Lesson 4 – Solving Linear Equations

For the past three lessons, we learned about functions. A function is a rule that tells you what to do with an *x* in order to get a *y*-value. But how are functions different from **equations**?

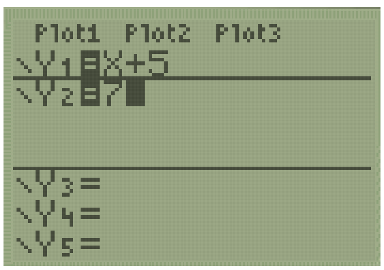
When told to solve an equation for a variable, such as solving $\mathbf{3x + 2 = x - 1}$, we are looking for a single value (sometimes more) that will makes both sides of the equation have the same *y*-value.

Most students solve *algebraically*. They do operations on both sides of the equation until the variable is by itself and a number is on the other side of the equal sign. However, all equations can be solved using graphs and tables as well. This lesson, we will practice solving linear equations.

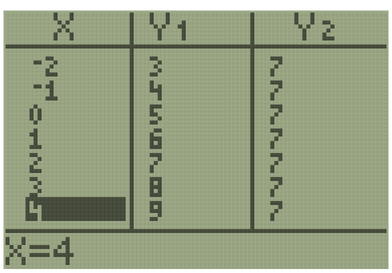
Solving (mostly) linear equations

Consider how to solve the equation $\mathbf{x + 5 = 7}$.

Method 1 – Using a table



Press **Y=**, <u>enter one side of the</u> **=** sign as **Y1** and <u>the other side as</u> **Y2**

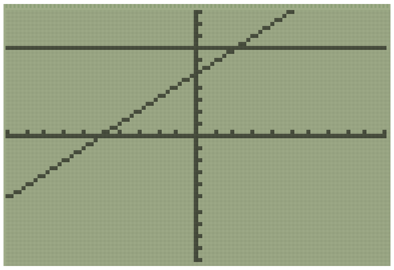


Find the x-value where **Y1** equals **Y2**

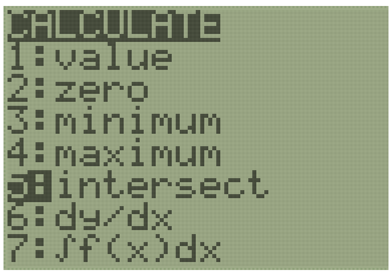
Notice that Y1 = Y2 when x = 2. **The x-value is the solution to the equation**.

<u>Method 2 – Using a graph</u>

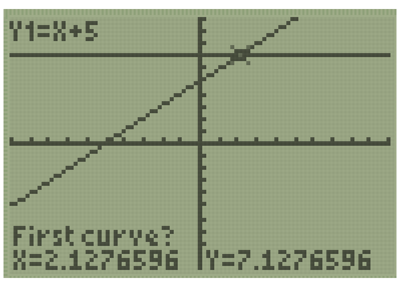
Do the same first step as before: Press **Y=**, <u>enter one side of the</u> **=** sign as **Y1** and <u>the other side as</u> **Y2**.



Press **GRAPH**. **The solution is the point where the graphs intersect**.

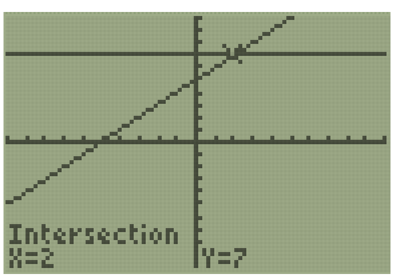


Press **2nd**, **TRACE**, use the **down arrow** to choose choice **"5: intersect,"** and press **ENTER**



Use the left or right arrow to move the blinker close to the desired point.

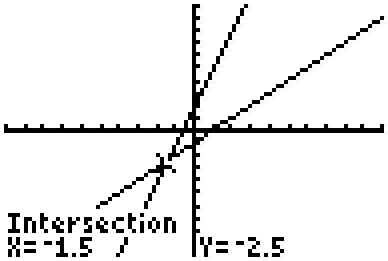
Then press **ENTER**, **ENTER**, **ENTER**.



The graph intersects at the point **(2,7).** The solution to the equation is the **x-value 2.**

We need both the table method and the graph method to solve equations. The table method is fast and simple, but if the solution to the equation is a decimal or a fraction, then the solution will not show up in the standard table. The graphing method will always work, but it is a little more complicated.

1. Solve the equation $\mathbf{3x + 2 = x - 1}$ using a table or graph. Show evidence of your method to support your answer.

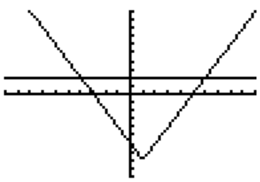


The solution is $x = -1.5$

Note that the solution is not obvious from the table.

You also have the tools to solve equations involving the other nonlinear function families using graphs or tables. **Solve** the following equations using graphs or tables in your calculator. Some equations have more than one solution. ***One equation has no solution!***

2. $2|x - 1| - 8 = 2$

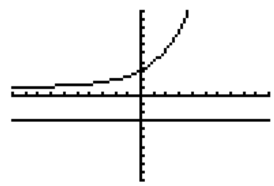


X	Y1	Y2
-4	2	2
6	2	2

$x = -4$ *and* $x = 6$

To get absolute value bars, you can press ALPHA, WINDOW, ENTER

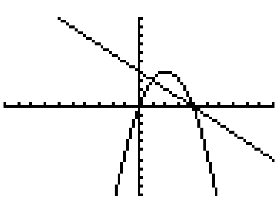
4. $2(1.5)^x + 1 = -3$



no solution

(The graphs do not intersect.)

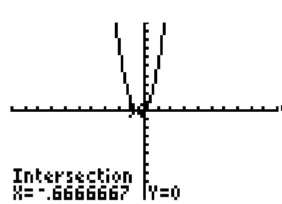
3. $-x^2 + 4x = 4 - x$

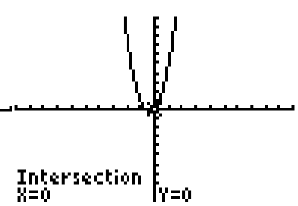


X	Y1	Y2
1	3	3
4	0	0

$x = 1$ *and* $x = 4$

5. $3x^2 + 2x = 0$





$x = -0.\overline{6}$ *and* $x = 0$

6. Summarize how to determine a solution to an equation using a graph or table. Explain how you can tell when an equation has no solution.

On a graph, the solutions are the x-values where the graphs intersect. On a table, the solutions are the x-values that have matching y-values in Y1 and Y2.

There is no solutions if the graphs do not intersect.

Method 3 – Solving linear equations algebraically

Marcos solved the equation $\mathbf{7x - 3(x + 1) = 2(x + 4)}$ algebraically below.

7. ***Explain each step of his process.***

$7x - 3(x + 1) = 2(x + 4)$

Distributive Property

$7x - 3x - 3 = 2x + 8$

Combine like terms ($7x - 3x = 4x$)

$4x - 3 = 2x + 8$

Subtract $2x$ from both sides.

$2x - 3 = 11$

Add 3 to both sides.

$2x = 14$

Divide both sides by 2. (Multiply both sides by $\frac{1}{2}$)

$x = 7$

8. Show how Marcos could check that his answer is correct using a graphing calculator.

Students should that the graphs of $Y1 = 7x - 3(x + 1)$ and $Y2 = 2(x + 4)$ intersect when $x = 7$ or that the y-values are both the same number (22) at $x = 7$ on a table.

Students may be tempted to enter Y1 and Y2 based on steps further down in their algebraic solution. This a choice that they have, but you should have a conversation with them that since it is possible they made a mistake in the solution process, choosing the original equation may be the best choice when checking.

9. Solve each equation algebraically. Check that your answer is correct using a graphing calculator.

a. $4x + 10 = 2x + 7$

$\boldsymbol{x = -1.5}$

c. $12 - 2(x + 5) = 4$

$\boldsymbol{x = -1}$

b. $2(5x - 3) = 3(4x + 1)$

$\boldsymbol{x = -4.5}$

d. $17 - 3(3x - 5) = 2 + x$

$\boldsymbol{x = 3}$

10. Write a general summary explaining how to solve linear equations algebraically.

Answers will vary.
In general:

Do the distributive property to eliminate parentheses, then combine like terms. Next, work to get the variables on one side of the equal sign and constant terms on the other side by performing opposite operations on each side of the equation.

If students are already familiar with the acronym PEMDAS, they might benefit from understanding that solving equations is the reversal of this process.

You are also expected to solve linear equations algebraically that contain fractions.

11. Explain each step of the process to solve $\mathbf{6-\frac{2}{3}(x+5)=4x}$.

$$6-\frac{2}{3}(x+5)=4x$$

Distributive property

$$6-\frac{2}{3}x-\frac{10}{3}=4x$$

Combine like terms $\left(6-\frac{10}{3}=\frac{8}{3}\right)$

$$\frac{8}{3}-\frac{2}{3}x=4x$$

Operations with fractions are often a weakness of Algebra I students. Emphasize the idea that when solving equations, fractions behaves just as whole numbers, and that the calculator can perform operations with them. To type a fraction, press ALPHA, Y=, ENTER. For example, while the calculator cannot do $x+\frac{2}{3}x$, a student can type $4+\frac{2}{3}$ in the home screen.

Add $\frac{2}{3}x$ to both sides.

$$\frac{8}{3}=\frac{14}{3}x$$

Divide by $\frac{14}{3}$ on both sides (or multiply by $\frac{3}{14}$ on both sides)

$$x=\frac{4}{7}\ or\ .5714285714$$

12. Show how to check that the answer is correct using the graphing calculator.

Since the answer is a fraction, the solution is easier to find with a graph. Note that the intersection of the graph occurs at $x=.57142857$. Students can see this is equivalent to $\frac{4}{7}$ if they type $4\div 7$ [ENTER] into their home screen.

13. Solve each linear equation for the variable algebraically. Check that your answers are correct using a graphing calculator.

a. $-\frac{2}{3}(x+12)+\frac{2}{3}x=-\frac{5}{4}x+2$

$\boldsymbol{x=8}$

b. $\frac{3}{4}x-6=\frac{1}{2}x-9$

$\boldsymbol{x=-12}$

c. $\frac{1}{6}y+8=10-\frac{2}{3}y$

$\boldsymbol{x=2.4\ or\ x=\frac{12}{5}}$