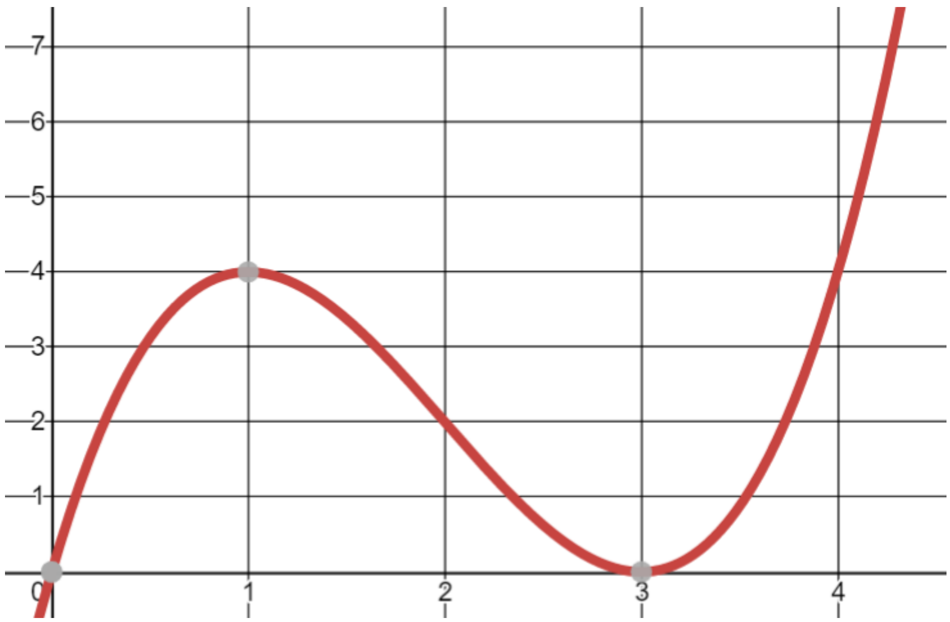
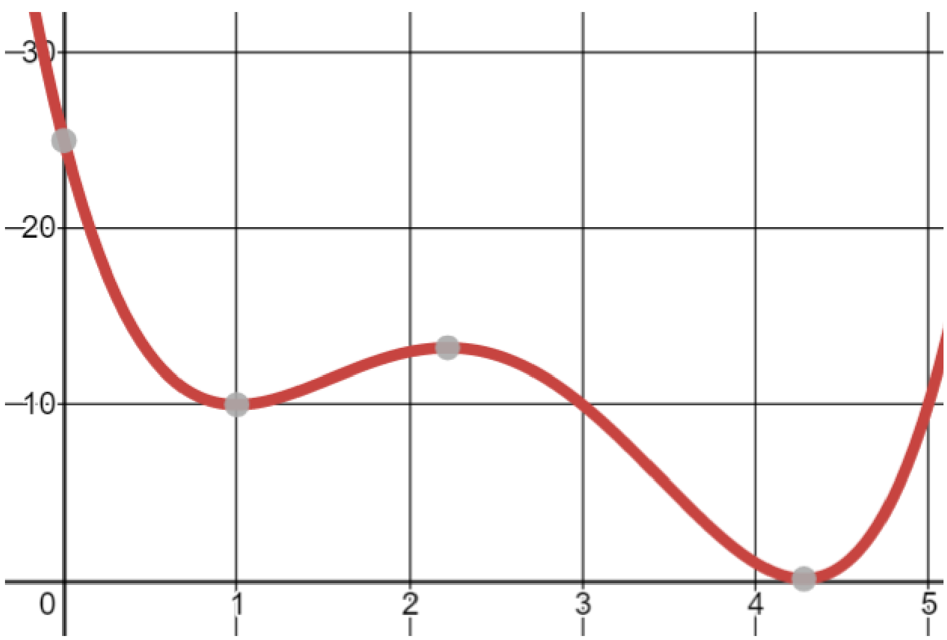
Lesson 6 – An Overview of Polynomials

Francisca is an engineer who designs roller coasters. She is designing two sections of a new roller coaster for Six Flags theme park. Using ***regression***, she generates two **polynomials** to model her track designs.

Section 1 $\boldsymbol{f(x) = x^3 - 6x^2 + 9x}$



Section 2 $\boldsymbol{g(x) = x^4 - 10x^3 + 32x^2 - 38x + 25}$



Polynomials are a very important family of functions that include two types of functions you have studied so far. All linear and quadratic functions are part of the polynomial function family. Polynomial rules can be written in **standard** or sometimes in **factored** form. Both $\boldsymbol{f(x)}$ and $\boldsymbol{g(x)}$ are standard form. Consider all the following examples written in standard form.

$f(x) = x^3 - 6x^2 + 9x$ **Degree 3, l.c. is 1**

$g(x) = 4x^4 - 10x^3 + 32x^2 - 38x + 25$ **Degree 4, l.c. is 4**

$h(x) = -2x^2 - 7x + 15$ **Degree 2, l.c. is -2**

$k(x) = 5x + 1$ **Degree 1, l.c. is 5**

$m(x) = x^6 + 4x^3 - 7x$ **Degree 6, l.c. is 1**

$n(t) = 16t^8 - 4$ **Degree 8, l.c. is 16**

1) What do you believe are the important characteristics of a standard form polynomial?

Students should notice that all polynomials have whole number positive exponents on the variable and multiple terms separated by addition and subtraction. Note that linear functions, such as k(x) have an invisible exponent of 1 on the x.

2) You may have noticed that polynomials all have exponents on the variable. Even $\boldsymbol{k(x)}$ has an exponent of 1 on the **x** that is usually not written. A polynomial's **degree** is the value of its highest exponent. Next to each polynomial function above, write its degree next to its function rule. For example, you can write ***"degree 3"*** or "**deg 3**" next to $\boldsymbol{f(x)}$.

The **highest degree term** is the term that has the biggest exponent on the variable. Standard form polynomials are always written from the highest degree term to the lowest degree term. A term with no variable is called a **constant**.

3) A coefficient is the constant that multiplies the variable. The coefficient of $\boldsymbol{-6x^5}$ is $\boldsymbol{-6}$. The coefficient of the highest degree term is called the **leading coefficient**. Write leading coefficient for each standard form polynomial above. For example, on page 1, in Section 2, the leading coefficient of $\boldsymbol{g(x)}$ is **4** so you could write "**l.c. is 4**" next to the rule for $\boldsymbol{g(x)}$**.**

4) Rewrite each polynomial below in standard form. Then state its degree and the value of its leading coefficient.

a. $4x^3 + 5x^5 - 4 - 2x^2$

$\boldsymbol{5x^5 + 4x^3 - 2x^2 - 4}$**, degree 5, l.c. is 5**

b. $12 - 3x^2 + 7x - 3x^4$

$\boldsymbol{-3x^4 - 3x^2 + 7x + 12}$**, degree is 4, l.c. is -3**

Here are a selection of polynomials written in **factored form.**

$$a(x) = (x + 2)(x - 7)$$

$$b(x) = x(x + 3)(x + 5)$$

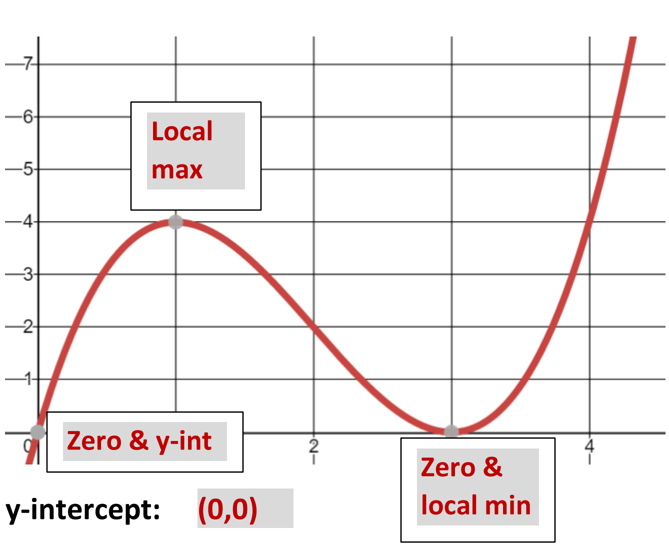
$$c(x) = (x - 4)(x + 2)(x - 1)$$

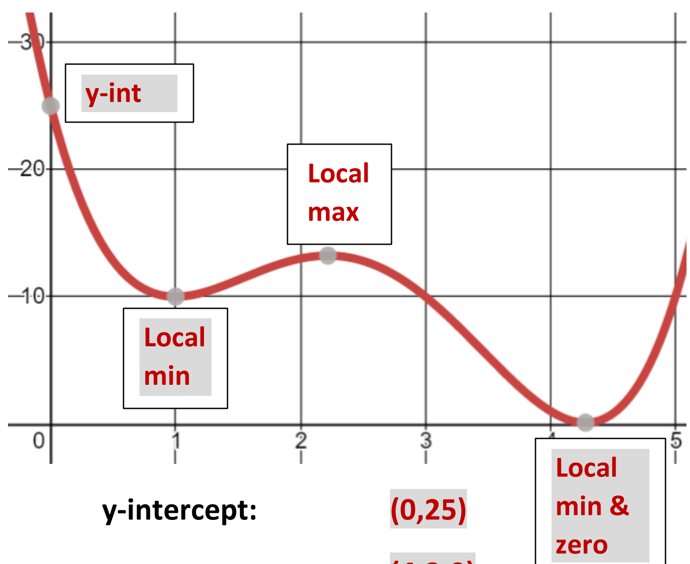
What do you think are the important characteristics of a factored form polynomial?

Factored form are linear functions, usually in parentheses, that are separated by multiplication.

Polynomials have some important features. Recall the vertex of a quadratic function was its turning point. Polynomial graphs can have many turning points. When a turning point makes a little "hill" it is called a **local maximum**. When a turning point makes a "valley" we call it a **local minimum**. Similarly, polynomials also have a **y-intercept**, and their **x-intercepts** are called **zeros** or **roots**.

5) Label Francisca's roller coaster track designs' local minimum(s), local maximum(s), roots, and zeros below and write their coordinates.





y-intercept: **(0,0)**

zeros: **(0,0) and (3,0)**

local maximum(s): **(1,4)**

local minimum(s): **(3,0)**

y-intercept: **(0,25)**

zero: **(4.3,0)**

local minimum(s): **(1,10) and (4.3,0)**

local maximum(s): **(2.2, 13)**

(Okay to estimate)

6) Try to notice some patterns.

a. What do the coordinates of the y-intercepts share in common?

The x-coordinate is always zero

b. What do the coordinates of the zeros share in common?

The y-coordinate it always zero

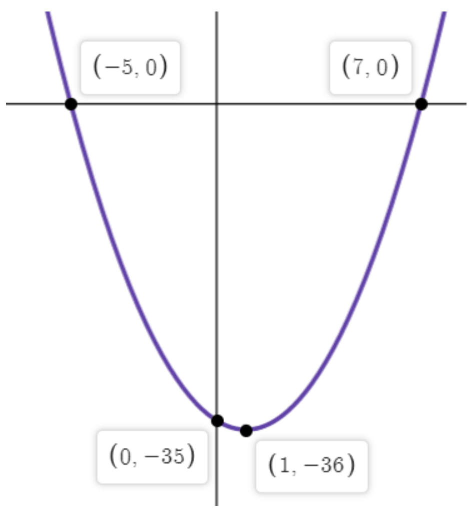
The important features of polynomials can be located using the graphing calculator and analyzing either a table or graph. The next problem seeks to highlight some of the advantages to representing polynomials in either factored or standard form.

7) In each part below, you will be given the standard form and equivalent factored form of a polynomial function. Then use a calculator table or graph to locate the coordinates of the function's y-intercept, root(s), and local maximum and/or minimum points. State the degree and the value of its leading coefficient. Sketch a graph of each polynomial function. Provide evidence for how you found your coordinates.

a. $f(x) = (x+5)(x-7)$ = $x^2 - 2x - 35$

Degree: **2** **Leading Coefficient:** **1**

Sketch:



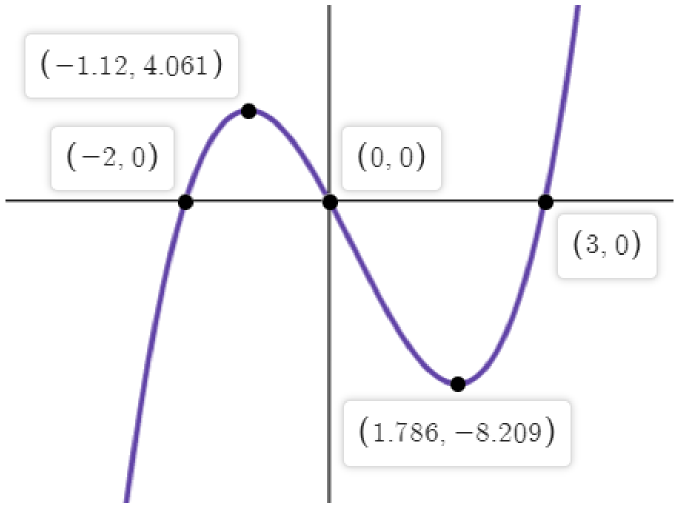
y-intercept: **(0,-35)** **zero(s):** **(-5,0) and (7,0)**

Local maximum(s): **none** **Local minimum(s):** **(1,-36)**

b. $g(x) = x(x+2)(x-3)$ = $x^3 - x^2 - 6x$

Degree: 3 **Leading Coefficient:** 1

Sketch:



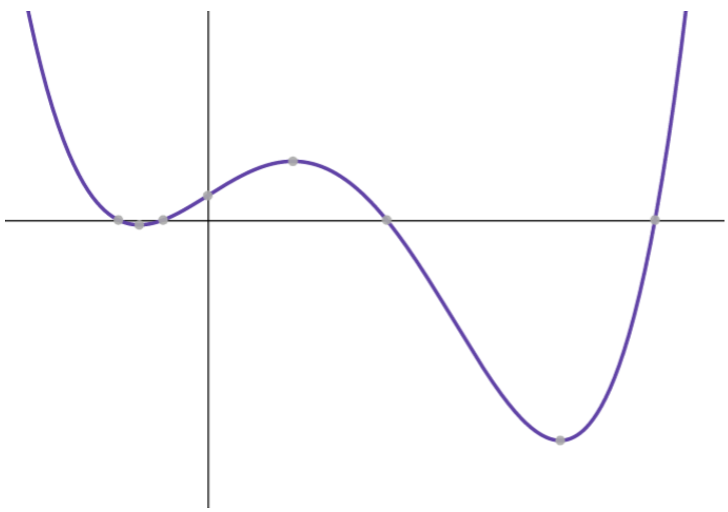
y-intercept: (0,0) **zero(s):** (-2,0), (0,0), and (3,0)

Local maximum(s): (-1.12,4.061) **Local minimum(s):** (1.786,-8.209)

c. $k(x) = 2x^4 - 11x^3 + 23x + 10$ = $(x+1)(2x+1)(x-5)(x-2)$

Degree: 4 **Leading Coefficient:** 2

Sketch:



y-intercept: (0,10) **zero(s):** (-1,0), (-0.5,0), (2,0), (5,0)

Local maximum(s): (0.952, 24.048) **Local minimum(s):** (3.94, -90.209)

8) Jared and Karen were working together on problem 7. Jared notices "you can find the zeros if you look at the factored form. Just switch the sign in the middle and it tells you." Karen looked at her work and noticed "that only sometimes work. Look at part c. When I checked with the graph the zeros were -1, 5, 2, but also -0.5."

a. Do you believe that the factored form $\boldsymbol{k(x) = (x+1)(2x+1)(x-5)(x-2)}$ can be helpful to determine that the zeros are -1, 5, 2, and -0.5? Which factor helps to find each zero?

-1 goes with (x+1), 5 goes with (x-5), 2 goes with (x-2), and -0.5 goes with (2x+1)

b. Substitute each number into the factor that you believe it belongs with. What number do you get? Why do you think these numbers are called zeros?

You get zero. These x-values are called zeros because the function's y-value is zero when you plug in each of these x-values.

c. Karen had an idea. "We are looking for the *x* that makes each factor zero. What if I make each factor an equation that equals zero and solve for $\boldsymbol{x}$." Verify that this strategy works for each factor of $\boldsymbol{k(x)}$.

-1 + 1 = 0, 2(0.5) + 1 = 0, 5 – 5 = 0, 2 – 2 = 0

9) Analyze your results from problem seven. Identify which form is best for finding a polynomial's degree, leading coefficient, x-intercepts, y-intercept, and coordinates of its local maximum and/or minimum point(s). Briefly explain how you can use that form to identify each important characteristic.

In general, standard form is best for determining the degree, leading coefficient, and y-intercept. The y-intercept is the value of the constant term. The factored form is best for finding the zeros/roots. To find them, set each factor equal to zero and solve for x. The graph is the best way to locate the local max and min points. To find them when viewing a graph, press 2nd TRACE, select minimum or maximum, then choose a point to the left, right, and near the max or min point.

10) How can you use the calculator table or graph to find a polynomial's zeros and y-intercept?

In the table, the zeros are the x-values where y is zero. On a graph, the zeros are the x-intercepts.
The y-intercept is the y-value where x is zero on the table. On a graph, it is the location where the graph intersects the y-axis.

11) Write a polynomial function that has zeros at $x = -2$, $x = 4$, and $x = 1$. Use your calculator to verify that the function rule you write has these zeros.

$$y = (x + 2)(x - 4)(x - 1)$$

12) Answer the following Regents questions. Use your knowledge of algebra as well as your graphing calculator to reach the correct answer.

a.

Which expression is equivalent to $2(x^2 - 1) + 3x(x - 4)$?

(1) $5x^2 - 5$

(2) $5x^2 - 6$

(3) $5x^2 - 12x - 1$

(4) $5x^2 - 12x - 2$

Choice 4

Enter $2(x^2 - 1) + 3x(x - 4)$ into Y1, and check each answer choice in Y2 for tables that match

b.

Josh graphed the function $f(x) = -3(x - 1)^2 + 2$. He then graphed the function $g(x) = -3(x - 1)^2 - 5$ on the same coordinate plane. The vertex of $g(x)$ is

(1) 7 units below the vertex of $f(x)$

(2) 7 units above the vertex of $f(x)$

(3) 7 units to the right of the vertex of $f(x)$

(4) 7 units to the left of the vertex of $f(x)$

Choice 1

c.

The expression $16x^2 - 81$ is equivalent to

(1) $(8x - 9)(8x + 9)$

(2) $(8x - 9)(8x - 9)$

(3) $(4x - 9)(4x + 9)$

(4) $(4x - 9)(4x - 9)$

Choice 3

d.

A ball is thrown into the air from the top of a building. The height, $h(t)$, of the ball above the ground t seconds after it is thrown can be modeled by $h(t) = -16t^2 + 64t + 80$. How many seconds after being thrown will the ball hit the ground?

(1) 5 (3) 80

(2) 2 (4) 144

Choice 1

Enter h(t) as Y1 and view the table to find the x-value when Y1 = 0

e.

Which equation is equivalent to $y = x^2 + 24x - 18$?

(1) $y = (x + 12)^2 - 162$ (3) $y = (x - 12)^2 - 162$

(2) $y = (x + 12)^2 + 126$ (4) $y = (x - 12)^2 + 126$

Choice 1

f.

When $(x)(x - 5)(2x + 3)$ is expressed as a polynomial in standard form, which statement about the resulting polynomial is true?

(1) The constant term is 2.

(2) The leading coefficient is 2.

(3) The degree is 2.

(4) The number of terms is 2.

Choice 2

g.

The quadratic functions $r(x)$ and $q(x)$ are given below.

x	r(x)
−4	−12
−3	−15
−2	−16
−1	−15
0	−12
1	7

$$q(x) = x^2 + 2x - 8$$

The function with the *smallest* minimum value is

(1) $q(x)$, and the value is -9

(2) $q(x)$, and the value is -1

(3) $r(x)$, and the value is -16

(4) $r(x)$, and the value is -2

Choice 3