

Lesson 3 – Piecing it Together

You are becoming more familiar with linear and exponential functions. This lesson, we will introduce two more important families: **quadratic** and **absolute value** functions. Finally, we will learn about **piecewise** functions.

Quadratic Functions

The third major function family, quadratic functions, is also **nonlinear**. Situations that are modeled with quadratic function rules many times involve gravity or motion. Here are a few examples:

- Jonathan kicks a football into the air. The height in feet at any given time t can be modeled by the function rule $h(t) = -16t^2 + 60t + 3$
- A skydiver jumps from a height of 3,500 meters. The total distance fallen at any given time t can be modeled by the function rule $d(t) = 3500 - 9.8t^2$
- A concert venue's revenue depends on the ticket price, x . The concert revenue can be modeled by the quadratic function rule $R(x) = x(200 - 20x)$
- The path of a comet as it slingshots around planet Earth can be modeled by the quadratic function $p(x) = (x - 3)^2 + 4$

What similarities and differences do you notice about these function rules?

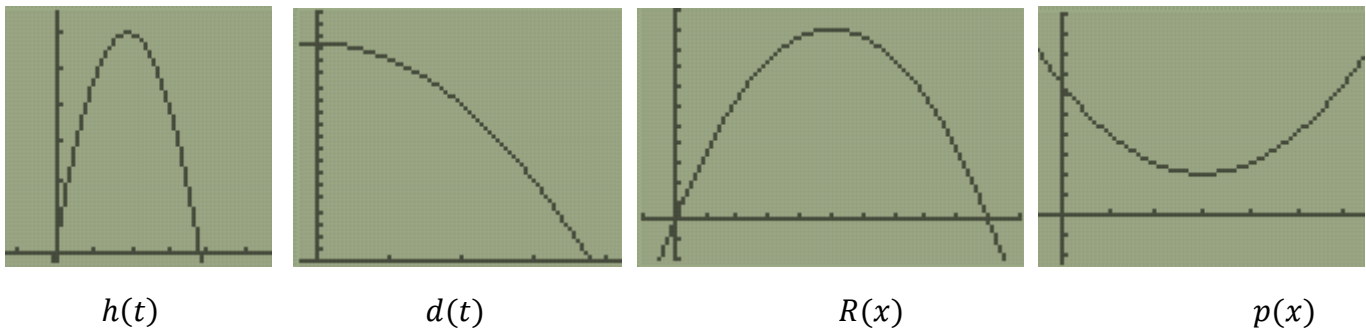
Three rules have an exponent of 2. One rule does not.

Educator Note: Students may point out that when you distribute the x in $R(x)$ that it will in fact have an exponent of 2.

Based on the situations what shape do you predict quadratic graphs to have? Make a sketch.

Students should draw upward or downward facing U-shapes (parabolas).

Quadratic function rules appear in three major forms. We are going to learn how to convert between these forms later, but it makes it more difficult to recognize what exactly makes function rules quadratic. Did you notice how three rules involved an exponent of 2? Quadratic function rules do have x^2 in them. It is less obvious that $R(x) = x(200 - 20x)$ is quadratic, but it is. Let's take a look at each of their graphs.



Which graphs are similar? How are they similar? Which graph is different? How is it different?

Students may point out that d is different because it only appears to decrease, or that p is different because it opens upward.

Students might say that h , R , and p are all similar because they all increase and decrease.

Karla answered the previous question by saying that $d(t)$ is the graph that is different. She said,

"I think $d(t)$ is different because it decreases the whole time. Every other graph has one part that increases and another part that decreases."

Robert and Karla exchange papers and read each other's responses. Robert disagrees with Karla and tells her,

"If you graph it on the left, $d(t)$ is similar to h and R . I think p is the different one because it goes up on both sides."

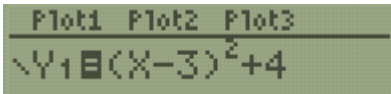
Explain Robert's thinking and expand upon it in your own words. What evidence could Robert have shown Karla to help support his response?

Robert extended the graph on the left side, making it symmetrical to the part that is showing.

Looking at a graph will always reveal whether a rule is quadratic. These U-shaped graphs are called **parabolas**. Notice that it is not obvious that $d(t)$ is a parabola. We will discuss how to adjust the calculator **WINDOW** in greater detail in the future.

Notice that linear and exponential functions either always increase or always decrease from left to right. Parabolas change directions. This turning point is called the **vertex**. If the quadratic graph opens up (shaped like a cup), the vertex is the minimum of the graph. If the parabola opens down (shaped like a frown), the vertex is a maximum point.

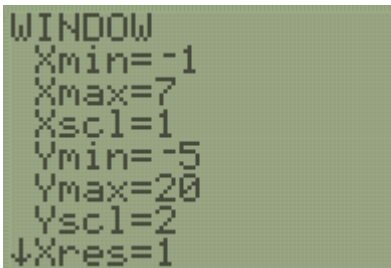
Suppose you want to find the minimum point of $p(x)$.



Press **Y=** and enter the function rule for **Y1**

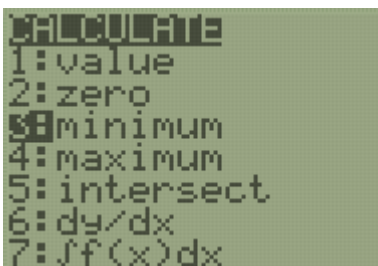
| X | Y1 |
|---|----|
| 0 | 13 |
| 1 | 8 |
| 2 | 5 |
| 3 | 4 |
| 4 | 5 |
| 5 | 8 |
| 6 | 13 |

To get an idea of the correct window settings, press **2nd**, **GRAPH**. Decide on the lowest and highest values for x- and y-values.

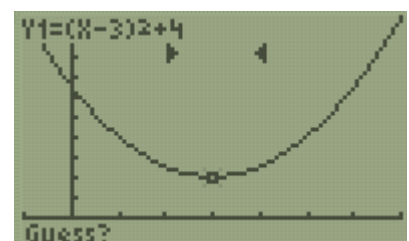
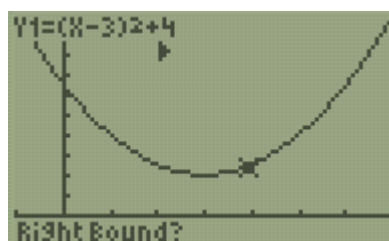
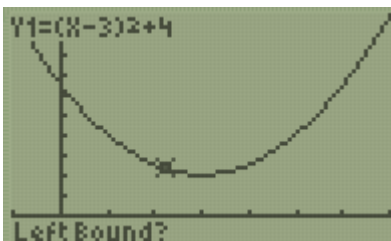


To follow along with this example, press **WINDOW** and enter the following settings.

Are these choices close to the ones you made based on looking at the table?



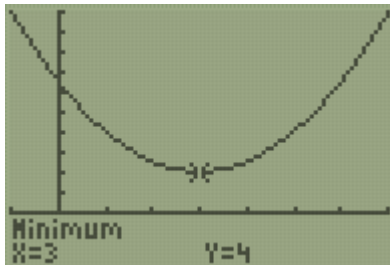
Press **2nd**, **TRACE**, and choose choice **3** for the minimum finder.



Move the cursor using the arrow keys until it is on the left side of the minimum, then press **ENTER**.

Move the cursor to the right side of the minimum and press **ENTER**

Finally, move the cursor to somewhere close to the minimum point and press **ENTER**.



The coordinates of the minimum point of $p(x)$ are $(3, 4)$.

This process works nearly the same for finding a maximum.

Try to find the coordinates of the maximum point on $h(t) = -16t^2 + 60t + 3$. Note that when you put the rule into **Y=** you always use x as your variable.

```
WINDOW
Xmin=-1
Xmax=7
Xscl=1
Ymin=-5
Ymax=65
Yscl=10
↓Xres=1
```

These settings produce a nice viewing window for h . Again, we will discuss strategies for how to do this in the future.

Use these Window settings and write the coordinates of the maximum point of $h(t) = -16t^2 + 60t + 3$.

(1.87, 59.25)

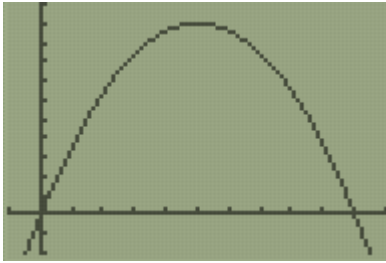
The Regents Exam also loves to ask about the **zeros** of quadratic functions. **Zeros** are x -values for which $y = 0$. Sometimes we can find zeros on a table.

Locate the zeros of $R(x) = x(200 - 20x)$ using your table.

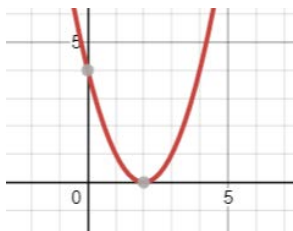
The zeros occur when x is 0 and when x is 10.

From this example, we see that quadratics can have two zeros.

The zeros of a function are the **x-intercepts** of its graph. Notice how the graph of $R(x)$ intersects the x-axis at two locations, $x = 0$ and $x = 10$

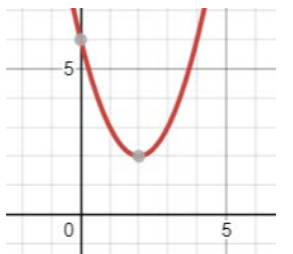


Sketch the graph of a quadratic function that has one zero.



The graph touches the x-axis only once

Sketch the graph of a quadratic function that has no zeros.



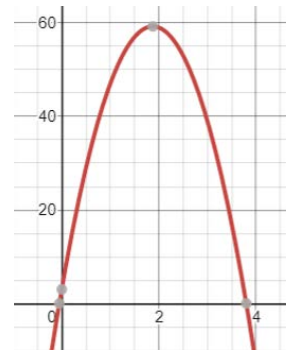
The graph does not intersect the x-axis

Is it possible for a quadratic function to have more than two zeros? Explain your reasoning.

Since a quadratic only changes direction one time, it is only possible for a quadratic function to increase through the x-axis then decrease through the x-axis a second time, or vice versa. Therefore, it is not possible for a quadratic to have more than two zeros.

Consider the function $h(t) = -16t^2 + 60t + 3$. View a graph of this function. How many zeros does h have?

Two zeros



Fill out the table of values for $h(t)$ for integers $x = -2$ through $x = 5$

| x | -2 | -1 | 0 | 1 | 2 | 3 | 4 | 5 |
|--------|------|-----|---|----|----|----|-----|-----|
| $h(x)$ | -181 | -73 | 3 | 47 | 59 | 39 | -13 | -97 |

It's not immediately obvious what exact x -values the zeros are. Between what values of x must the zeros be "hiding"? Explain how you can tell.

The zeros are between -1 and 0 and also between 3 and 4. You can tell because the y -values change between positive and negative at these locations.

This example shows that we can't rely on tables to find the zeros of every quadratic function.

To find the zeros on a graph, press **2nd**, **TRACE**, and choose option **2: zero**.

Then, follow the same process to locate a max or min.

Use the following **WINDOW** for a nice picture of the graph.

```

WINDOW
Xmin=-2
Xmax=6
Xscl=1
Ymin=-5
Ymax=70
Yscl=10
↓Xres=1
  
```

Try this process now to locate the zeros of $h(t) = -16t^2 + 60t + 3$. Note that they will be decimal values.

$x = -0.049$ and $x = 3.799$

Absolute Value Functions

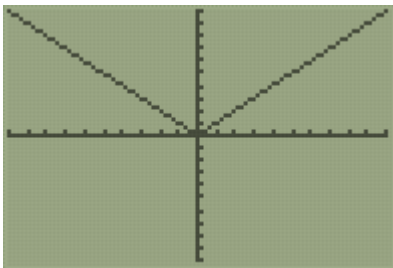
There is another family, in addition to linear, exponential, and quadratic, that is worth knowing – the absolute value function.

The **absolute value** of a number a , written as $|a|$, is that number's distance from 0. Since distance is always positive, the absolute value of a number is always positive.

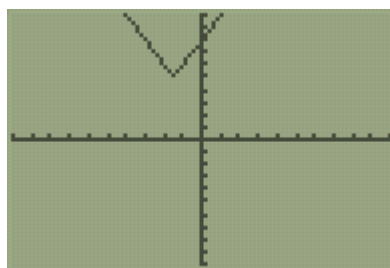
For example, the absolute value of negative four is four because negative four is four units away from zero on a number line. This is written as $|-4| = 4$

Absolute value functions have similarities in the shape of their graphs.

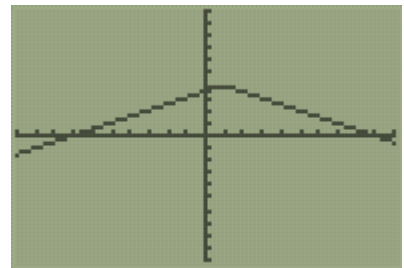
$$y = |x|$$



$$y = |2x + 3| + 5$$



$$y = -0.5|x - 1| + 4$$



Get the absolute value bars in the calculator by pressing **ALPHA, WINDOW, ENTER**.

What are some similarities between each absolute value function rule?

Each rule has the absolute value bars (vertical lines) that are around the variable. Each rule is slightly different based on the numbers that are added, subtracted, or multiplied inside or outside of the absolute value bars.

What similarities do you notice in each graph?

Each graph is “V-shaped.” It is made of two linear sections and has either a minimum or maximum point.

What similarities and differences do you notice between absolute value functions and quadratic function graphs? Rules?

Absolute value and quadratic functions both increase and decrease and have a maximum or minimum point. Absolute value functions change by a constant amount on either side of the min or max whereas quadratic functions do not. Their rules are mostly different as quadratics involve squaring the variable and absolute value functions have their own unique notation.

Domain and Graphing

In your own words, explain the idea of a function's domain and range.

Domain is the set of inputs or x-values for a function.

Range is the set of outputs or y-values for a function.

For many situations, there are only certain input values that make sense. For example, consider the example from the beginning of the lesson involving revenue:

- A concert venue's revenue depends on the ticket price, x . The concert revenue can be modeled by the quadratic function rule $R(x) = x(200 - 20x)$

What does x represent in this situation?

x is the ticket price

What is one value of x that does not make sense for this situation?

Zero or negative values.

Select the inequality that best represents the domain for this situation. Explain your reasoning for your choice.

$$\{x|x = 5\}$$

$$\{x|x > 0\}$$

$$\{x|x \geq 0\}$$

$$\{x|x < 0\}$$

$\{x|x > 0\}$ means "x-values that are greater than zero."

Tyrece is exploring $R(x) = x(200 - 20x)$ using a table of values on his calculator. He notices a pattern and writes it down.

As the ticket price increases, the money or revenue increases for a little while, then it decreases and becomes negative.

Do you agree with the pattern Tyrece noticed?

Students should view $R(x)$ in the table and agree.

Explain why it makes sense in the context of the problem that as ticket price increases, the revenue will eventually decrease.

As ticket price increases, fewer people will purchase tickets and the concert won't make as much money.

Explain why it makes sense in the context of the problem that revenue will eventually be negative.

Eventually the concert venue will not make enough money to cover its costs and will end up owing money to its employees, for its bills, etc.

Tyrece wants to consider the domain of ticket prices for which revenue is positive. Using words, which ticket prices give positive revenues?

All ticket prices between \$0 and \$10

Which inequality below represents the domain you described? Explain your reasoning.

$$\{0 < x < 10\}$$

$$\{x < 10\}$$

$$\{0 > x > 10\}$$

$\{0 < x < 10\}$

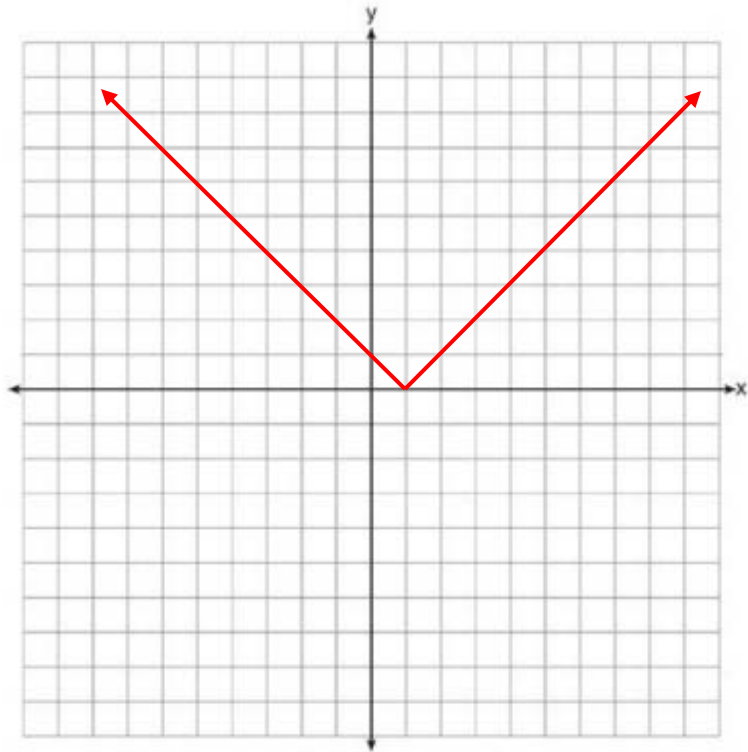
Literally, 0 is less than x , which is less than 10. This means all the x 's between 0 and 10.

Many students struggle to interpret these types of inequalities. To help you, fill in the chart below:

| Inequality | Literally | In my own words |
|-----------------|--|---|
| $0 < x < 10$ | Zero is less than x which is less than ten | All the x-values between zero and ten, not including zero or ten |
| $2 \leq x < 5$ | 2 is less than or equal to x which is less than 5 | All the x's between 2 and 5, including 2 but not including 5 |
| $-1 < x \leq 2$ | -1 is less than x which is less than or equal to 2 | All the numbers between -1 and 2 not including -1 and including 2 |
| $x > 10$ | x is greater than 10 | All the numbers more than 10 but not 10 itself |
| $4 < x \leq 7$ | 4 is less than x which is less than or equal to 7 | All x-values between 4 and 7, not including 4 but including 7 |
| $x < 8$ | x is less than 8 | All x-values smaller than 8 |

Complete the following Regents question.

On the set of axes below, graph the function $y = |x + 1|$.



State the range of the function. State the domain over which the function is increasing.

Range: $\{y \geq 0\}$

The function is increasing for the domain $\{x > -1\}$

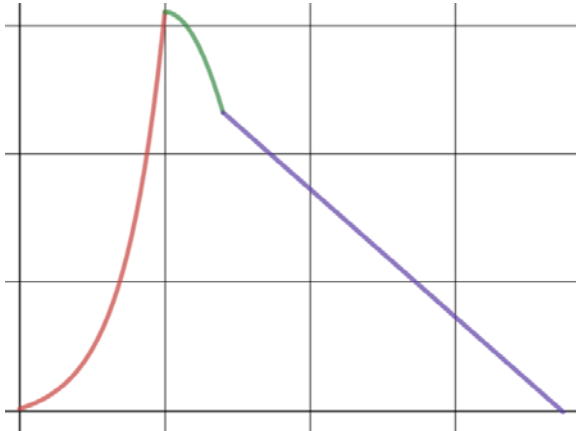
Piecewise Functions

Brendalee receives a model rocket kit for her birthday. The rocket has a GPS signal that allows her to track its distance from the ground using a phone app.

- For the first five seconds, the engine causes the rocket's distance from the ground in meters to increase according to the exponential function $y = 5(2)^t - 4.5$.
- When the fuel runs out, the rocket free falls toward the ground for two seconds. Its distance from the ground decreases according to the quadratic function $y = -9.8(t - 5)^2 + 155.5$
- Finally, the parachute opens, and the rocket's distance from the ground decreases at a constant rate of -10 meters per second according to the linear function $y = 186.3 - 10t$ until the rocket hits the ground.



Let $D(t)$ represent the distance from the ground of the rocket for the entire time between launch to when it hits the ground. Sketch what you think the graph of $D(t)$ looks like.



Encourage students to identify whether each function rule is quadratic, exponential, or linear. The goal is not for them to make an accurate graph of individual points, but to think about a general graph shape. This is difficult, so it's okay if they are not able to visualize this graph shape right away.

During which times will Brendalee's rocket follow the exponential function rule? The quadratic function rule? The linear function rule? Write inequalities to represent each of these intervals.

From 0 to 5 seconds, $0 < t < 5$, the rocket will follow the exponential rule $y = 5(2)^t - 4.5$

From 5 to 7 seconds, $5 < t < 7$, the rocket will follow the quadratic rule $y = -9.8(x - 5)^2 + 155.5$

From times more than 7 seconds, $t > 7$, the rocket will follow the linear rule $y = 186.3 - 10x$

Brendalee uses a **piecewise function** to define $D(t)$.

$$D(t) = \begin{cases} 5(2)^t - 4.5 & 0 \leq t \leq 5 \\ -9.8(t - 5)^2 + 155.5 & 5 < t \leq 7 \\ 186.3 - 10t & 7 < t < 18.6 \end{cases}$$

Explain what this notation means in your own words.

The function rule is on the left and when to use it is on the right. Use the top rule when t is between 0 and 5 including 0 and 5. Use the middle rule when t is between 5 and 7. Use the bottom rule when t is between 7 and 18.6

According to the piecewise function Brendalee wrote, which of the three function rules should be applied when $t = 2$? What about when $t = 6$? How do you know?

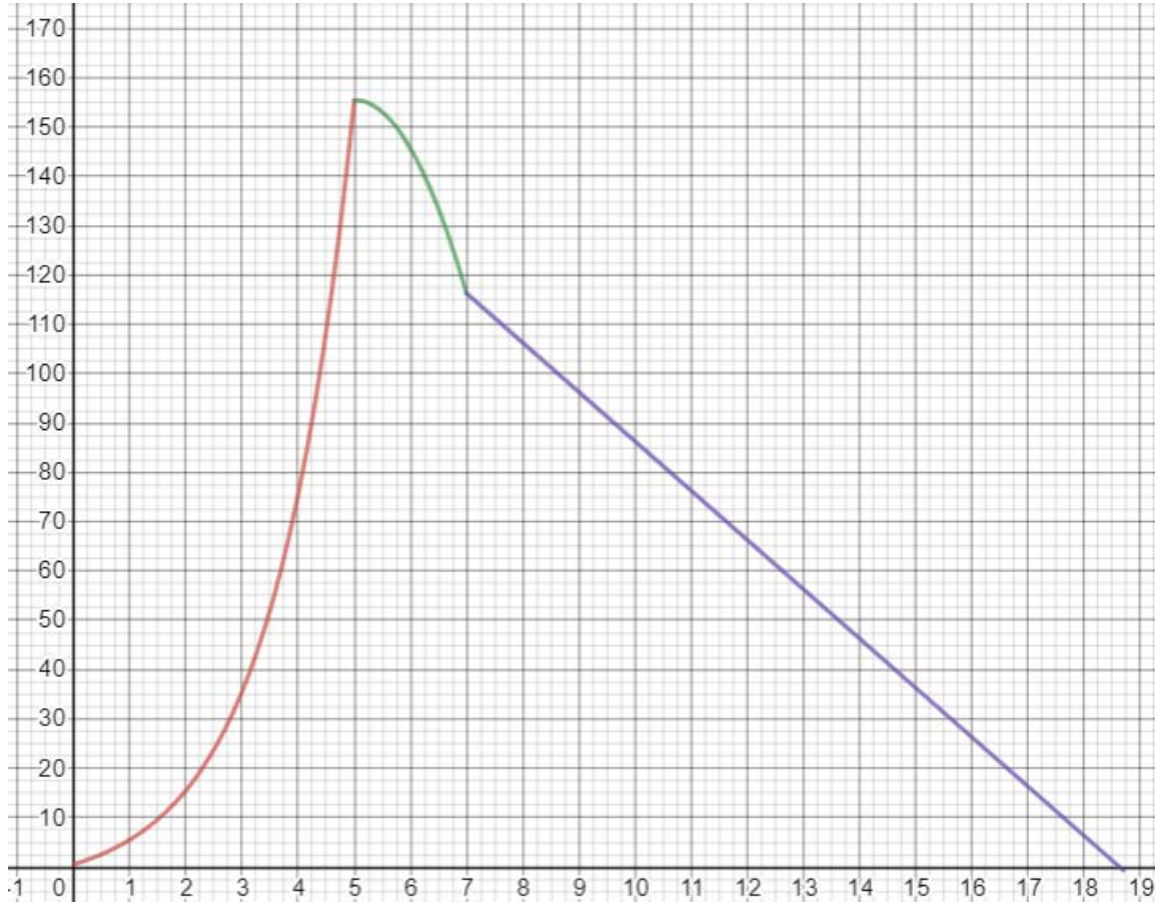
When $t = 2$, Brendalee should use the top rule $5(2)^t - 4.5$.

When $t = 6$, she should use the middle rule, $-9.8(t - 5)^2 + 155.5$

What is the domain of $D(t)$?

$$\mathbf{0 \leq t \leq 18.6}$$

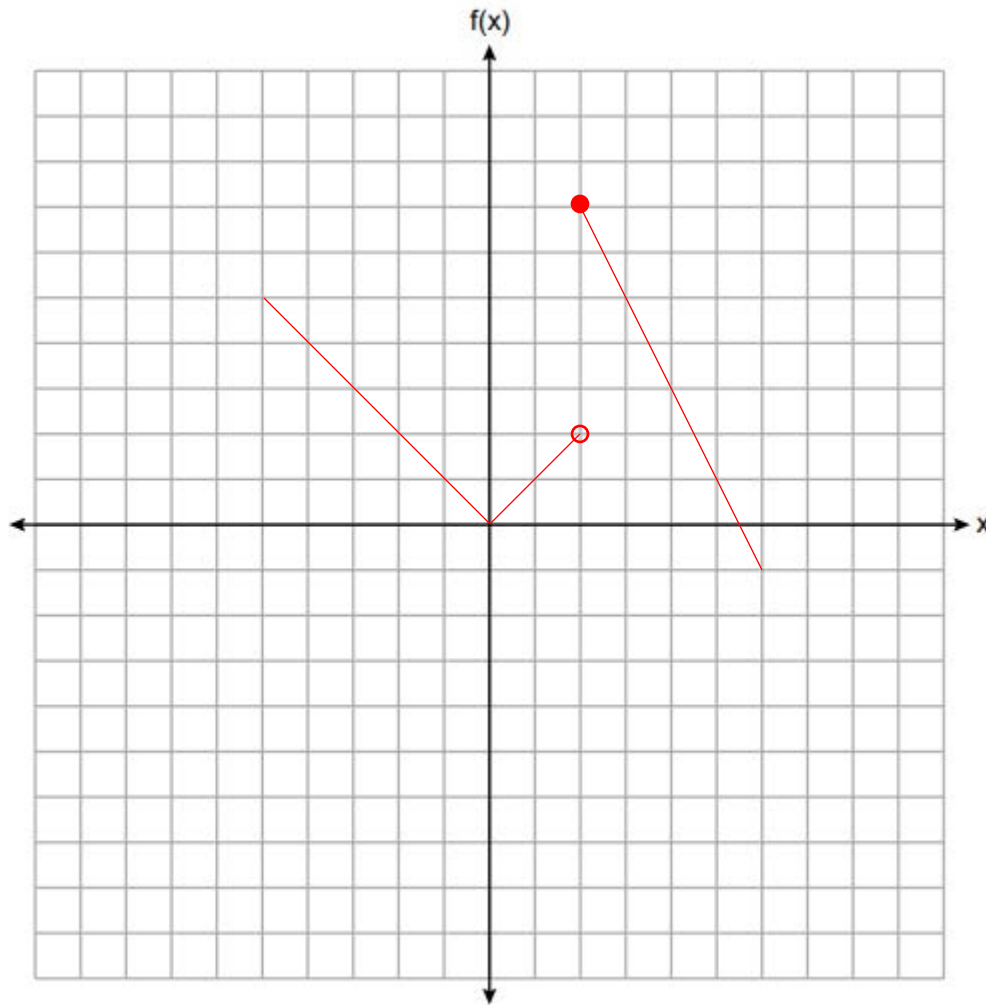
Make a graph of $D(t)$ below.



Did you notice that each rule matched up to form one continuous graph? Not all piecewise functions do this.

27 Graph the following piecewise function on the set of axes below.

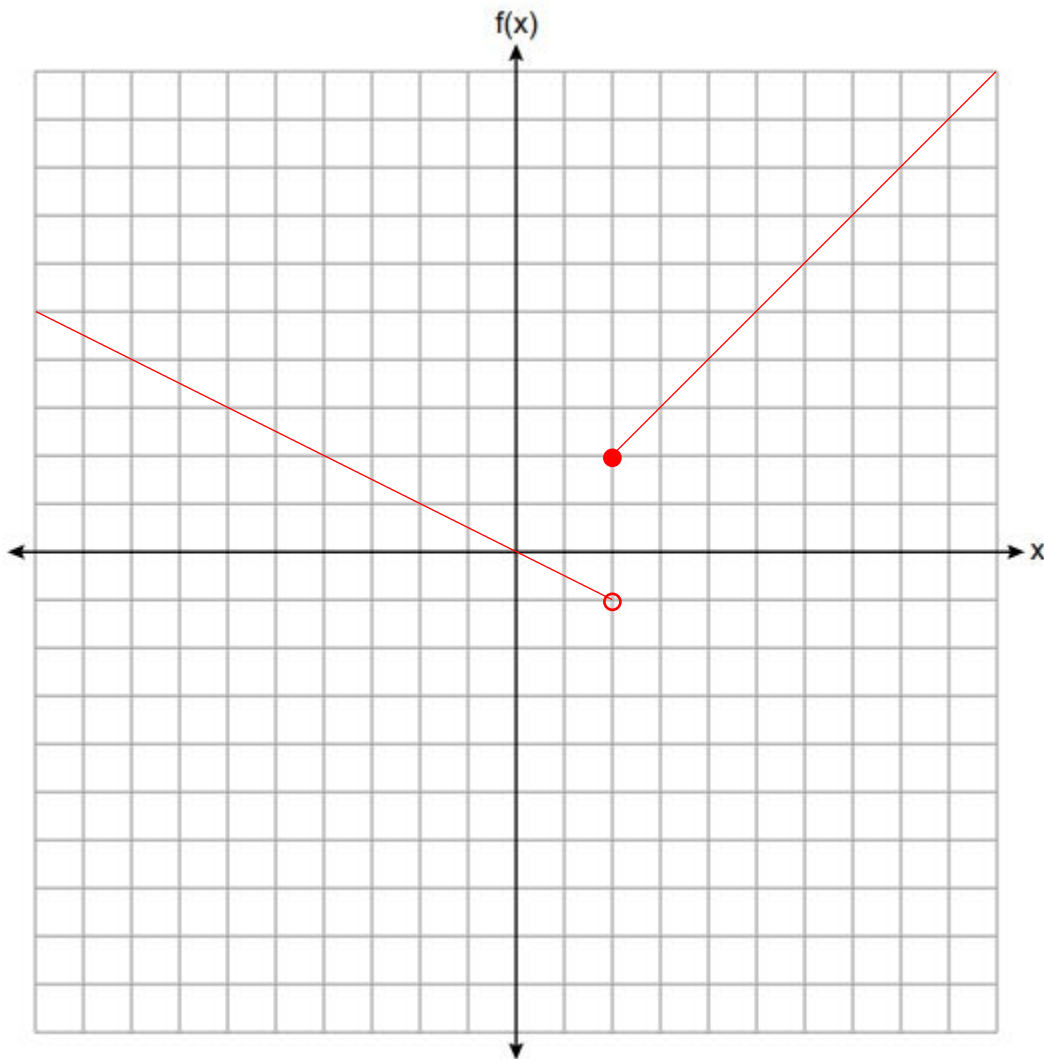
$$f(x) = \begin{cases} |x|, & -5 \leq x < 2 \\ -2x + 10, & 2 \leq x \leq 6 \end{cases}$$



Educator Note: Notice that when graphing “ $-5 \leq x < 2$ ” there is an open circle for point (2, 2) because x is less than 2, but not equal to 2.

32 On the set of axes below, graph the piecewise function:

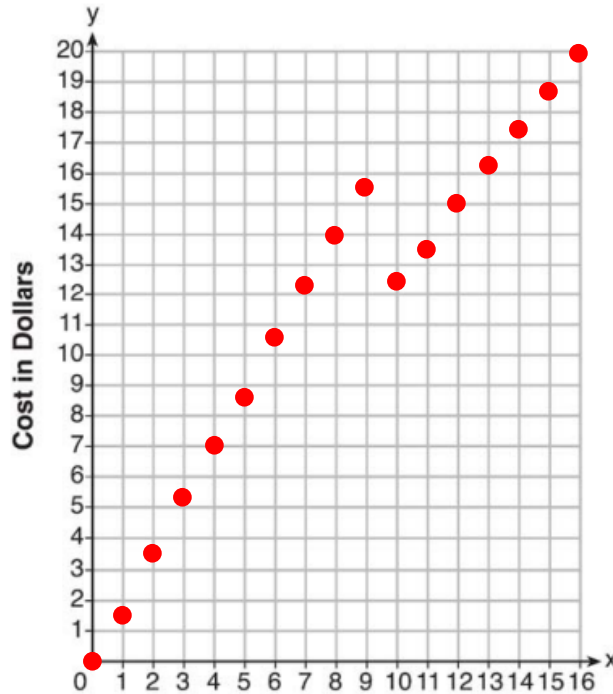
$$f(x) = \begin{cases} -\frac{1}{2}x, & x < 2 \\ x, & x \geq 2 \end{cases}$$



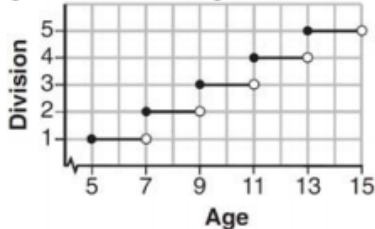
At an office supply store, if a customer purchases fewer than 10 pencils, the cost of each pencil is \$1.75. If a customer purchases 10 or more pencils, the cost of each pencil is \$1.25. Let c be a function for which $c(x)$ is the cost of purchasing x pencils, where x is a whole number.

$$c(x) = \begin{cases} 1.75x, & \text{if } 0 \leq x \leq 9 \\ 1.25x, & \text{if } x \geq 10 \end{cases}$$

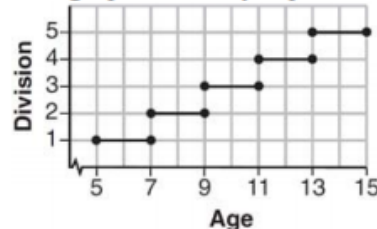
Create a graph of c on the axes below.



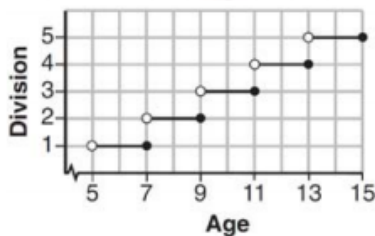
Morgan can start wrestling at age 5 in Division 1. He remains in that division until his next odd birthday when he is required to move up to the next division level. Which graph correctly represents this information?



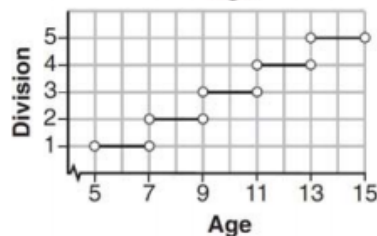
a.



c.



b.



d.

Choice a