Lesson 3 – Piecing it Together

You are becoming more familiar with linear and exponential functions. This lesson, we will introduce two more important families: **quadratic** and **absolute value** functions. Finally, we will learn about **piecewise** functions.

Quadratic Functions

The third major function family, quadratic functions, is also <u>nonlinear</u>. Situations that are modeled with quadratic function rules many times involve gravity or motion. Here are a few examples:

- Jonathan kicks a football into the air. The height in feet at any given time t can be modeled by the function rule $h(t) = -16t^2 + 60t + 3$
- A skydiver jumps from a height of 3,500 meters. The total distance fallen at any given time t can be modeled by the function rule $d(t) = 3500 - 9.8t^2$
- A concert venue's revenue depends on the ticket price, x. The concert revenue can be modeled by the quadratic function rule $R(x) = x(200 - 20x)$
- The path of a comet as it slingshots around planet Earth can be modeled by the quadratic function $p(x) = (x - 3)^2 + 4$

What similarities and differences do you notice about these function rules?

Based on the situations what shape do you predict quadratic graphs to have? Make a sketch.

Quadratic function rules appear in three major forms. We are going to learn how to convert between these forms later, but it makes it more difficult to recognize what exactly makes function rules quadratic. Did you notice how three rules involved an exponent of 2? Quadratic function rules do have x^2 in them. It is less obvious that $R(x) = x(200 - 20x)$ is quadratic, but it is. Let's take a look at each of their graphs.

$$h(t) \qquad\qquad d(t) \qquad\qquad\qquad R(x) \qquad\qquad\qquad p(x)$$
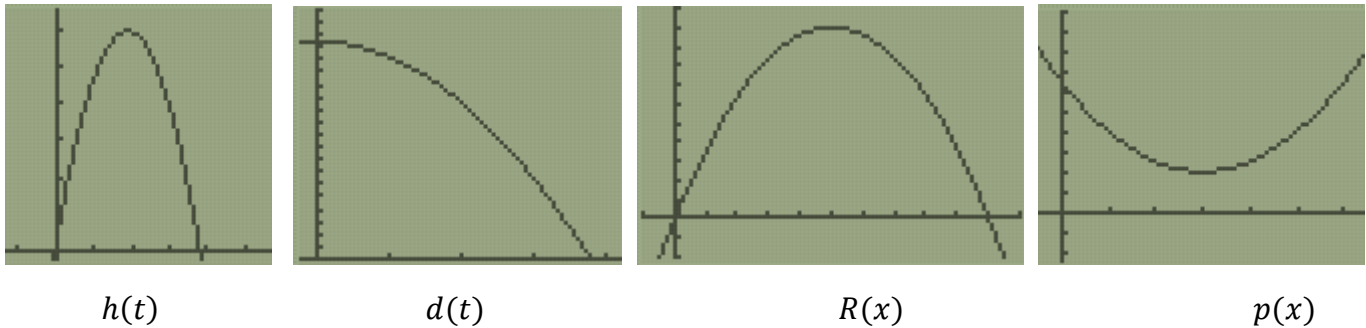
Which graphs are similar? How are they similar? Which graph is different? How is it different?

Karla answered the previous question by saying that $d(t)$ is the graph that is different. She said,

"I think $d(t)$ is different because it decreases the whole time. Every other graph has one part that increases and another part that decreases."

Robert and Karla exchange papers and read each other's responses. Robert disagrees with Karla and tells her,

"If you graph it on the left, $d(t)$ is similar to h and R. I think p is the different one because it goes up on both sides."

Explain Robert's thinking and expand upon it in your own words. What evidence could Robert have shown Karla to help support his response?

Looking at a graph will always reveal whether a rule is quadratic. These U-shaped graphs are called **parabolas**. Notice that it is not obvious that $d(t)$ is a parabola. We will discuss how to adjust the calculator **WINDOW** in greater detail in the future.

Notice that linear and exponential functions either always increase or always decrease from left to right. Parabolas change directions. This turning point is called the **vertex**. If the quadratic graph opens up (shaped like a cup), the vertex is the minimum of the graph. If the parabola opens down (shaped like a frown), the vertex is a maximum point.
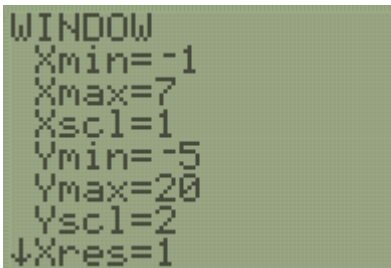
Suppose you want to find the minimum point of $p(x)$.

Plot1 Plot2 Plot3
\Y1▉(X−3)²+4
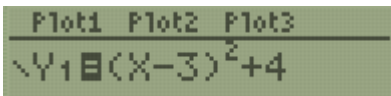
Press **Y=** and enter the function rule for **Y1**

X	Y1
0	13
1	8
2	5
3	4
4	5
5	8
6	13

To get an idea of the correct window settings, press **2ⁿᵈ, GRAPH**. Decide on the lowest and highest values for *x*- and *y*-values.

WINDOW
 Xmin=-1
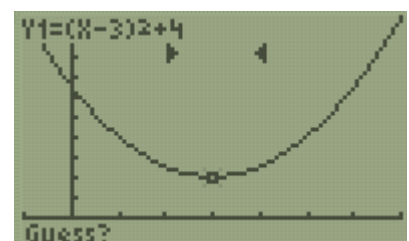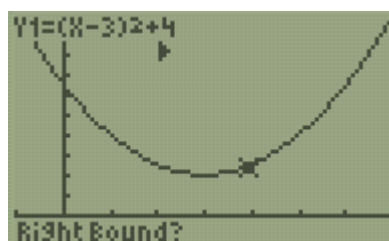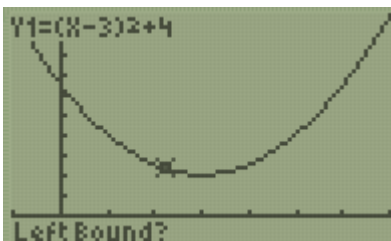 Xmax=7
 Xscl=1
 Ymin=-5
 Ymax=20
 Yscl=2
↓Xres=1

To follow along with this example, press **WINDOW** and enter the following settings.

Are these choices close to the ones you made based on looking at the table?

CALCULATE
1:value
2:zero
3:minimum
4:maximum
5:intersect
6:dy/dx
7:∫f(x)dx
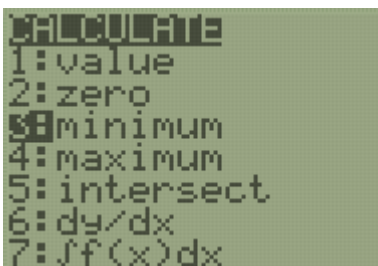
Press **2ⁿᵈ**, **TRACE**, and choose choice **3** for the minimum finder.

Y1=(X−3)2+4

Left Bound?

Y1=(X−3)2+4

Right Bound?

Y1=(X−3)2+4

Guess?

Move the cursor using the arrow keys until it is on the left side of the minimum, then press **ENTER**.

Move the cursor to the right side of the minimum and press **ENTER**

Finally, move the cursor to somewhere close to the minimum point an press **ENTER.**
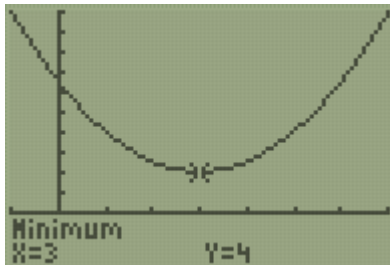
The coordinates of the minimum point of $p(x)$ are $(3, 4)$.

This process works nearly the same for finding a maximum.

Try to find the coordinates of the maximum point on $h(t) = -16t^2 + 60t + 3$. Note that when you put the rule into **Y=** you always use *x* as your variable.

These settings produce a nice viewing window for **h**. Again, we will discuss strategies for how to do this in the future.

Use these Window settings and write the coordinates of the maximum point of
$h(t) = -16t^2 + 60t + 3$.

The Regents Exam also loves to ask about the **zeros** of quadratic functions. **Zeros** are x-values for which y = 0. Sometimes we can find zeros on a table.
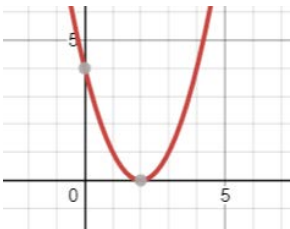
Locate the zeros of $R(x) = x(200 - 20x)$ using your table.

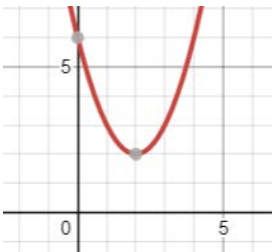From this example, we see that quadratics can have two zeros.

The zeros of a function are the **x-intercepts** of its graph. Notice how the graph of $R(x)$ intersects the x-axis at two locations, **x = 0** and **x = 10**
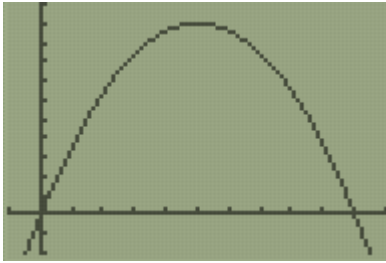
Sketch the graph of a quadratic function that has one zero.
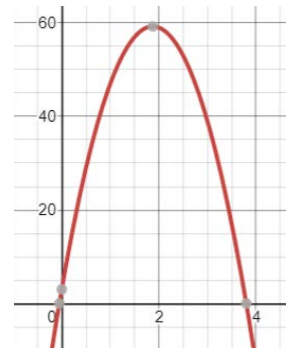

Sketch the graph of a quadratic function that has no zeros.

Is it possible for a quadratic function to have more than two zeros? Explain your reasoning.

Consider the function $h(t) = -16t^2 + 60t + 3$. View a graph of this function. How many zeros does h have?

Fill out the table of values for $h(t)$ for integers **x = -2** through **x = 5**

x	-2	-1	0	1	2	3	4	5
$h(x)$								

It's not immediately obvious what exact *x*-values the zeros are. Between what values of **x** must the zeros be "hiding"? Explain how you can tell.

This example shows that we can't rely on tables to find the zeros of every quadratic function.

To find the zeros on a graph, press **2ⁿᵈ**, **TRACE**, and choose option **2: zero**.

Then, follow the same process to locate a max or min.

Use the following **WINDOW** for a nice picture of the graph.

```
WINDOW
 Xmin=-2
 Xmax=6
 Xscl=1
 Ymin=-5
 Ymax=70
 Yscl=10
↓Xres=1
```

Try this process now to locate the zeros of $h(t) = -16t^2 + 60t + 3$. Note that they will be decimal values.
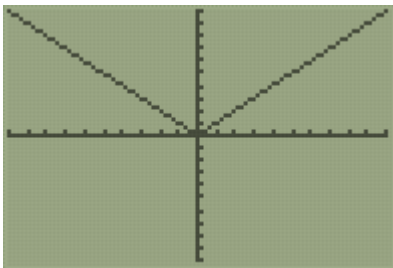
Absolute Value Functions

There is another family, in addition to linear, exponential, and quadratic, that is worth knowing – the absolute value function.

The **absolute value** of a number a, written as $|a|$, is that number's <u>distance from 0</u>. Since distance is always positive, the absolute value of a number is always positive.
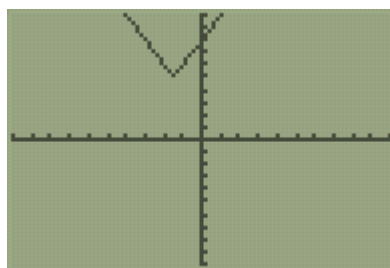
For example, the absolute value of negative four is four because negative four is four units away from zero on a number line. This is written as $|-4| = 4$

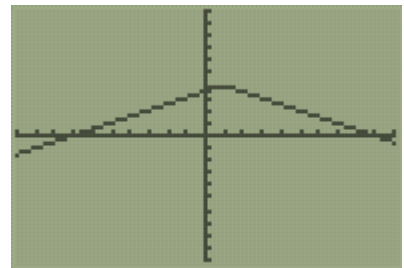Absolute value functions have similarities in the shape of their graphs.

$y = |x|$ $y = |2x + 3| + 5$ $y = -0.5|x - 1| + 4$

Get the absolute value bars in the calculator by pressing **ALPHA**, **WINDOW**, **ENTER**.

What are some similarities between each absolute value function rule?

What similarities do you notice in each graph?

What similarities and differences do you notice between absolute value functions and quadratic function graphs? Rules?

Domain and Graphing

In your own words, explain the idea of a function's <u>domain</u> and <u>range</u>.

For many situations, there are only certain input values that make sense. For example, consider the example from the beginning of the lesson involving revenue:

- A concert venue's revenue depends on the ticket price, **x.** The concert revenue can be modeled by the quadratic function rule $R(x) = x(200 - 20x)$

What does **x** represent in this situation?

What is one value of **x** that does <u>not</u> make sense for this situation?

Select the inequality that best represents the domain for this situation. Explain your reasoning for your choice.

$$\{x|x = 5\} \qquad \{x|x > 0\} \qquad \{x|x \geq 0\} \qquad \{x|x < 0\}$$

Tyrece is exploring $R(x) = x(200 - 20x)$ using a table of values on his calculator. He notices a pattern and writes it down.

As the ticket price increases, the money or revenue increases for a little while, then it decreases and becomes negative.

Do you agree with the pattern Tyrece noticed?

Explain why it makes sense in the context of the problem that as ticket price increases, the revenue will eventually decrease.

Explain why it makes sense in the context of the problem that revenue will eventually be negative.

Tyrece wants to consider the domain of ticket prices for which revenue is positive. Using words, which ticket prices give positive revenues?

Which inequality below represents the domain you described? Explain your reasoning.

$$\{0 < x < 10\} \qquad \{x < 10\} \qquad \{0 > x > 10\}$$

Many students struggle to interpret these types of inequalities. To help you, fill in the chart below:

Inequality	Literally	In my own words
$0 < x < 10$	Zero is less than x which is less than ten	All the *x*-values between zero and ten, not including zero or ten
$2 \leq x < 5$		
$-1 < x \leq 2$		
$x > 10$		
		All *x*-values between 4 and 7, not including 4 but including 7
		All x-values smaller than 8

Complete the following Regents question.

On the set of axes below, graph the function $y = |x + 1|$.
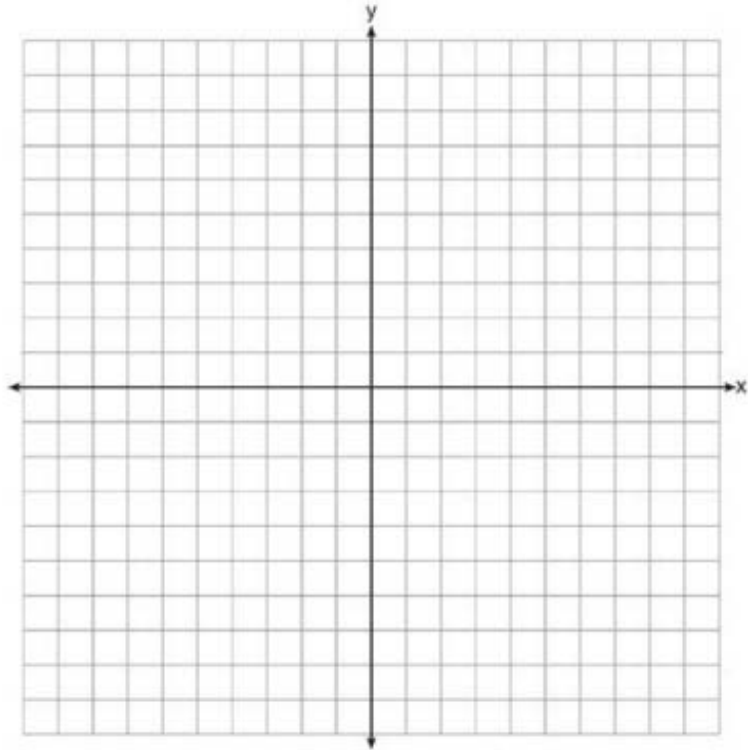
State the range of the function. State the domain over which the function is increasing.

Piecewise Functions

Brendalee receives a model rocket kit for her birthday. The rocket has a GPS signal that allows her to track its distance from the ground using a phone app.

- For the first five seconds, the engine causes the rocket's distance from the ground in meters to increase according to the exponential function $= 5(2)^t - 4.5$.
- When the fuel runs out, the rocket free falls toward the ground for two seconds. Its distance from the ground decreases according to the quadratic function
$y = -9.8(t - 5)^2 + 155.5$
- Finally, the parachute opens, and the rocket's distance from the ground decreases at a constant rate of -10 meters per second according to the linear function
$y = 186.3 - 10t$ until the rocket hits the ground.

Let $D(t)$ represent the distance from the ground of the rocket for the entire time between launch to when it hits the ground. Sketch what you think the graph of $D(t)$ looks like.

During which times will Brendalee's rocket follow the exponential function rule? The quadratic function rule? The linear function rule? Write inequalities to represent each of these intervals.

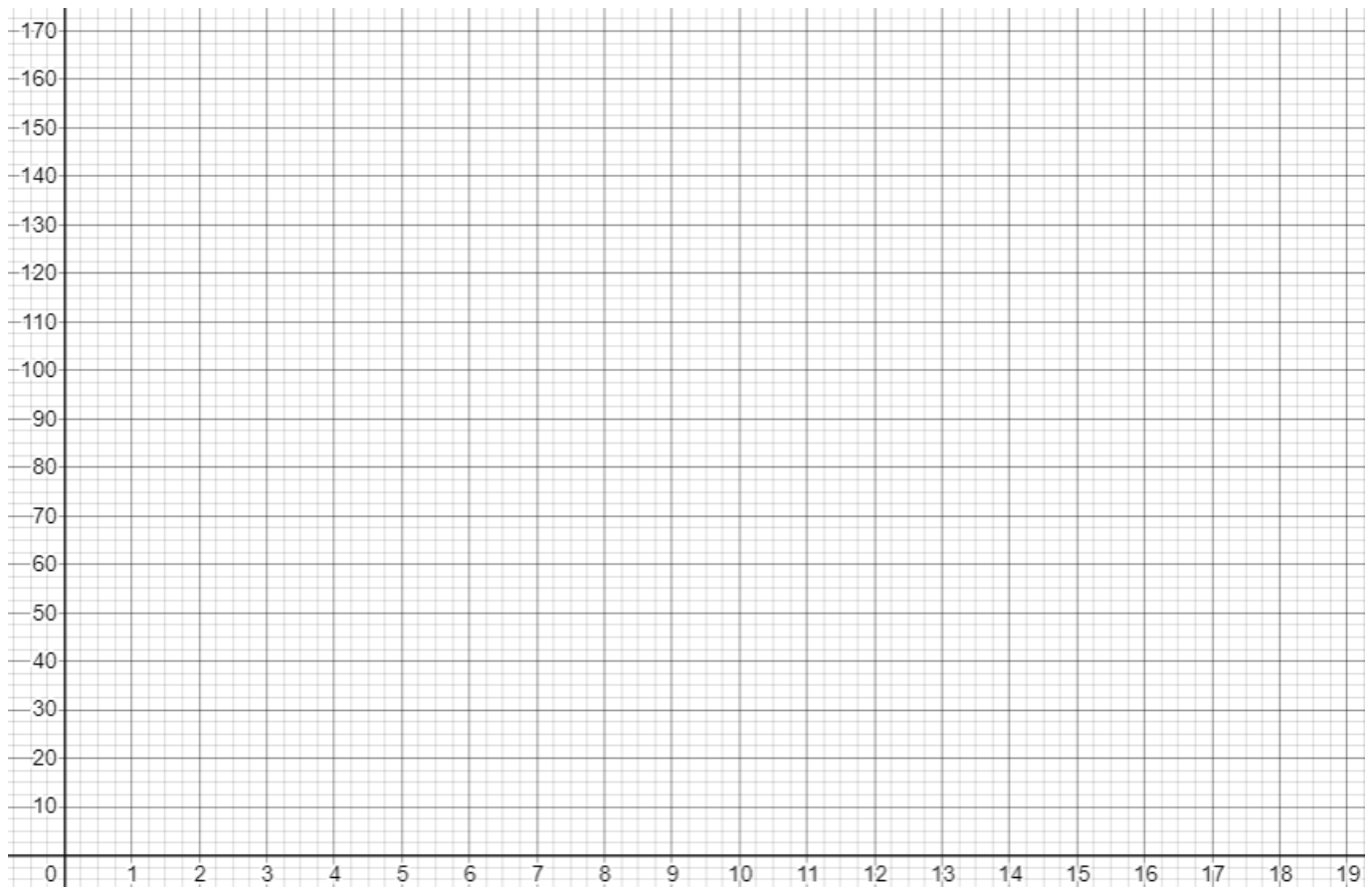Brendalee uses a **piecewise function** to define $D(t)$.

$$D(t) = \begin{cases} 5(2)^t - 4.5 & 0 \le t \le 5 \\ -9.8(t-5)^2 + 155.5 & 5 < t \le 7 \\ 186.3 - 10t & 7 < t < 18.6 \end{cases}$$

Explain what this notation means in your own words.

According to the piecewise function Brendalee wrote, which of the three function rules should be applied when $t = 2$? What about when $t = 6$? How do you know?
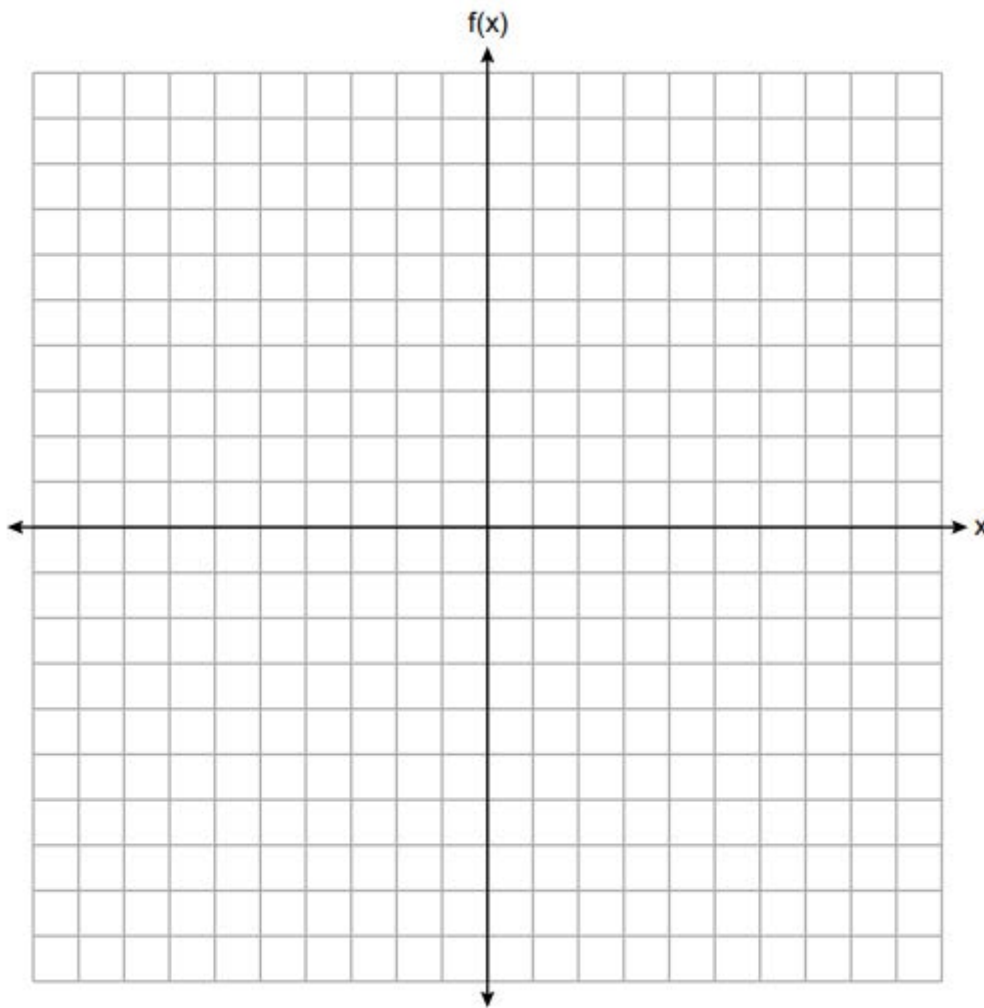
What is the domain of $D(t)$?

Make a graph of $D(t)$ below.

Did you notice that each rule matched up to form one continuous graph? Not all piecewise functions do this.

27 Graph the following piecewise function on the set of axes below.

$$f(x) = \begin{cases} |x|, & -5 \le x < 2 \\ -2x+10, & 2 \le x \le 6 \end{cases}$$

f(x)

**32** On the set of axes below, graph the piecewise function:

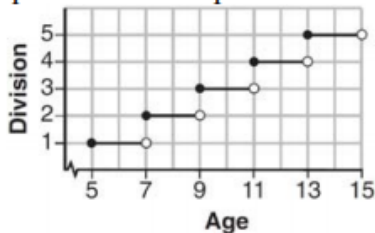$$f(x) = \begin{cases} -\dfrac{1}{2}x, & x < 2 \\ x, & x \geq 2 \end{cases}$$
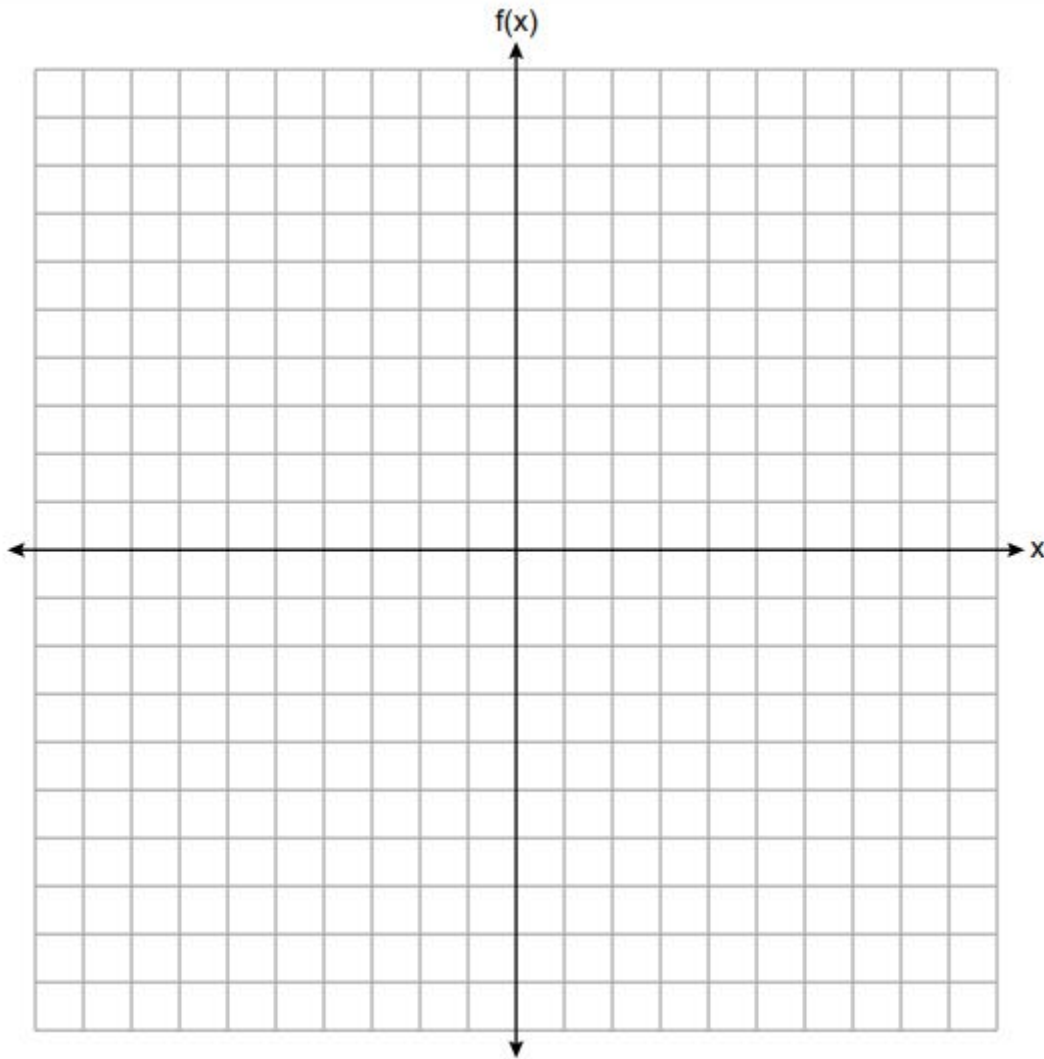
At an office supply store, if a customer purchases fewer than 10 pencils, the cost of each pencil is $1.75. If a customer purchases 10 or more pencils, the cost of each pencil is $1.25. Let c be a function for which $c(x)$ is the cost of purchasing x pencils, where x is a whole number.

$$c(x) = \begin{cases} 1.75x, & \text{if } 0 \le x \le 9 \\ 1.25x, & \text{if } x \ge 10 \end{cases}$$
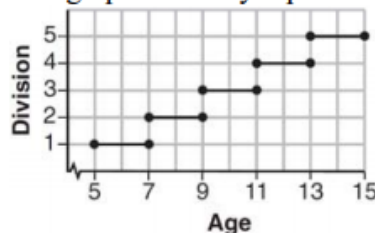
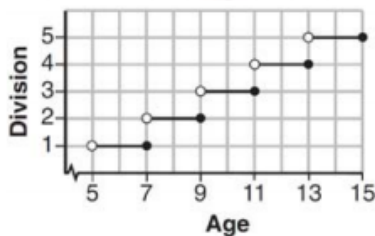Create a graph of c on the axes below.
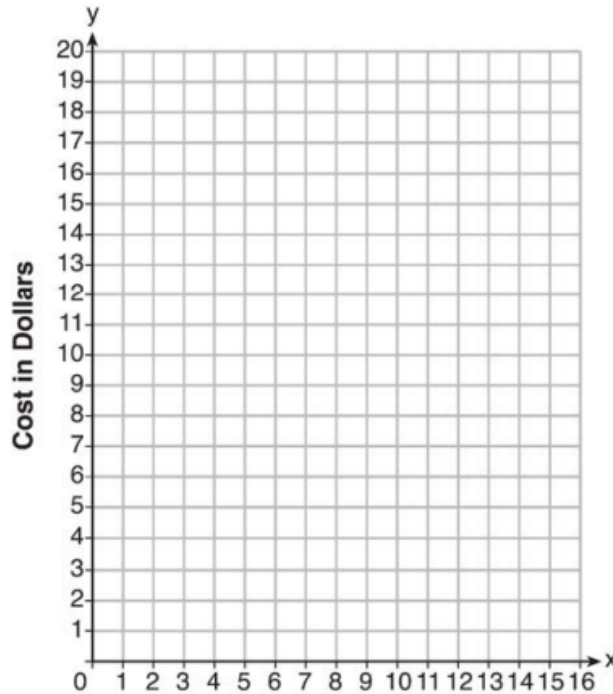
Morgan can start wrestling at age 5 in Division 1. He remains in that division until his next odd birthday when he is required to move up to the next division level. Which graph correctly represents this information?
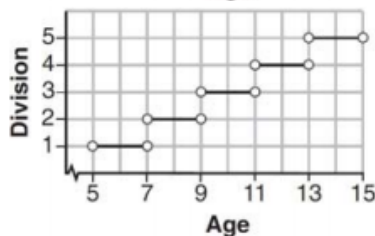

a.

b.

c.

d.