Lesson 5 – Systems of Equations & Inequalities

In the last lesson, you practiced solving a single linear equation. In this lesson, we will focus on **systems** of two equations. A **system of equations** are two or more equations. The *solution* to a system of equations are the numbers for each variable that make all equations true at once.

This lesson, we will go over three methods to solve a system of equations, then practice solving systems of inequalities graphically.

A group of students were given the following situation and asked to solve:

A baseball team is planning a special promotion at its first game. Fans who arrive early will get a team athletic bag or cap, as long as supplies last. The promotion manager from the team can buy athletic bags for $9 each and caps for $5 each. The total budget for buying bags and caps is $25,500. The team plans to give a bag or cap, but not both, to the first 3,500 fans. The promotion manager wants to know: *How many caps and bags should be given away?*

1. When given a problem in context, the first step is to always translate it into math equations.
   a. All students first wrote two equations: one for the total cost of bags and caps, and one for the total number of bags and caps. Fill in the coefficients below.

$$\_\_\, b + \_\_\, c = \$25{,}500$$
$$\_\_\, b + \_\_\, c = 3{,}500$$

   b. What is the meaning of the variables ***b*** and ***c***?

2. Tyrece chose to solve for *b* and *c* using the **substitution method**.
   a. Follow Tyrece's work and explain his process in your own words.

$b + c = 3500$

$b = 3500 - c$

$9b + 5c = 25500$

$9(3500 - c) + 5c = 25500$

$31500 - 9c + 5c = 25500$

$31500 - 4c = 25500$

$6000 = 4c$

$c = 1500$

$b + c = 3500$

$b + 1500 = 3500$

$b = 2000$

b. Write a sentence to explain the meaning of Tyrece's answer.

c. Another student, Clair, also used the substitution method to solve this problem. However, her first step was to write $\boldsymbol{c = 3500 - b}$. Is she also correct? Use the substitution method to show that Clair will reach the same answer as Tyrece.

4. Eddie and Monica used a different method, the **<u>elimination method</u>**, to solve the same system of equations. Eddie's work is shown below. Follow Eddie's work and discuss his process.

$9b + 5c = 25500$
$b + c = 3500$ → Multiply the second equation by -9 → $9b + 5c = 25500$
$-9b - 9c = -31500$

Add these equations together

$$-4c = -6000$$

$$c = 1500$$

$$b + 1500 = 3500$$

$$b = 2000$$

a. Explain why you think Eddie chose to multiply the second equation by **-9**.

b. Monica looks at Eddie's work and says, "I got the same answer, but I eliminated the c's first." Show the work that Monica must have used to get the same answer as Eddie.

c. Eddie and Monica are trying to solve the following systems of equations using **elimination,** but they are stuck on what to multiply by. Their teacher gives them a hint, "Sometimes you have to multiply each equation by a different number."

$$-2a + 6b = 6$$
$$-7a + 8b = -5$$

i. Eddie looks at the ***a*** coefficients and thinks aloud to his partner, "What if we multiplied the first equation by negative 7 and the second equation by positive 2?"

What new system of equations will result from following Eddie's idea? Will this first step be productive in finding the correct answer? Explain.

ii. Monica's idea is slightly different. She wants to eliminate the ***b*'s**. What should she multiply by in order to help eliminate the ***b*'s**?

iii. Use either Eddie or Monica's first step and solve for ***a*** and ***b*** using the elimination method.

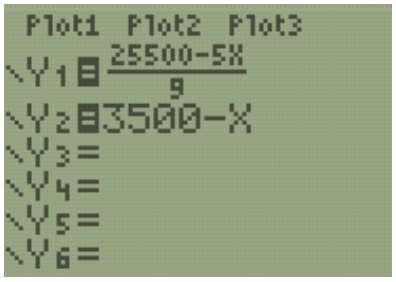
5. Rita decides to solve the original system of equation **graphically**.
$\boldsymbol{9b + 5c = 25500 \quad b + c = 3500}$

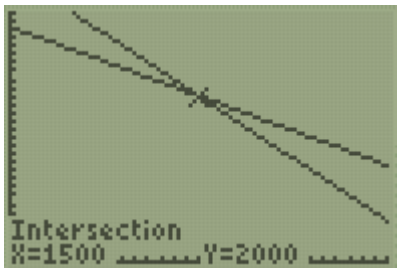
a. Follow Rita's work and explain her process.

$9b + 5c = 25500$ $\qquad$ $b + c = 3500$

$9b = 25500 - 5c$ $\qquad$ $b = 3500 - c$

$b = \frac{25500-5c}{9}$





$\boldsymbol{c = 1500}$ **and** $\boldsymbol{b = 2000}$

b. Based on her calculator work, explain how Rita knew that **c** equaled 1500 and not the other way around.

6. After reviewing the **substitution**, **elimination**, and **graphing** method, identify one advantage and one disadvantage of each method. When might it make the most sense to use each method?

7. Solve each problem involving a system of two linear equations algebraically by using either substitution or elimination.

a. $2x - 6y = 28$
$-x - y = 14$

b. $-2x + 2y = -16$
$-3x - 2y = 11$

c. $-5x - 14y = -9$
$-10x - 7y = 3$

d. $y = 4x - 5$
$-x - 8y = 7$

8. To participate in a school trip, Kim had to earn $85 in one week. Kim could earn $8 per hour babysitting and $15 dollars per hour for yard work. Kim's parents limit work time to 8 hours per week. How many hours should Kim work at each job in order to meet her income goal and work exactly eight hours?

9. Solve the following Regents question from the June 2019 exam.

When visiting friends in a state that has no sales tax, two families went to a fast-food restaurant for lunch. The Browns bought 4 cheeseburgers and 3 medium fries for $16.53. The Greens bought 5 cheeseburgers and 4 medium fries for $21.11.

Using c for the cost of a cheeseburger and f for the cost of medium fries, write a system of equations that models this situation.

The Greens said that since their bill was $21.11, each cheeseburger must cost $2.49 and each order of medium fries must cost $2.87 each. Are they correct? Justify your answer.

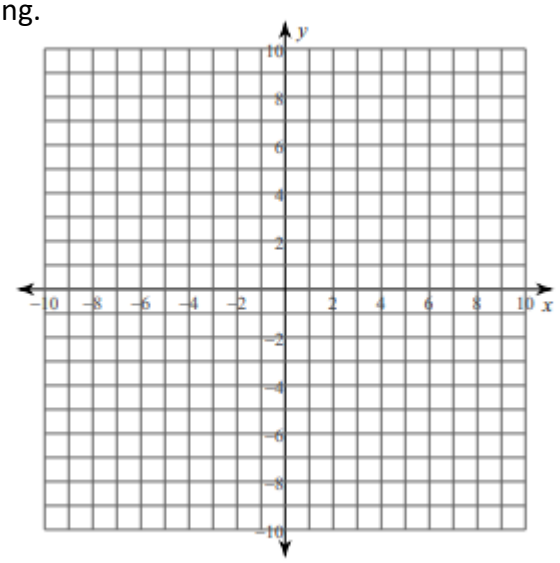
Using your equations, algebraically determine both the cost of one cheeseburger and the cost of one order of medium fries.

10. Solve the following systems using graphing.

a.

$$y = -\frac{5}{6}x + 2$$

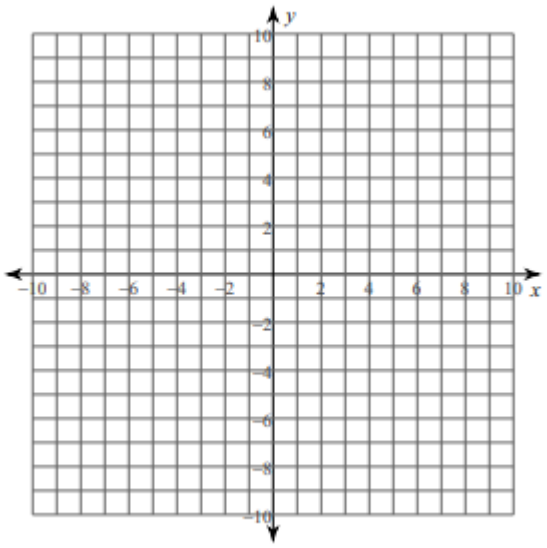
$$y = \frac{1}{6}x + 8$$



b.

$$y = \frac{3}{2}x + 1$$

$$y = -\frac{1}{4}x - 6$$



Jess was reading the following part from problem 8:

To participate in a school trip, Kim had to earn $85 in one week. Kim could earn $8 per hour babysitting and $15 dollars per hour for yard work. Kim's parents limit work time to 8 hours per week.

11. Jess thought aloud to her partner, "Kim wants to earn at least $85, but she could make more than that. Also, her parents limit her work to 8 hours, but she could work less than that if she wanted."

a. Assuming Kim makes $8 babysitting and $15 per hour for yard work, determine three different ways she could work to earn at least $85 and also work 8 hours or less.

b. The system of equations from problem 8 is written below, but the equal signs have been removed. Replace them with the proper inequality symbol <, >, ≤, ≥.

$$b + y \quad 8$$

$$8b + 15y \quad 85$$

c. In Algebra 1, we solve **systems of inequalities** graphically. The graph to this system of inequalities is below. On the graph, plot your three answers to part a as points on the graph. What do you notice about the location of your three points?

d. One of the inequality graphs uses a dotted line and the other uses a solid line. Why do you think this is?

e. The dotted-line-inequality graph is shaded below the dotted line and the solid-line-inequality graph is shaded above the solid line. What features about the algebraic inequalities from part b explain why this is?

f. The student who produced the graph in part c, Ernest, explained his process. "First, I solved each inequality for *y* just like I would solve a normal equation. Then I typed those into Y= and looked at the table to plot the points of each line graph. Then I used the inequality symbols to decide which line had to be dotted or solid, and which way to shade."

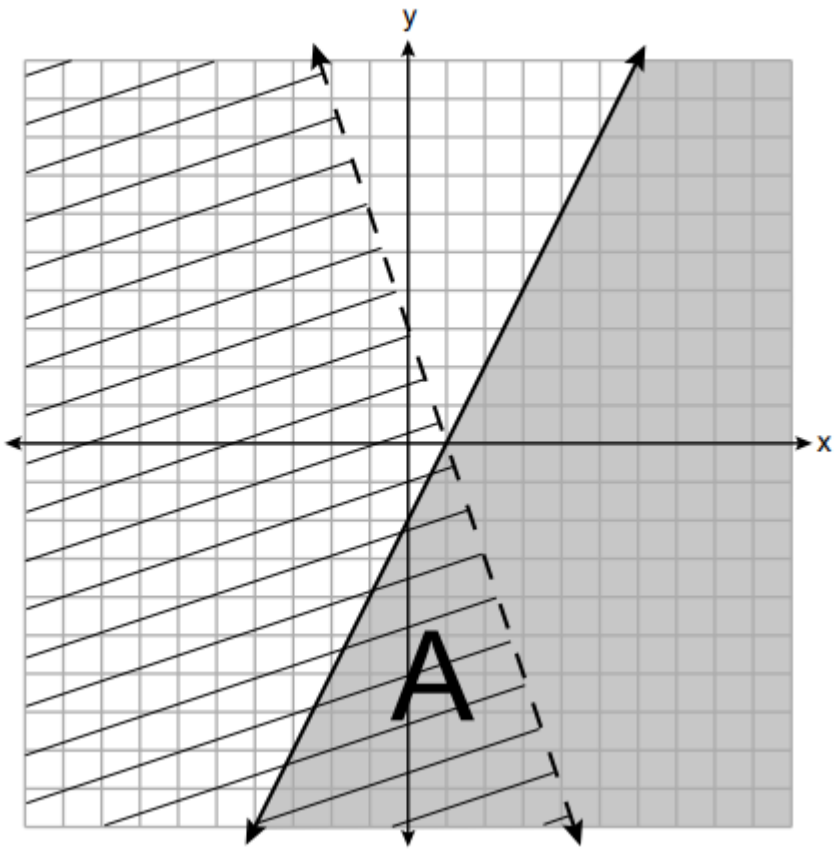
i. Show the algebraic work Ernest might have used to solve each inequality for *y*.

ii. What did Ernest mean when he wrote, "I used the inequality symbols to decide which line had to be dotted or solid, and which way to shade"?

g. The double-shaded region on the graph is called the **solution set**. Explain what the solution set of this graph represents in the context of Kim's school trip situation.

12. Solve the following question from the June 2019 Regents exam.

A system of inequalities is graphed on the set of axes below.



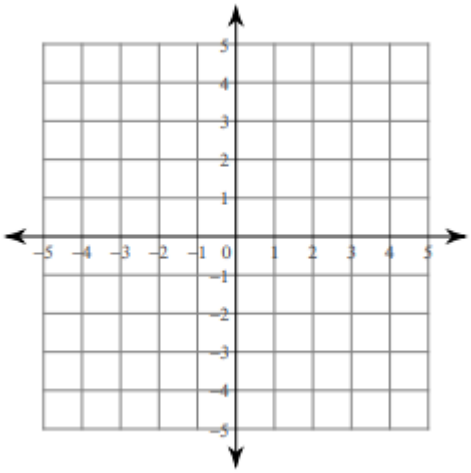
State the system of inequalities represented by the graph.

State what region A represents.

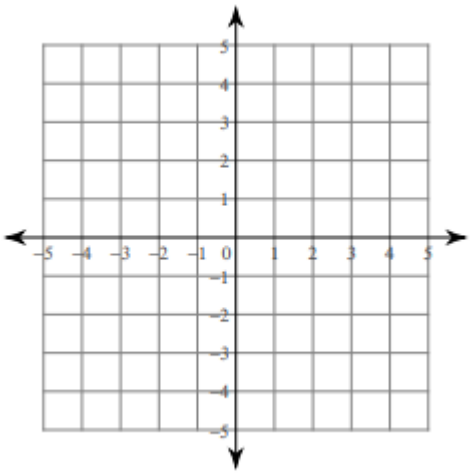
State what the entire gray region represents.

13. Sketch the solution to each system of inequalities. **Label each solution set with the letter S**.

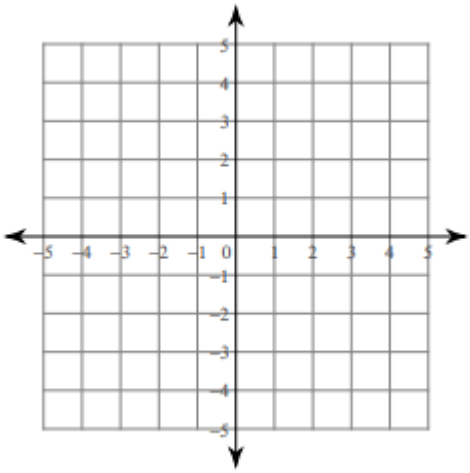
a. $y > 4x - 3$
$y \geq -2x + 3$



b. $4x - 3y < 9$
$x + 3y > 6$



c. $x + y \geq 2$
$4x + y \geq -1$



d. $3x + y \geq -3$
$x + 2y \leq 4$