Topic 3 – Function Families

Congratulations! You are nearing the end of this four topic series. This lesson will aim to deepen your understanding of functions by grouping them into three main "families."

Think about your own family. Everyone in your family has things about them that make them unique. But now think about what makes your family similar. Maybe it's the way you speak, or dress, or behave that makes your family similar.

Just like you, certain functions belong to a family. In Algebra I, we focus on three main families: linear, exponential, and quadratic.

Linear Functions

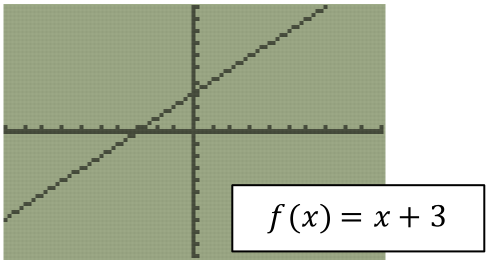
The following four situations are examples of linear functions.

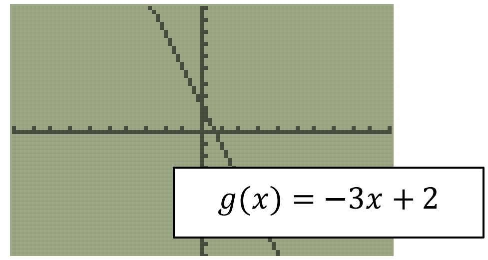
- Pedro earns $12 per hour at his job.
- A plant's height increases 3 inches per week.
- Water is emptying out of a bathtub such that its volume is decreasing at a rate of 2 gallons per minute.
- A car's distance traveled is changing at a constant speed of 55 miles per hour.

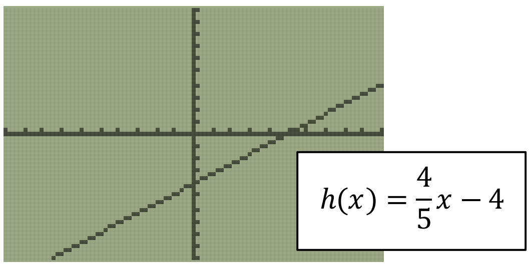
Write all the ways these situations are similar and how they are different.

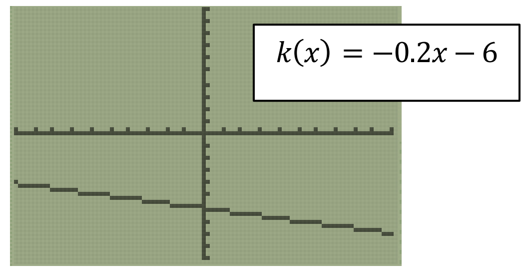
Similar	**Different**

The following graphs and rules are linear functions.









What is similar about the graphs of linear functions? What are some differences between the graphs?

Similar	**Different**

How are the linear function rules similar to one another? How are the linear function rules different?

Similar	**Different**

Use your calculator the fill in tables for f, g, h, and k from $x = -5$ to $x = 5$.

x	$f(x) = x + 3$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

x	$g(x) = -3x + 2$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

x	$h(x) = \frac{4}{5}x - 4$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

?	$f(x) = -0.2x - 6$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

What similarities and differences do you notice about the tables of linear function rules?

Similar

Different

Every linear function rule can be written in the form

$$y = mx + b$$

The *m* and *b* represent numbers. Here are important ideas about them that you may have noticed from the similarities and differences work you just did.

m

- Also called the rate of change.
- It is the slope of the graph.
- How much the *y*-value increases or decreases by when the *x*-value increases by 1
- When *m* is positive, the graph increases as *x* increases from left to right.
- When *m* is negative, the graph decreases as *x* increases from left to right.

b

- The value of *y* when *x* = 0.
- It is where the graph intersects the *y*-axis.
- The starting amount for linear situations.

Exponential Functions

Exponential functions are a type of nonlinear function, because their patterns do not result in straight lines. The following situations are examples of exponential functions.

A. 500 bacteria double every hour.

B. A car bought for $25,000 is worth half of its value each year.

C. A savings account has $1000 initially and gains 5% interest annually.

D. A school with 900 students is decreasing its enrollment by 2% each year.

What do each of these situations have in common? How are they different?

Same	Different

Here are rules for each of the situations A through D.

$$A(x) = 500(2)^x$$

$$B(x) = 25000\left(\frac{1}{2}\right)^x$$

$$C(x) = 1000(1.05)^x$$

$$D(x) = 900(0.98)^x$$

How are these exponential rules similar? How are they different?

Similar	**Different**

What connections do you see between the situation described and its function rule?

A.

B.

C.

D.

Here are tables for each of the rules.

$A(x) = 500(2)^x$

X	Y1
0	500
1	1000
2	2000
3	4000
4	8000
5	16000
6	32000

$B(x) = 25000\left(\frac{1}{2}\right)^x$

X	Y1
0	25000
1	12500
2	6250
3	3125
4	1562.5
5	781.25
6	390.63

$C(x) = 1000(1.05)^x$

X	Y1
0	1000
1	1050
2	1102.5
3	1157.6
4	1215.5
5	1276.3
6	1340.1

$D(x) = 900(0.98)^x$

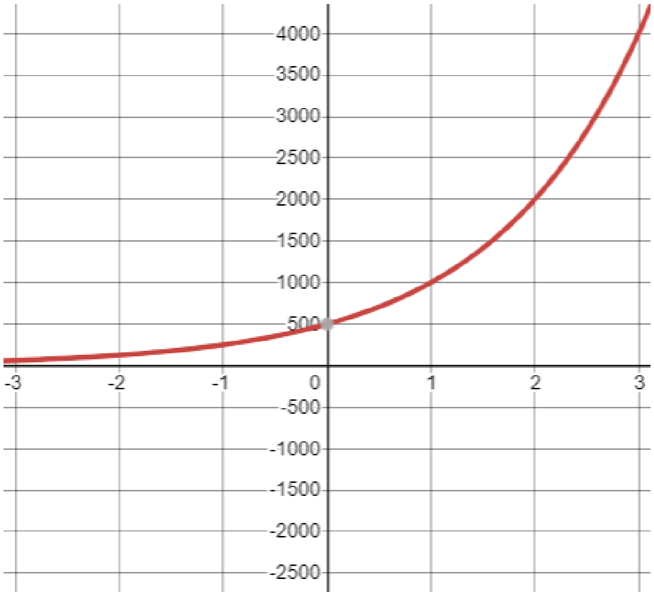
X	Y1
0	900
1	882
2	864.36
3	847.07
4	830.13
5	813.53
6	797.26

What similarities and differences do you notice between the exponential patterns in the tables?

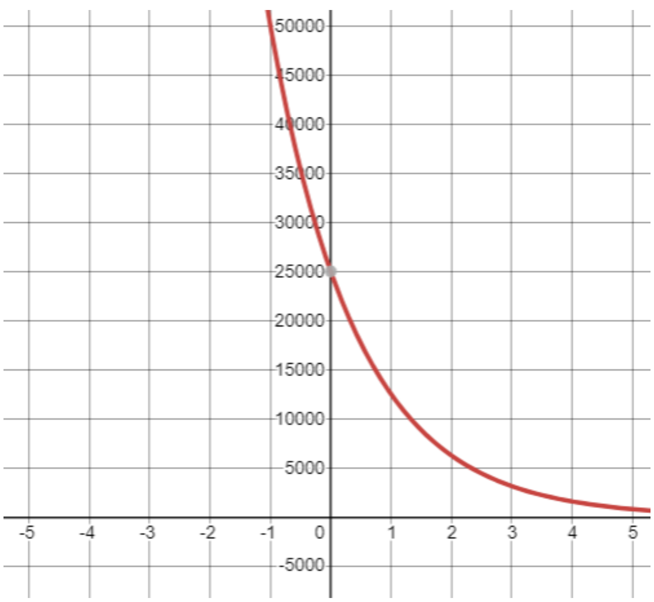
Similar	**Different**

What connections do you notice about the function rule and the table of values?

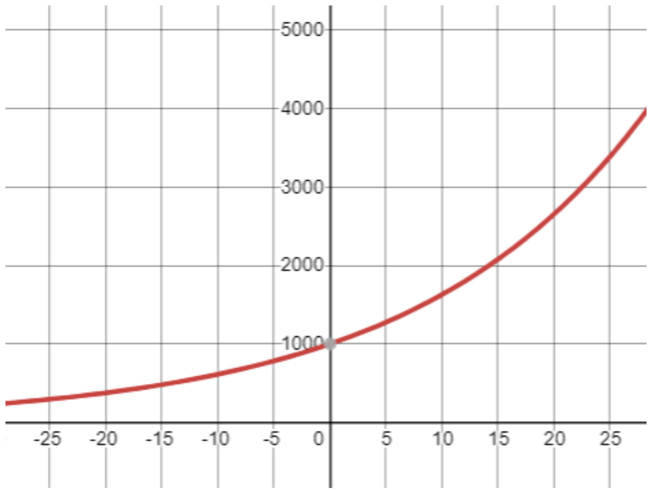
Here are graphs of each exponential function.



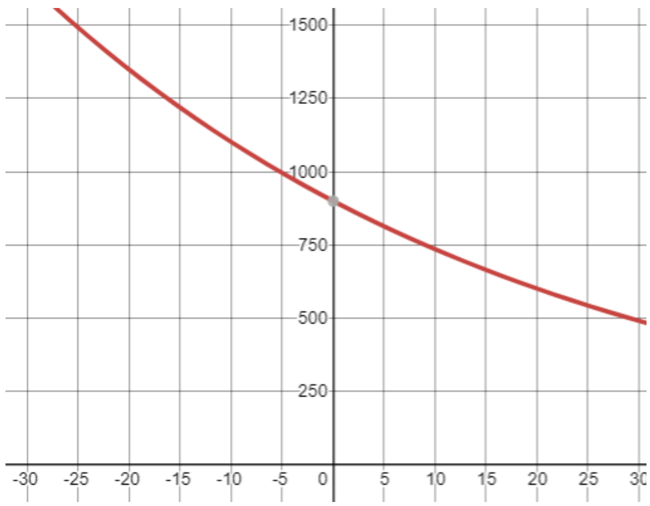
$$A(x) = 500(2)^x$$



$$B(x) = 25\,000\left(\frac{1}{2}\right)^x$$



$C(x) = 1000(1.05)^x$



$D(x) = 900(0.98)^x$

What are some similarities and differences you notice about how each graph is shaped?

Similar	**Different**

How do the exponential graphs compare to the linear graphs?

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What are some observations you can make about the how the exponential function rules and their graphs are related?

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Every exponential function rule can be written in the form

$$y = a(b)^x$$

Where a and b are numbers. Here are some important ideas about how these numbers affect the behavior of tables and graphs of exponential function rules.

a

- This is the initial (starting) value of the function.
- On a table, this is the *y*-value when *x* = 0.
- On a graph, this is where the graph will intersect the y-axis (vertical axis).

b

- This is the multiplier of the function. If the function is growing or shrinking by a percent, *b* will be equal to $1 + \frac{\%}{100}$ or $1 - \frac{\%}{100}$
- To get the next *y*-value in the table, multiply the current *y*-value by *b*.
- If *b* is greater than 1, the graph will be increasing from left to right.
- If *b* is between 0 and 1, the graph will be decreasing from left to right.

Quadratic Functions

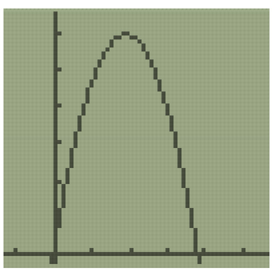
The third major function family, quadratic functions, are also nonlinear. Situations that are modeled with quadratic function rules typically involve gravity or motion. For example:

- Jonathan kicks a football into the air. The height in feet at any given time (*t*) can be modeled by the function rule $h(t) = -16t^2 + 60t + 3$.
- A skydiver jumps from a height of 3,500 meters. The total distance fallen at any given time (*t*) can be modeled by the function rule $d(t) = 3500 - 9.8t^2$.
- A concert venue's revenue depends on the ticket price, *x*. The concert revenue can be modeled by the quadratic function rule $R(x) = x(200 - 20x)$.
- The path of a comet as it slingshots around planet Earth can be modeled by the quadratic function $p(x) = (x - 3)^2 + 4$.

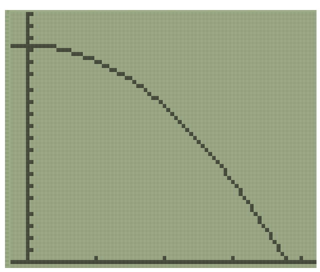
What similarities and differences do you notice about these function rules?

Similar	**Different**

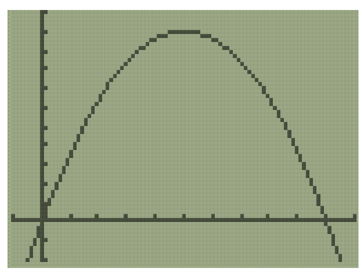
Quadratic function rules appear in three major forms. We are not going to focus on how to convert between these forms, but it makes it more difficult to recognize what exactly makes function rules quadratic. Did you notice how three rules involved an exponent of 2? Quadratic function rules do have x^2 in them. It is less obvious that $R(x) = x(200 - 20x)$ is quadratic, but it is. Let's take a look at each of their graphs.



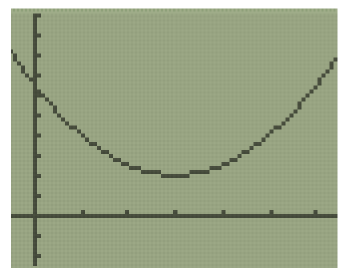
$h(t)$



$d(t)$



$R(x)$



$p(x)$

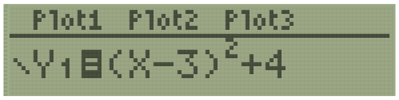
Which graphs are similar? How are they similar? Which graph is different? How is it different?

Similar	**Different**

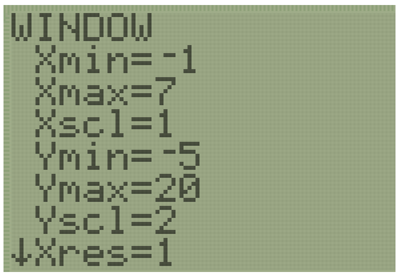
Looking at a graph will always reveal whether a rule is quadratic. These U-shaped graphs are called **parabolas**. Notice that it is not obvious that $d(t)$ is a parabola. Next lesson, we will discuss how to adjust the calculator WINDOW in greater detail.

Notice that linear and exponential functions either always increase or always decrease from left to right. Parabolas change directions. This turning point is called the **<u>vertex</u>**. If the quadratic graph opens up (shaped like a cup), the vertex is the minimum of the graph. If the parabola opens down (shaped like a frown), the vertex is a maximum point.

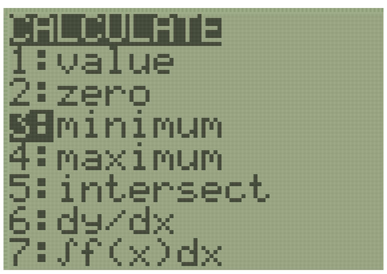
Suppose you want to find the minimum point of $p(x)$.



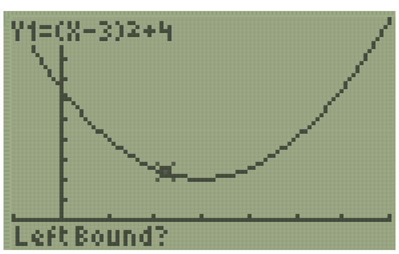
Press Y= and enter the function rule for Y1.

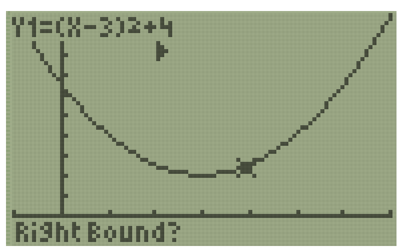


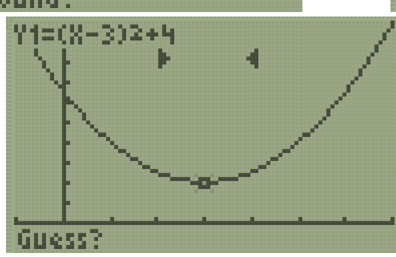
To follow along with this example, press WINDOW and enter the following settings.



Press **2nd**, **TRACE**, and choose choice **3** for the minimum finder.



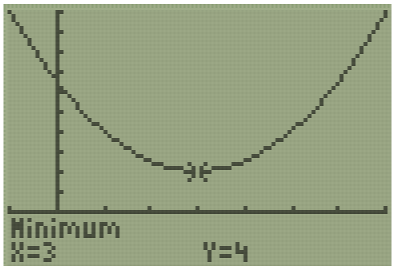




Move the cursor using the arrow keys until it is on the left side of the minimum, then press **ENTER**.

Move the cursor to the right side of the minimum and press **ENTER.**

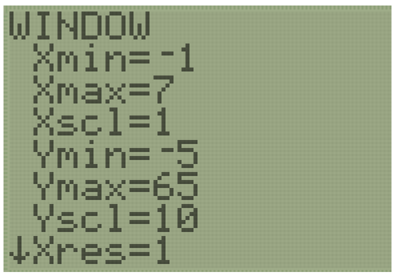
Finally, move the cursor to somewhere close to the minimum point an press **ENTER.**



The coordinates of the minimum point of $p(x)$ are $(3{,}4)$.

This process works nearly the same for finding a maximum.

Try to find the coordinates of the maximum point on $h(t) = -16t^2 + 60t + 3$. Note that when you put the rule into **Y=** you always use *x* as your variable.



These settings produce a nice viewing window for *h*. Again, we will discuss strategies for how to do this next lesson.

Use these Window settings and write the coordinates of the maximum point of $h(t) = -16t^2 + 60t + 3$.

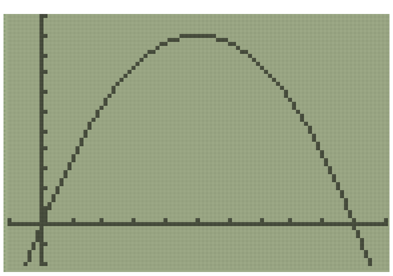
The Regents exam also loves to ask about the **zeros** of quadratic functions. **Zeros** are *x*-values for which *y* = 0. Sometimes we can find zeros on a table.

Locate the zeros of $R(x) = x(200 - 20x)$ using your table.

Hopefully you noticed that the zeros are *x* = 0 and *x* = 10.

From this example, we see that quadratics can have two zeros.

The zeros of a function are the ***x*-intercepts** of its graph. Notice how the graph of $R(x)$ intersects the *x*-axis at two locations, *x* = 0 and *x* = 10

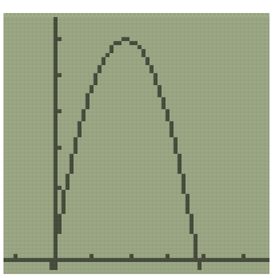


Sketch the graph of a quadratic function that has one zero.

Sketch the graph of a quadratic function that has no zeros.

Is it possible for a quadratic function to have more than two zeros? Explain your reasoning.

Consider the function $h(t) = -16t^2 + 60t + 3$. By looking at its graph, we know that $h(t)$ has two zeros.



However, when we examine a table of $h(t)$ values, the location of the zeros are not obvious.

Fill out the table of values for $h(t)$ for integers x = -2 through x = 5

x	-2	-1	0	1	2	3	4	5
$h(x)$								

Between what values of x must the zeros be "hiding"? Explain how you can tell.

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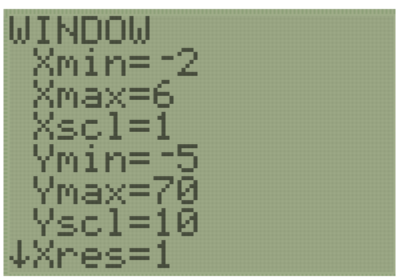
This example shows that we can't rely on tables to find the zeros of every quadratic function.

To find the zeros on a graph, press **2nd**, **TRACE**, and choose option **2: zero**.

Then, follow the same process to locate a max or min.

Use the following WINDOW for a nice picture of the graph.



Try this process now to locate the zeros of $h(t) = -16t^2 + 60t + 3$. Note that they will be decimal values.

Great job so far! You have made it through the third topic. Next, particular questions have been chosen from the August 2018 Regents exam that you should be able to correctly answer by applying strategies and ideas from each lesson so far. Some questions may look a little different than what you have experienced so far. That's okay; use your strategies and what you know to pick answers that make the most sense. You will notice that not all questions from the test are here. Next lesson, you will be expected to decide which questions to leave out. Good luck, and remind yourself that you only need at least ten correct answers! **You can do this!!**

1 The number of bacteria grown in a lab can be modeled by $P(t) = 300 \bullet 2^{4t}$, where t is the number of hours. Which expression is equivalent to $P(t)$?

(1) $300 \bullet 8^t$ (3) $300^t \bullet 2^4$

(2) $300 \bullet 16^t$ (4) $300^{2t} \bullet 2^{2t}$

2 During physical education class, Andrew recorded the exercise times in minutes and heart rates in beats per minute (bpm) of four of his classmates. Which table best represents a linear model of exercise time and heart rate?

Student 1

Exercise Time (in minutes)	Heart Rate (bpm)
0	60
1	65
2	70
3	75
4	80

(1)

Student 3

Exercise Time (in minutes)	Heart Rate (bpm)
0	58
1	65
2	70
3	75
4	79

(3)

Student 2

Exercise Time (in minutes)	Heart Rate (bpm)
0	62
1	70
2	83
3	88
4	90

(2)

Student 4

Exercise Time (in minutes)	Heart Rate (bpm)
0	62
1	65
2	66
3	73
4	75

(4)

3 David correctly factored the expression $m^2 - 12m - 64$. Which expression did he write?

(1) $(m - 8)(m - 8)$ (3) $(m - 16)(m + 4)$

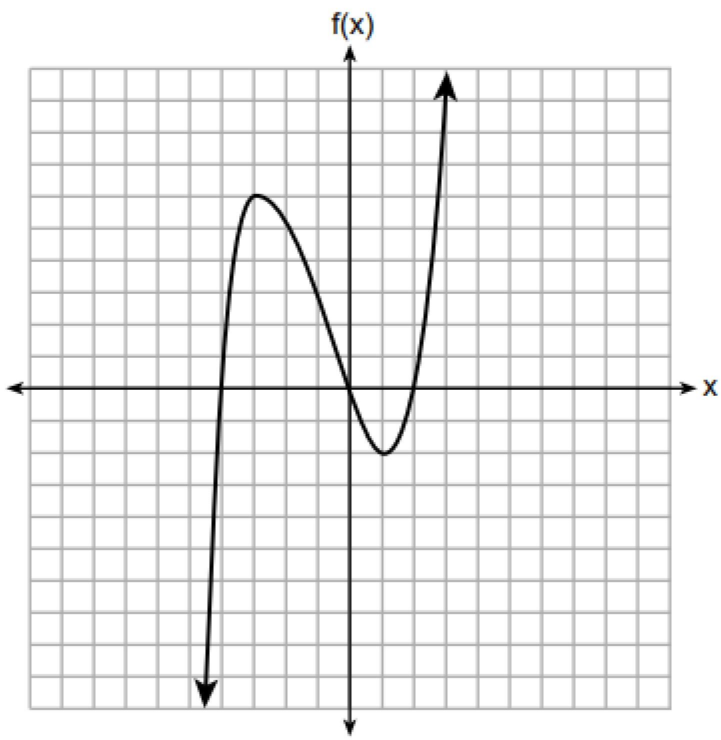
(2) $(m - 8)(m + 8)$ (4) $(m + 16)(m - 4)$

4 The solution to $-2(1 - 4x) = 3x + 8$ is

(1) $\frac{6}{11}$ (3) $-\frac{10}{7}$

(2) 2 (4) -2

5 The graph of $f(x)$ is shown below.



What is the value of $f(-3)$?

(1) 6 (3) -2

(2) 2 (4) -4

6 If the function $f(x) = x^2$ has the domain $\{0, 1, 4, 9\}$, what is its range?

(1) $\{0, 1, 2, 3\}$ (3) $\{0, -1, 1, -2, 2, -3, 3\}$

(2) $\{0, 1, 16, 81\}$ (4) $\{0, -1, 1, -16, 16, -81, 81\}$

7 The expression $4x^2 - 25$ is equivalent to

(1) $(4x - 5)(x + 5)$ (3) $(2x + 5)(2x - 5)$

(2) $(4x + 5)(x - 5)$ (4) $(2x - 5)(2x - 5)$

8 Compared to the graph of $f(x) = x^2$, the graph of $g(x) = (x - 2)^2 + 3$ is the result of translating $f(x)$

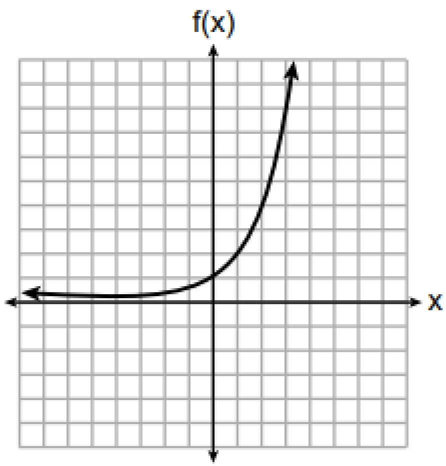
(1) 2 units up and 3 units right

(2) 2 units down and 3 units up

(3) 2 units right and 3 units up

(4) 2 units left and 3 units right

11 Three functions are shown below.



$g(x) = 3^x + 2$

x	h(x)
–5	30
–4	14
–3	6
–2	2
–1	0
0	–1
1	–1.5
2	–1.75

Which statement is true?

(1) The y-intercept for $h(x)$ is greater than the y-intercept for $f(x)$.

(2) The y-intercept for $f(x)$ is greater than the y-intercept for $g(x)$.

(3) The y-intercept for $h(x)$ is greater than the y-intercept for both $g(x)$ and $f(x)$.

(4) The y-intercept for $g(x)$ is greater than the y-intercept for both $f(x)$ and $h(x)$.

13 If $y = 3x^3 + x^2 - 5$ and $z = x^2 - 12$, which polynomial is equivalent to $2(y + z)$?

(1) $6x^3 + 4x^2 - 34$

(2) $6x^3 + 3x^2 - 17$

(3) $6x^3 + 3x^2 - 22$

(4) $6x^3 + 2x^2 - 17$

16 If $f(x) = 2x^2 + x - 3$, which equation can be used to determine the zeros of the function?

(1) $0 = (2x - 3)(x + 1)$
(3) $0 = 2x(x + 1) - 3$
(2) $0 = (2x + 3)(x - 1)$
(4) $0 = 2x(x - 1) - 3(x + 1)$

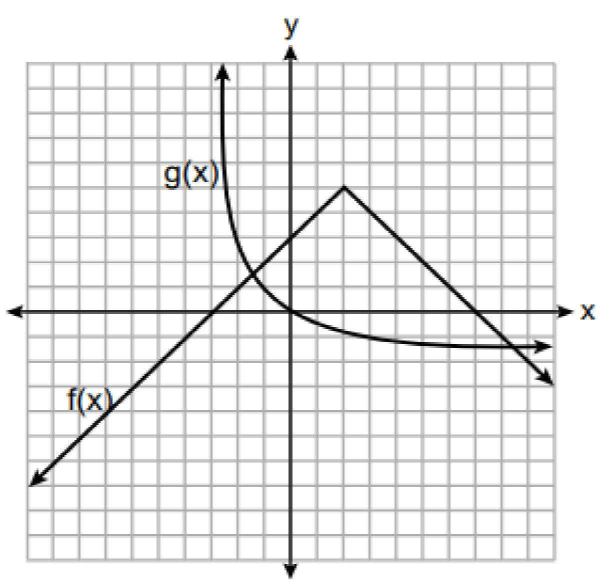
17 Each day, a local dog shelter spends an average of \$2.40 on food per dog. The manager estimates the shelter's daily expenses, assuming there is at least one dog in the shelter, using the function $E(x) = 30 + 2.40x$.

Which statements regarding the function $E(x)$ are correct?

I. x represents the number of dogs at the shelter per day.
II. x represents the number of volunteers at the shelter per day.
III. 30 represents the shelter's total expenses per day.
IV. 30 represents the shelter's nonfood expenses per day.

(1) I and III
(3) II and III
(2) I and IV
(4) II and IV

19 The functions $f(x)$ and $g(x)$ are graphed below.



Based on the graph, the solutions to the equation $f(x) = g(x)$ are

(1) the x-intercepts
(2) the y-intercepts
(3) the x-values of the points of intersection
(4) the y-values of the points of intersection

20 For the sequence $-27, -12, 3, 18, \ldots$, the expression that defines the nth term where $a_1 = -27$ is

(1) $15 - 27n$

(2) $15 - 27(n - 1)$

(3) $-27 + 15n$

(4) $-27 + 15(n - 1)$

23 Which of the three situations given below is best modeled by an exponential function?

I. A bacteria culture doubles in size every day.
II. A plant grows by 1 inch every 4 days.
III. The population of a town declines by 5% every 3 years.

(1) I, only

(2) II, only

(3) I and II

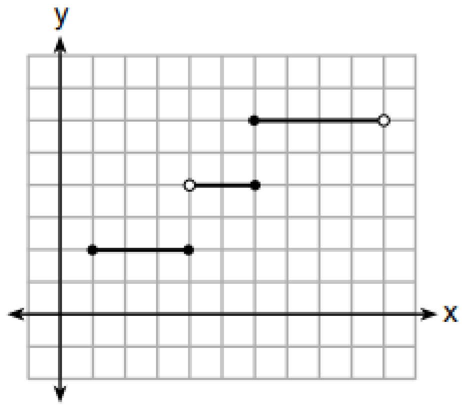
(4) I and III

Next are some "gettable" long response questions. We have not spent much time discussing the long response questions. This is because the majority of your time and effort should be spent making sure you are answering multiple choice questions correctly. But long response questions can only help you earn points. There is also partial credit available. If a question says "explain" it means that you must write words. If a question says "justify" it means to use words and/or show **evidence**. The words you write can describe calculator steps and the evidence you show can be tables and sketches of graphs that your calculator produces. You should try your best on the long response, and don't worry if some of them seem challenging to you. These are considered your "reach" questions to get as many points as possible. Try the ones you know and don't get discouraged! Show your thinking on paper.

25 Explain how to determine the zeros of $f(x) = (x + 3)(x - 1)(x - 8)$.

State the zeros of the function.

26 Four relations are shown below.



I

x	y
–4	1
0	3
4	5
6	6

III

$\{(1,2), (2,5), (3,8), (2,-5), (1,-2)\}$

II

$y = x^2$

IV

State which relation(s) are functions.

Explain why the other relation(s) are *not* functions.

30 Solve the following equation by completing the square:

$$x^2 + 4x = 2$$

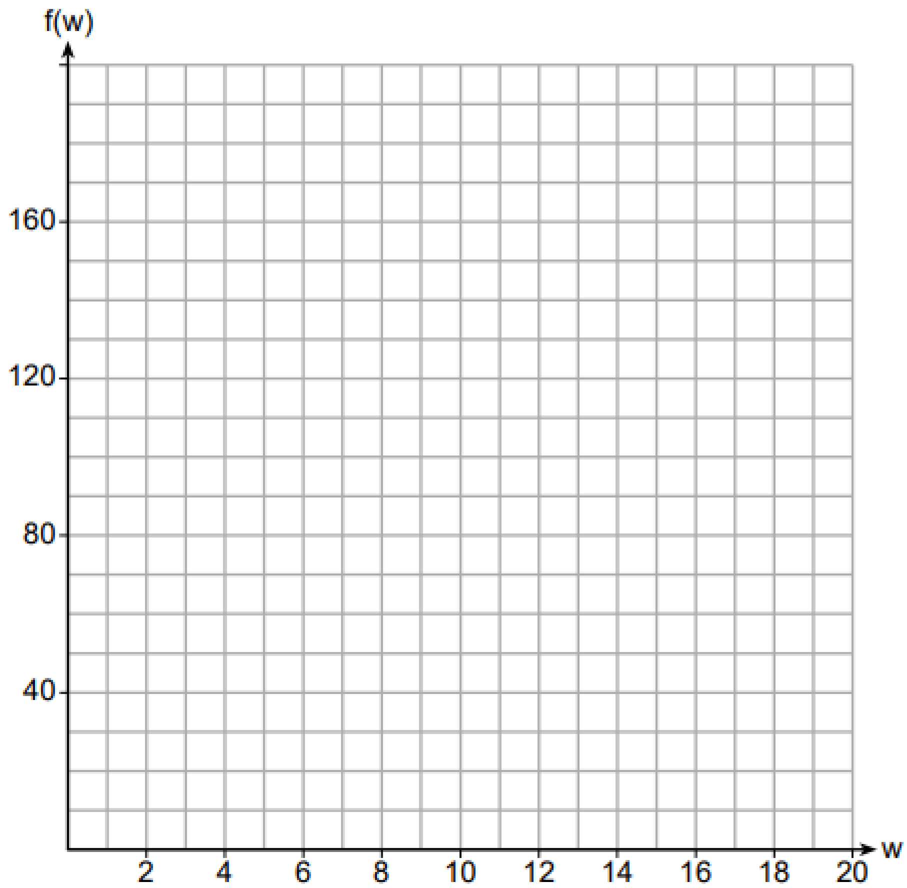
34 A car was purchased for \$25,000. Research shows that the car has an average yearly depreciation rate of 18.5%.

Create a function that will determine the value, $V(t)$, of the car t years after purchase.

Determine, to the *nearest cent*, how much the car will depreciate from year 3 to year 4.

36 Paul plans to have a rectangular garden adjacent to his garage. He will use 36 feet of fence to enclose three sides of the garden. The area of the garden, in square feet, can be modeled by $f(w) = w(36 - 2w)$, where w is the width in feet.

On the set of axes below, sketch the graph of $f(w)$.



Explain the meaning of the vertex in the context of the problem.