer for locating the zeros is written in the student packets. *"Hopefully you notice…"*

Hopefully you noticed that the zeros are *x* = 0 and *x* = 10.

From this example, we see that quadratics can have two zeros.

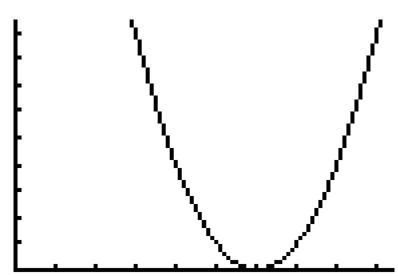
The zeros of a function are the ***x*-intercepts** of its graph. Notice how the graph of $R(x)$ intersects the *x*-axis at two locations, *x* = 0 and *x* = 10



Sketch the graph of a quadratic function that has one zero.

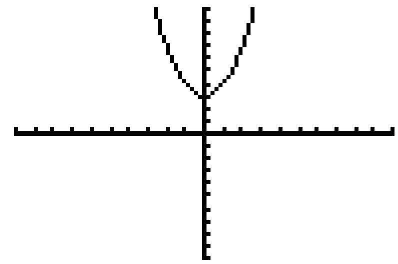
Explanation: Sketches will vary, but the graph's vertex will touch the x-axis. (Graphs can intersect the y-axis)

Note: Encourage students to sketch different graphs that meet Each criteria.



Sketch the graph of a quadratic function that has no zeros.

Explanation: The graph will not intersect the x-axis.

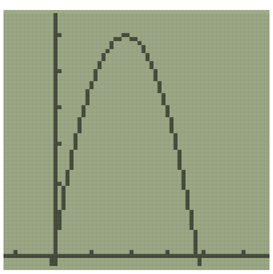


Is it possible for a quadratic function to have more than two zeros? Explain your reasoning.

Answer: It is not.

Reasoning: One possible explanation is since quadratic functions change directions only once, it is only possible for the function to decrease past zero, then increase past zero a second time (or vice versa). For a quadratic to have three zeros, the function would need to change directions a second time.

Consider the function $h(t) = -16t^2 + 60t + 3$. By looking at its graph, we know that $h(t)$ has two zeros.



However, when we examine a table of $h(t)$ values, the location of the zeros are not obvious.

Fill out the table of values for $h(t)$ for integers *x* = -2 through *x* = 5

x	-2	-1	0	1	2	3	4	5
$h(x)$	**-181**	**-73**	**3**	**47**	**59**	**39**	**-13**	**-97**

Between what values of *x* must the zeros be "hiding"? Explain how you can tell.

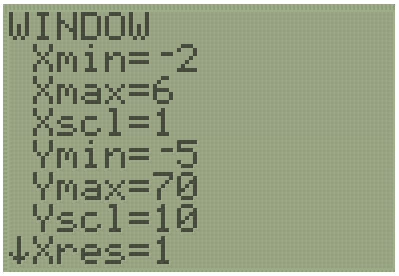
Explanation: The zeros are hiding between -1 and 0 and between 3 and 4. We can tell because the y-values change between positive and negative. Since there are no breaks in the graph of a quadratic function, the y-values must cross through zero in order to change their sign.

This example shows that we can't rely on tables to find the zeros of every quadratic function.

To find the zeros on a graph, press **2nd**, **TRACE**, and choose option **2: zero**.

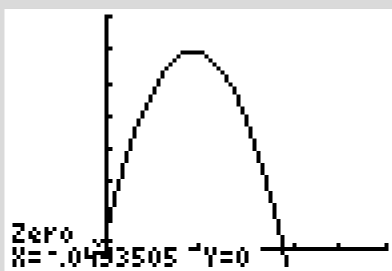
Then, follow the same process to locate a max or min.

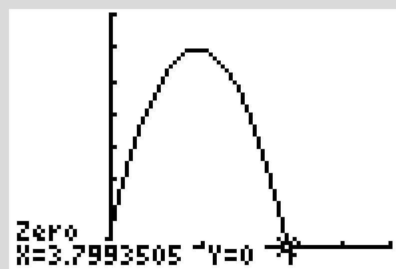
Use the following WINDOW for a nice picture of the graph.



Try this process now to locate the zeros of $h(t) = -16t^2 + 60t + 3$. Note that they will be decimal values.

Answer:





The zeros are approximately x = -0.049 and x = 3.80

Note: You can improve the viewing window by choosing a lower value for Ymin such as -50

Great job so far! You have made it through the third topic. Next, particular questions have been chosen from the August 2018 Regents exam that you should be able to correctly answer by applying strategies and ideas from each lesson so far. Some questions may look a little different than what you have experienced so far. That's okay; use your strategies and what you know to pick answers that make the most sense. You will notice that not all questions from the test are here. Next lesson, you will be expected to decide which questions to leave out. Good luck, and remind yourself that you only need at least ten correct answers! **You can do this!!**

1 The number of bacteria grown in a lab can be modeled by $P(t) = 300 \bullet 2^{4t}$, where t is the number of hours. Which expression is equivalent to $P(t)$?

(1) $300 \bullet 8^t$ (3) $300^t \bullet 2^4$

(2) $300 \bullet 16^t$ (4) $300^{2t} \bullet 2^{2t}$

Answer (2)

2 During physical education class, Andrew recorded the exercise times in minutes and heart rates in beats per minute (bpm) of four of his classmates. Which table best represents a linear model of exercise time and heart rate?

Student 1

Exercise Time (in minutes)	**Heart Rate** (bpm)
0	60
1	65
2	70
3	75
4	80

(1)

Student 3

Exercise Time (in minutes)	**Heart Rate** (bpm)
0	58
1	65
2	70
3	75
4	79

(3)

Student 2

Exercise Time (in minutes)	**Heart Rate** (bpm)
0	62
1	70
2	83
3	88
4	90

(2)

Student 4

Exercise Time (in minutes)	**Heart Rate** (bpm)
0	62
1	65
2	66
3	73
4	75

(4)

Answer (1)

3 David correctly factored the expression $m^2 - 12m - 64$. Which expression did he write?

(1) $(m - 8)(m - 8)$ (3) $(m - 16)(m + 4)$

(2) $(m - 8)(m + 8)$ (4) $(m + 16)(m - 4)$

Note: Factored form creates an <u>equivalent</u> function.

Answer (3)

4 The solution to $-2(1 - 4x) = 3x + 8$ is

(1) $\frac{6}{11}$ (3) $-\frac{10}{7}$

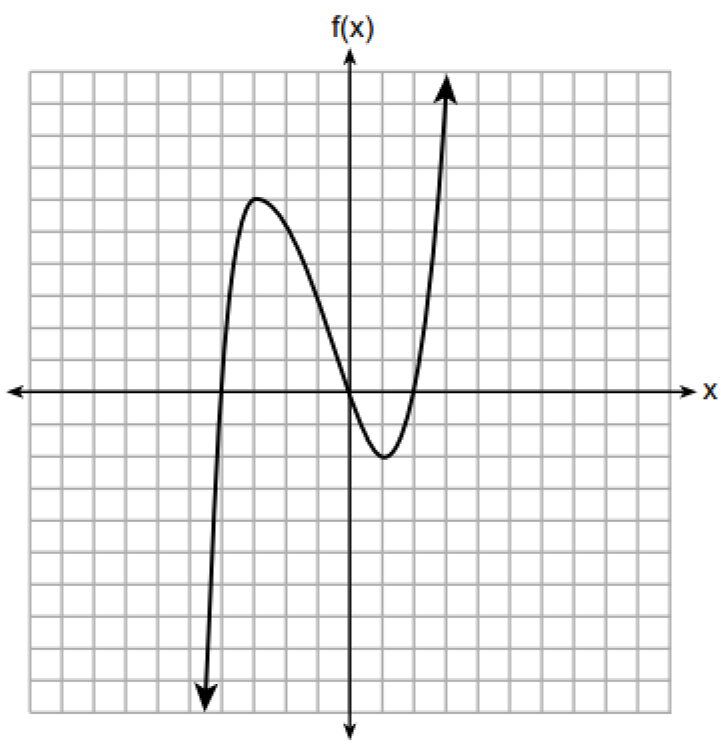
(2) 2 (4) -2

Note: Set $Y1 = -2(1-4x)$

$Y2 = 3x + 8$, use tables and graphs

Answer (2)

5 The graph of $f(x)$ is shown below.



What is the value of $f(-3)$?

(1) 6 (3) -2

(2) 2 (4) -4

Answer (1)

(The y-value when x = -3)

6 If the function $f(x) = x^2$ has the domain $\{0, 1, 4, 9\}$, what is its range?

(1) $\{0, 1, 2, 3\}$

(2) $\{0, 1, 16, 81\}$

(3) $\{0, -1, 1, -2, 2, -3, 3\}$

(4) $\{0, -1, 1, -16, 16, -81, 81\}$

Answer (2)

Note: These are the y-values for the given x-values in the domain.

7 The expression $4x^2 - 25$ is equivalent to

(1) $(4x - 5)(x + 5)$

(2) $(4x + 5)(x - 5)$

(3) $(2x + 5)(2x - 5)$

(4) $(2x - 5)(2x - 5)$

Answer (3)

8 Compared to the graph of $f(x) = x^2$, the graph of $g(x) = (x - 2)^2 + 3$ is the result of translating $f(x)$

(1) 2 units up and 3 units right

(2) 2 units down and 3 units up

(3) 2 units right and 3 units up

(4) 2 units left and 3 units right

Answer (3)
(compare graphs)

The y-intercept is the y-value when x = 0.

The y-int for *f* is 1

For *g*, the y-int is 2

For *h*, the y-int is -1

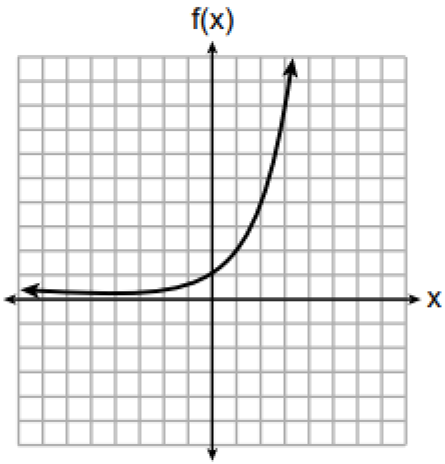
The y-intercept is the y-value when x = 0.

The y-int for *f* is 1

For *g*, the y-int is 2

For *h*, the y-int is -1

11 Three functions are shown below.



$g(x) = 3^x + 2$

x	h(x)
−5	30
−4	14
−3	6
−2	2
−1	0
0	−1
1	−1.5
2	−1.75

The y-intercept is the y-value when x = 0.

The y-int for *f* is 1

For *g*, the y-int is 2

For *h*, the y-int is -1

Which statement is true?

(1) The y-intercept for $h(x)$ is greater than the y-intercept for $f(x)$.

(2) The y-intercept for $f(x)$ is greater than the y-intercept for $g(x)$.

(3) The y-intercept for $h(x)$ is greater than the y-intercept for both $g(x)$ and $f(x)$.

(4) The y-intercept for $g(x)$ is greater than the y-intercept for both $f(x)$ and $h(x)$.

Answer (4)

13 If $y = 3x^3 + x^2 - 5$ and $z = x^2 - 12$, which polynomial is equivalent to $2(y + z)$?

(1) $6x^3 + 4x^2 - 34$ (3) $6x^3 + 3x^2 - 22$

(2) $6x^3 + 3x^2 - 17$ (4) $6x^3 + 2x^2 - 17$

Answer (1)

16 If $f(x) = 2x^2 + x - 3$, which equation can be used to determine the zeros of the function?

(1) $0 = (2x - 3)(x + 1)$ (3) $0 = 2x(x + 1) - 3$

(2) $0 = (2x + 3)(x - 1)$ (4) $0 = 2x(x - 1) - 3(x + 1)$

Answer (2)

17 Each day, a local dog shelter spends an average of \$2.40 on food per dog. The manager estimates the shelter's daily expenses, assuming there is at least one dog in the shelter, using the function $E(x) = 30 + 2.40x$.

Which statements regarding the function $E(x)$ are correct?

I. x represents the number of dogs at the shelter per day.
II. x represents the number of volunteers at the shelter per day.
III. 30 represents the shelter's total expenses per day.
IV. 30 represents the shelter's nonfood expenses per day.

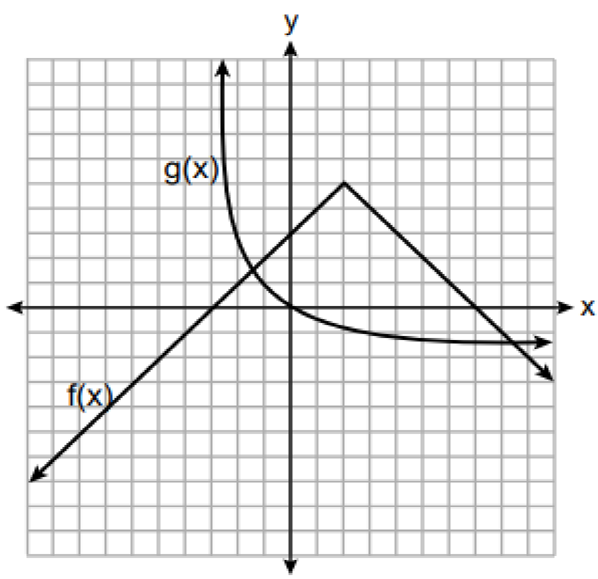
(1) I and III (3) II and III

(2) I and IV (4) II and IV

Answer (2)

Note: Question 17 is challenging. It may be worth discussing as a whole group. Allow students to explain why they think each part I through IV is true or not.

19 The functions $f(x)$ and $g(x)$ are graphed below.



Based on the graph, the solutions to the equation $f(x) = g(x)$ are

(1) the x-intercepts

(2) the y-intercepts

(3) the x-values of the points of intersection

(4) the y-values of the points of intersection

Answer (3)

20 For the sequence $-27, -12, 3, 18, \ldots$, the expression that defines the nth term where $a_1 = -27$ is

(1) $15 - 27n$ (3) $-27 + 15n$

(2) $15 - 27(n - 1)$ (4) $-27 + 15(n - 1)$

Answer (4)

Note: You may want to point out that a_1 is signaling that x = 1. Students should see which function rule produces the same number pattern (sequence) where -27 aligns with x = 1.

23 Which of the three situations given below is best modeled by an exponential function?

I. A bacteria culture doubles in size every day.
II. A plant grows by 1 inch every 4 days.
III. The population of a town declines by 5% every 3 years.

(1) I, only (3) I and II

(2) II, only (4) I and III

Answer (4)

Next are some "gettable" long response questions. We have not spent much time discussing the long response questions. This is because the majority of your time and effort should be spent making sure you are answering multiple choice questions correctly. But long response questions can only help you earn points. There is also partial credit available. If a question says "explain" it means that you must write words. If a question says "justify" it means to use words and/or show **evidence**. The words you write can describe calculator steps and the evidence you show can be tables and sketches of graphs that your calculator produces. You should try your best on the long response, and don't worry if some of them seem challenging to you. These are considered your "reach" questions to get as many points as possible. Try the ones you know and don't get discouraged! Show your thinking on paper.

25 Explain how to determine the zeros of $f(x) = (x + 3)(x - 1)(x - 8)$.

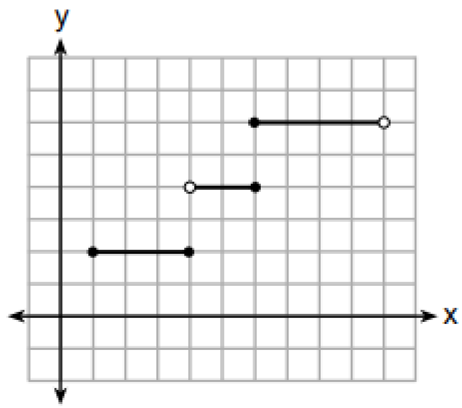
I entered f(x) as Y1 in my calculator and I looked for the x-values when y equals zero in the table.

X	Y
-3	0
1	0
8	0

State the zeros of the function.

The zeros are -3, 1, and 8

26 Four relations are shown below.



I

x	y
–4	1
0	3
4	5
6	6

III

$\{(1,2), (2,5), (3,8), (2,-5), (1,-2)\}$

II

$y = x^2$

IV

State which relation(s) are functions.

III and IV are functions

Explain why the other relation(s) are *not* functions.

I is not a function because the x-value 6 has two y-values, 4 and 6.

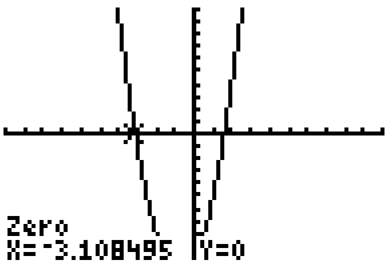
II is not a function because the x-value 2 has two y-values, 5 and -5.

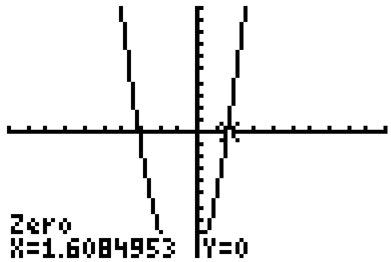
28 Is the solution to the quadratic equation written below rational or irrational? Justify your answer.

$$0 = 2x^2 + 3x - 10$$

Justification: *Rational numbers are numbers than can be written as a ratio or fraction of two whole numbers. They include terminating and repeating decimals.*

Irrational numbers result in decimals that never terminate (end) and never repeat in a pattern.



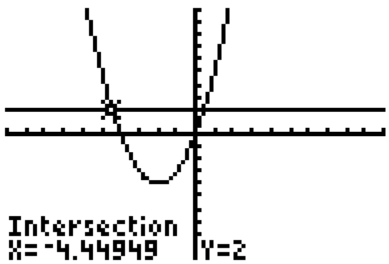


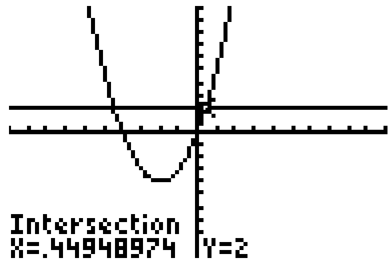
Answer: *These zeros are irrational. A graph sketch with zeros labeled is sufficient evidence.*

30 Solve the following equation by completing the square:

$$x^2 + 4x = 2$$

Note: We are not going to cover how to complete the square. This question was included for you to point out to your students that even if they are unsure about the algebraic process, they can still earn partial credit for showing evidence of a different solution method, such as graphically or with tables.





x = -4.44949 and x = 0.44948974

Note: Sketching a graph and labeling its solutions will earn students half credit.

34 A car was purchased for \$25,000. Research shows that the car has an average yearly depreciation rate of 18.5%.

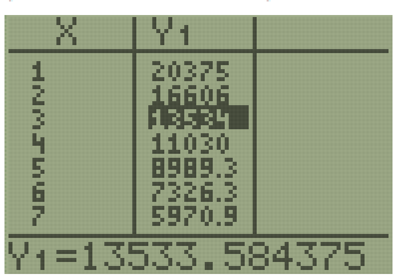
Create a function that will determine the value, $V(t)$, of the car t years after purchase.

Process: The car is losing 18.5% of its value. Another way to think of this is that it is worth

$100\% - 18.5\% = 81.5\%$ of its value each year.

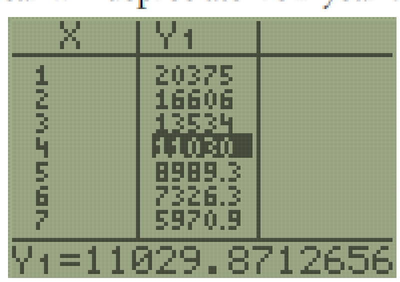
Answer: $\boldsymbol{V(t) = 25000(0.815)^t}$

Determine, to the *nearest cent*, how much the car will depreciate from year 3 to year 4.



X	Y_1
1	20375
2	16606
3	13534
4	11030
5	8989.3
6	7326.3
7	5970.9

Y_1=13533.584375



X	Y_1
1	20375
2	16606
3	13534
4	11030
5	8989.3
6	7326.3
7	5970.9

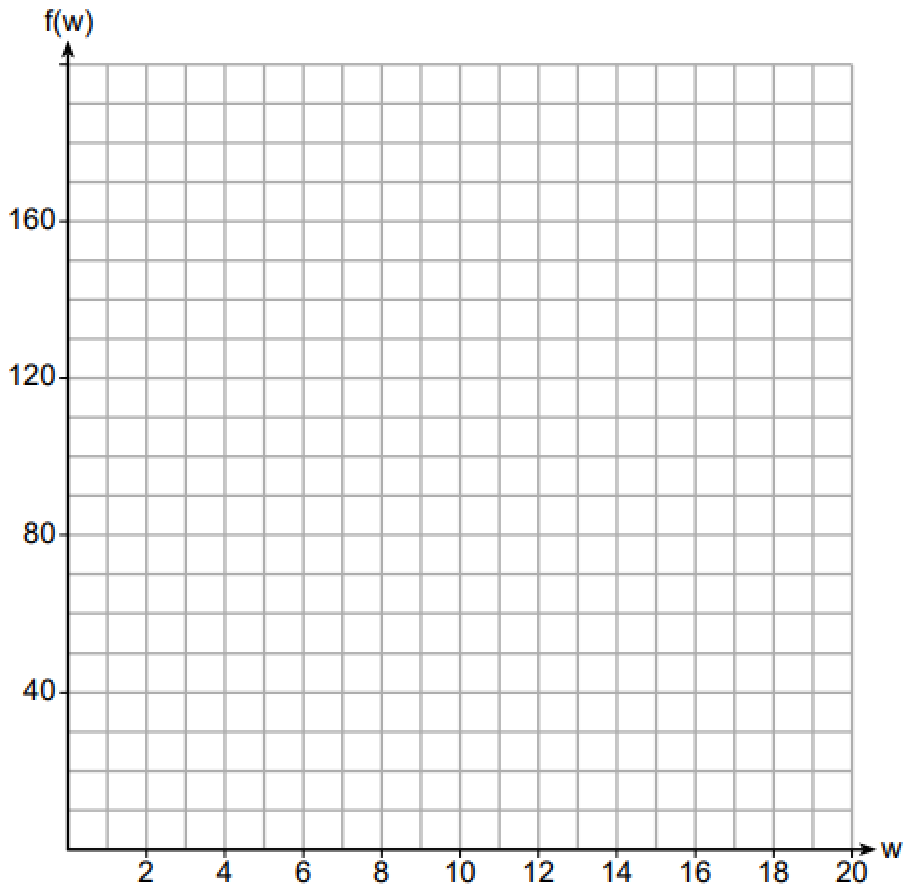
Y_1=11029.8712656

Process: View V(t) in the table. Highlight the y-values for when x = 3 and x = 4 so that you can see values to the nearest cent (second decimal place).

Year 3, the car was worth \$13,533.58. Year 4, the car was worth \$11,029.87. The difference is **Answer: 13533.58 – 11029.87 = \$2,503.71**

36 Paul plans to have a rectangular garden adjacent to his garage. He will use 36 feet of fence to enclose three sides of the garden. The area of the garden, in square feet, can be modeled by $f(w) = w(36 - 2w)$, where w is the width in feet.

On the set of axes below, sketch the graph of $f(w)$.



Explain the meaning of the vertex in the context of the problem.

Note: Encourage students to use the table in order to plot specific points on the paper. They must plot specific points and connect the graph to receive credit.

Students should locate the vertex using the graph or the table. It is the point (9, 162). In this context, it means that when the width of the garden is 9 feet, the area is 162 square feet.

Educator Note: Thank you for being one of the first to field-test the "Regents Prep for Algebra I in Four Sessions" materials. The Statewide Support Team looks forward to hearing your comments and suggestions for improvement.

More in depth study materials to prepare for the ***Algebra I Regents Exam*** are being developed for use in school year 2019.