Lesson 5 – Systems of Equations & Inequalities

In the last lesson, you practiced solving a single linear equation. In this lesson, we will focus on **systems of two equations**. A **system of equations** are two or more equations. The *solution* to a system of equations are the numbers for each variable that make all equations true at once.

This lesson, we will go over three methods to solve a system of equations, then practice solving systems of inequalities graphically.

A group of students were given the following situation and asked to solve:

A baseball team is planning a special promotion at its first game. Fans who arrive early will get a team athletic bag or cap, as long as supplies last. The promotion manager from the team can buy athletic bags for $9 each and caps for $5 each. The total budget for buying bags and caps is $25,500. The team plans to give a bag or cap, but not both, to the first 3,500 fans. The promotion manager wants to know: *How many caps and bags should be given away?*

1. When given a problem in context, the first step is to always translate it into math equations.
   a. All students first wrote two equations: one for the total cost of bags and caps, and one for the total number of bags and caps. Fill in the coefficients below.

$$9\,b + 5\,c = \$25{,}500$$
$$1\,b + 1\,c = 3{,}500$$

   b. What is the meaning of the variables ***b*** and ***c***?

b is the number of bags and c is the number of caps.

Including the language "number of" is important to emphasize that the variable stands for how many objects there are, and not the actual objects themselves.

2. Tyrece chose to solve for *b* and *c* using the **substitution method**.
   a. Follow Tyrece's work and explain his process in your own words.

$b + c = 3500$

Subtract c

$b = 3500 - c$

$9b + 5c = 25500$ **Rewrite the first equation**

$9(3500 - c) + 5c = 25500$ **Substitute for *b***

$31500 - 9c + 5c = 25500$ **Distribute**

$31500 - 4c = 25500$ **Combine -9c + 5c**

$6000 = 4c$	**Add 4c to both sides and subtract 25,500 from both sides.**
$c = 1500$	**Divide by 4**
$b + c = 3500$	**Rewrite the second original equation**
$b + 1500 = 3500$	**Substitute c = 1500**
$b = 2000$	**Subtract 1500 from both sides.**

b. Write a sentence to explain the meaning of Tyrece's answer.

The manager should give away 2,000 bags and 1,500 caps

c. Another student, Clair, also used the substitution method to solve this problem. However, her first step was to write $\boldsymbol{c = 3500 - b}$. Is she also correct? Use the substitution method to show that Clair will reach the same answer as Tyrece.

She is correct.

$$\mathbf{9}\boldsymbol{b} + \mathbf{5}\boldsymbol{c} = \mathbf{25500}$$
$$\mathbf{9}\boldsymbol{b} + \mathbf{5}(\mathbf{3500} - \boldsymbol{b}) = \mathbf{25500}$$
$$\mathbf{9}\boldsymbol{b} + \mathbf{17500} - \mathbf{5}\boldsymbol{b} = \mathbf{25500}$$
$$\mathbf{4}\boldsymbol{b} + \mathbf{17500} = \mathbf{25500}$$
$$\mathbf{4}\boldsymbol{b} = \mathbf{8000}$$
$$\boldsymbol{b} = \mathbf{2000}$$

$$\boldsymbol{b} + \boldsymbol{c} = \mathbf{3500}$$
$$\mathbf{2000} + \boldsymbol{c} = \mathbf{3500}$$
$$\boldsymbol{c} = \mathbf{1500}$$

4. Eddie and Monica used a different method, the **elimination method**, to solve the same system of equations. Eddie's work is shown below. Follow Eddie's work and discuss his process.

$9b + 5c = 25500$
$b + c = 3500$

→ Multiply the second equation by -9 →

$9b + 5c = 25500$
$-9b - 9c = -31500$

Add these equations together

$-4c = -6000$

Divide by -4

$c = 1500$

Substitute into b + c = 3500

$b + 1500 = 3500$

Subtract 1500

$b = 2000$

a. Explain why you think Eddie chose to multiply the second equation by **-9**.

By doing this, the b-terms in each equation had opposite coefficients. This causes the b-terms to "eliminate" when the equations are added together.

b. Monica looks at Eddie's work and says, "I got the same answer, but I eliminated the c's first." Show the work that Monica must have used to get the same answer as Eddie.

$\mathbf{9b + 5c = 25500}$
$\mathbf{b + c = 3500}$

(Multiply 2nd eq. by -5)

$\mathbf{9b + 5c = 25500}$
$\mathbf{-5b - 5c = -17500}$

Add these equations together

$\mathbf{4b = 8000}$

Divide by 4

$\mathbf{b = 2000}$

Substitute into b + c = 3500

$\mathbf{2000 + c = 3500}$

Subtract 2000

$\mathbf{c = 1500}$

c. Eddie and Monica are trying to solve the following systems of equations using **elimination,** but they are stuck on what to multiply by. Their teacher gives them a hint, "Sometimes you have to multiply each equation by a different number."

$$-2a + 6b = 6$$
$$-7a + 8b = -5$$

i. Eddie looks at the ***a*** coefficients and thinks aloud to his partner, "What if we multiplied the first equation by negative 7 and the second equation by positive 2?"
What new system of equations will result from following Eddie's idea? Will this first step be productive in finding the correct answer? Explain.

$$14a - 42b = -42$$
$$-14a + 16b = -10$$

This is a productive first step since the coefficients of the a-terms are now opposites.

ii. Monica's idea is slightly different. She wants to eliminate the ***b*'s**. What should she multiply by in order to help eliminate the ***b*'s**?

Multiply the first equation by negative 8 and the second equation by positive 6.

OR

Multiply the first equation by positive 8 and the second equation by negative 6.

iii. Use either Eddie or Monica's first step and solve for ***a*** and ***b*** using the elimination method.

$$a = 3, b = 2$$

5. Rita decides to solve the original system of equation **graphically**.

$\mathbf{9b + 5c = 25500} \quad \mathbf{b + c = 3500}$

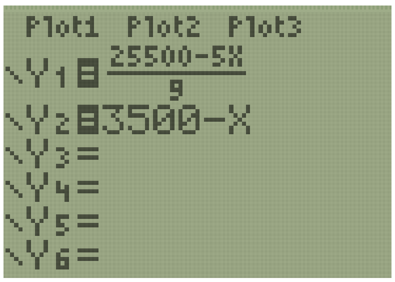
a. Follow Rita's work and explain her process.

$9b + 5c = 25500$ $\qquad$ $b + c = 3500$

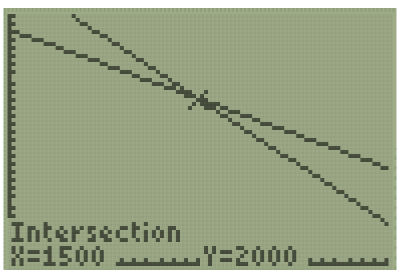
$9b = 25500 - 5c$ $\qquad$ $b = 3500 - c$

$b = \frac{25500-5c}{9}$

Rita solved each equation for *b*



She entered each equation into Y=



$\mathbf{c = 1500}$ **and** $\mathbf{b = 2000}$

b. Based on her calculator work, explain how Rita knew that **c** equaled 1500 and not the other way around.

Since she solved each equation for b in terms of c, the b-variable behaves as y and the c-variable is x.

6. After reviewing the **substitution**, **elimination**, and **graphing** method, identify one advantage and one disadvantage of each method. When might it make the most sense to use each method?

Responses to this will vary.

Students do not need to be masters of all three methods, although they must be able to perform the substitution or elimination method if a Regents Exam question asks for an algebraic solution. In my experience, students tend to prefer the elimination method, but the substitution method can sometimes be faster if one of the equations is already isolated for one of the variables. If both equations are already isolated for y, then graphing is a natural choice.

7. Solve each problem involving a system of two linear equations algebraically by using either substitution or elimination.

a. $2x - 6y = 28$
$-x - y = 14$

$\boldsymbol{x = -7, y = -7}$

c. $-5x - 14y = -9$
$-10x - 7y = 3$

$\boldsymbol{x = -1, y = 1}$

b. $-2x + 2y = -16$
$-3x - 2y = 11$

$\boldsymbol{x = 1, y = -7}$

d. $y = 4x - 5$
$-x - 8y = 7$

$\boldsymbol{x = 1, y = -1}$

8. To participate in a school trip, Kim had to earn $85 in one week. Kim could earn $8 per hour babysitting and $15 dollars per hour for yard work. Kim's parents limit work time to 8 hours per week. How many hours should Kim work at each job in order to meet her income goal and work exactly eight hours?

$$y = 3, b = 5$$

9. Solve the following Regents question from the June 2019 exam.

When visiting friends in a state that has no sales tax, two families went to a fast-food restaurant for lunch. The Browns bought 4 cheeseburgers and 3 medium fries for $16.53. The Greens bought 5 cheeseburgers and 4 medium fries for $21.11.

Using c for the cost of a cheeseburger and f for the cost of medium fries, write a system of equations that models this situation.

$$4c + 3f = 16.53$$

$$5c + 4f = 21.11$$

The Greens said that since their bill was $21.11, each cheeseburger must cost $2.49 and each order of medium fries must cost $2.87 each. Are they correct? Justify your answer.

$$No, 5(2.49) + 4(2.87) = 23.93$$

Using your equations, algebraically determine both the cost of one cheeseburger and the cost of one order of medium fries.

This question requires an algebraic solution, so only the use of substitution or elimination can receive full credit.

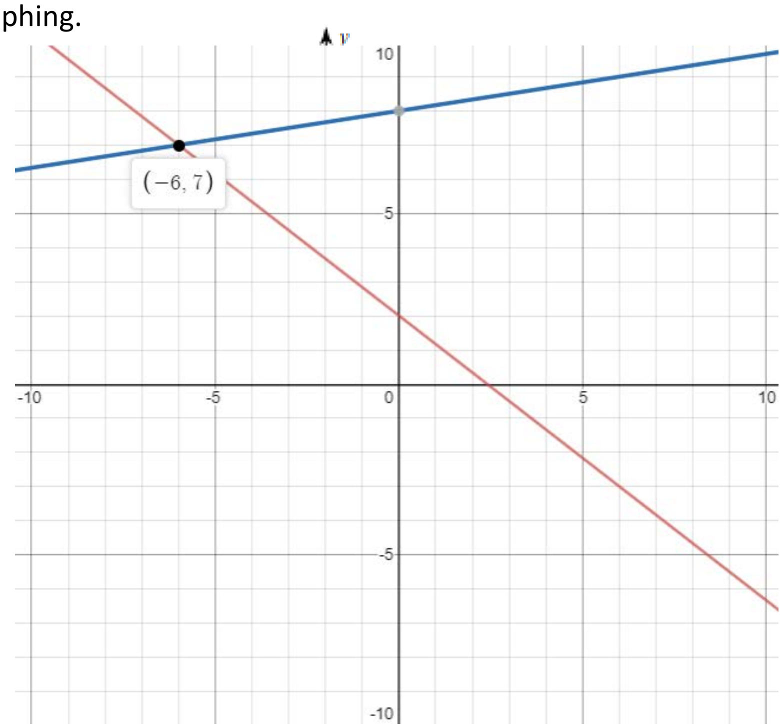
$$c = 2.79 \text{ and } f = 1.79$$

10. Solve the following systems using graphing.

a.

$$y = -\frac{5}{6}x + 2$$

$$y = \frac{1}{6}x + 8$$



Students should use the table to plot exact coordinates. They may skip points that do not have whole number coordinates.

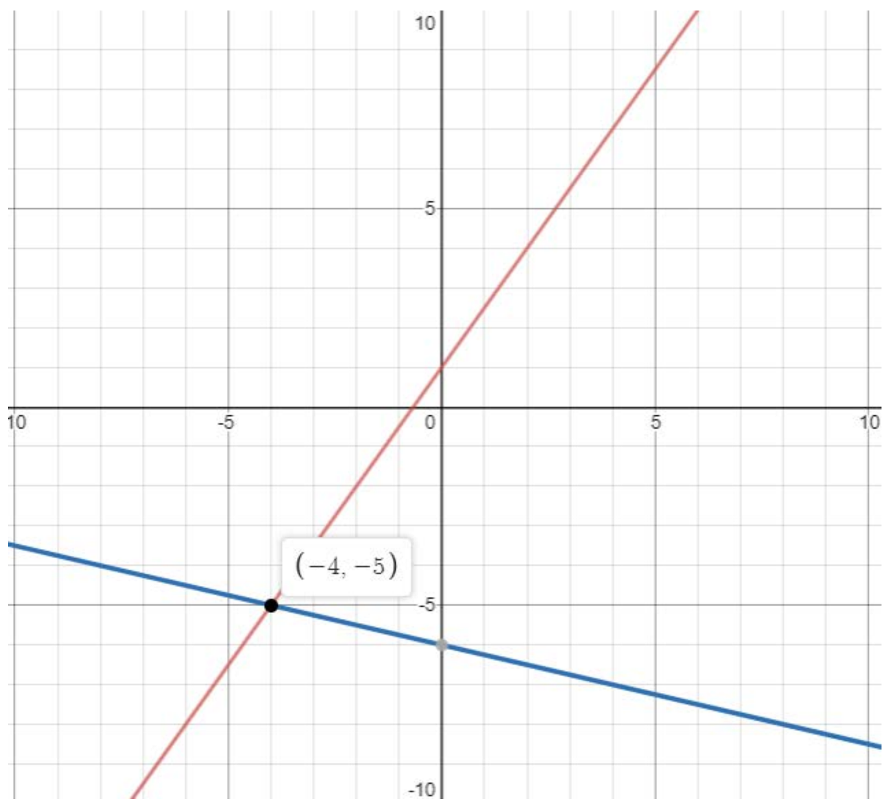
$x = -6, y = 7$

b.

$$y = \frac{3}{2}x + 1$$

$$y = -\frac{1}{4}x - 6$$

$x = -4, y = -5$



Jess was reading the following part from problem 8:

To participate in a school trip, Kim had to earn \$85 in one week. Kim could earn \$8 per hour babysitting and \$15 dollars per hour for yard work. Kim's parents limit work time to less than 8 hours per week.

11. Jess thought aloud to her partner, "Kim wants to earn at least \$85, but she could make more than that. Also, her parents limit her work to 8 hours, but she could work less than that if she wanted."

a. Assuming Kim makes \$8 babysitting and \$15 per hour for yard work, determine three different ways she could work to earn at least \$85 and also work less than 8 hours.

She could do yardwork only for six hours and earn \$90

She could do yardwork for five hours and babysit for two hours and earn \$91

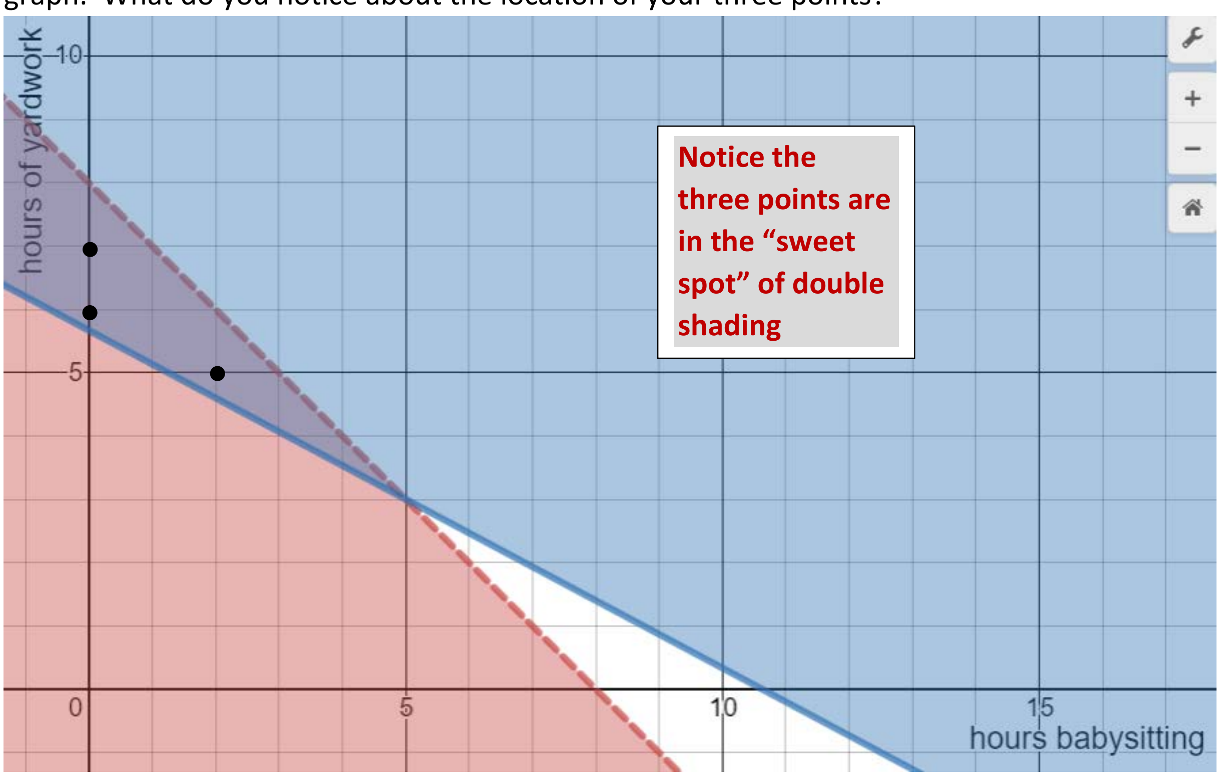
She could do yardwork for seven hours and earn \$105

b. The system of equations from problem 8 is written below, but the equal signs have been removed. Replace them with the proper inequality symbol <, >, ≤, ≥.

$$b + y < 8$$

$$8b + 15y \geq 85$$

c. In Algebra 1, we solve **systems of inequalities** graphically. The graph to this system of inequalities is below. On the graph, plot your three answers to part a as points on the graph. What do you notice about the location of your three points?



d. One of the inequality graphs uses a dotted line and the other uses a solid line. Why do you think this is?

The dotted line indicates < or >, but not equal to.

The solid line is for $\geq$ or $\leq$

e. The dotted-line-inequality graph is shaded below the dotted line and the solid-line-inequality graph is shaded above the solid line. What features about the algebraic inequalities from part b explain why this is?

For < or $\leq$, shade below the line

For > or $\geq$, shade above the line

f. The student who produced the graph in part c, Ernest, explained his process. "First, I solved each inequality for *y* just like I would solve a normal equation. Then I typed those into Y= and looked at the table to plot the points of each line graph. Then I used the inequality symbols to decide which line had to be dotted or solid, and which way to shade."

i. Show the algebraic work Ernest might have used to solve each inequality for *y*.

$b + y < 8$

$y < 8 - b$

$8b + 15y \geq 85$

$15y \geq 85 - 8b$

$y \geq \frac{85-8b}{15}$

ii. What did Ernest mean when he wrote, "I used the inequality symbols to decide which line had to be dotted or solid, and which way to shade"?

Since the first inequality is <, he used a dotted line and shaded below it

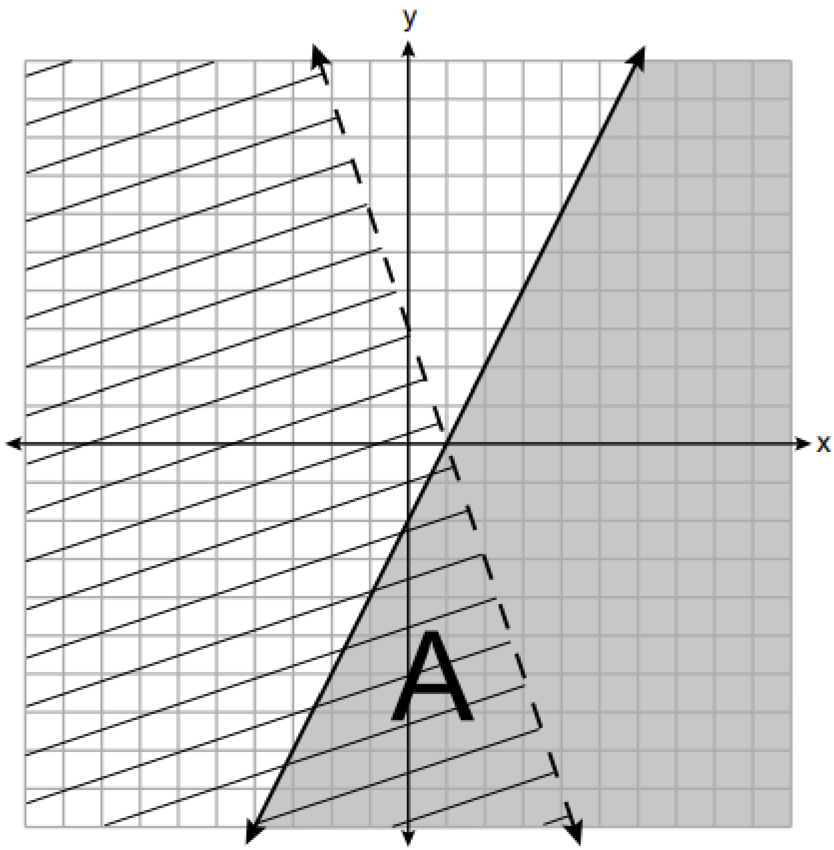
Since the second inequality is $\geq$, he used a solid line shaded above.

g. The double-shaded region on the graph is called the **solution set**. Explain what the solution set of this graph represents in the context of Kim's school trip situation.

The solution set represents all the combinations of hours spent doing yard work and babysitting that would satisfy both constraints – working fewer than 8 hours as well as earning at least $85.

12. Solve the following question from the June 2019 Regents exam.

A system of inequalities is graphed on the set of axes below.



State the system of inequalities represented by the graph.

$$y \leq 2x - 2$$

$$y < -3x + 3$$

One trick to generating equations is to use linear regression. Have students pick two point on the line and use regression.

It's also interesting to view how partial credit can be allocated. Visit http://www.jmap.org/JMAPRegentsExamArchives/ALGEBRAIEXAMS/0619ExamAI.pdf **and look at the anchor papers for this question starting on page 60.**

State what region A represents.

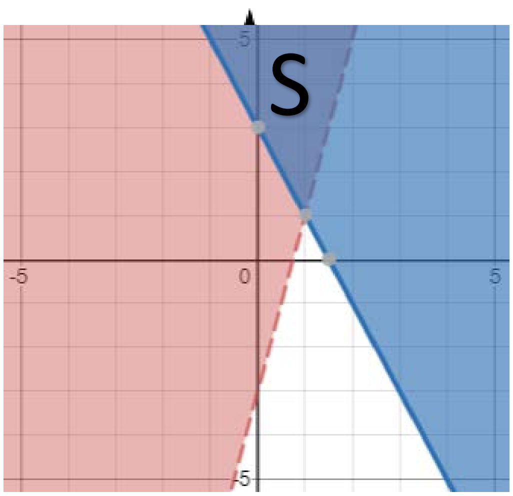
Region A is the solution set

State what the entire gray region represents.

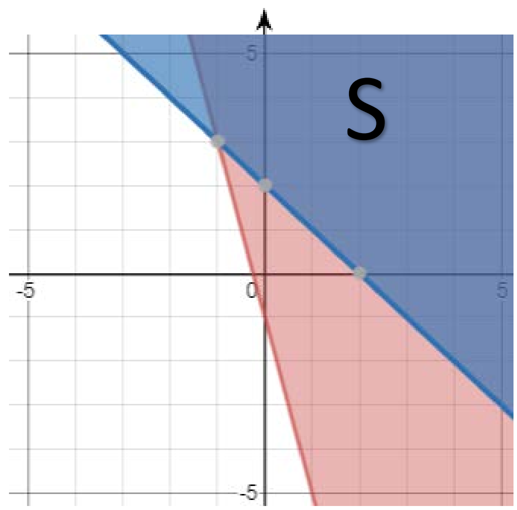
This is the solution to $y \leq 2x - 2$

13. Sketch the solution to each system of inequalities. **Label each solution set with the letter S**.

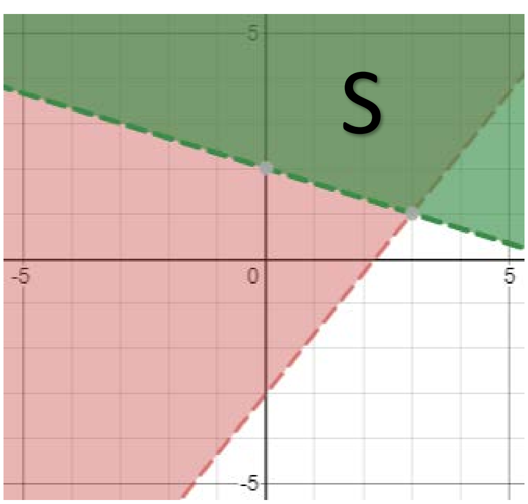
a. $y > 4x - 3$
$y \geq -2x + 3$



c. $x + y \geq 2$
$4x + y \geq -1$



b. $4x - 3y < 9$
$x + 3y > 6$



d. $3x + y \geq -3$
$x + 2y \leq 4$