

# Lesson 1 – Functions

## Function Definition and Representations

A **function** is one of the most important concepts in Algebra. Consider the following examples.

Tony and Maria attend different schools that each have a vending machine in the cafeteria.

- Tony’s favorite snack, potato chips, are in the location labeled A7. Each time he pays and inputs A7, potato chips come out.
- Maria’s favorite snack, chocolate bars, are in the location labeled B4. Each time she pays and inputs B4, the vending machine drops a chocolate bar, but also mixed nuts.



Something is a **function** if every x-value (or input) in the domain is assigned to only one y-value (or output) in the range.

1. Based on this definition, which person’s vending machine would be an example of something that is a function? Explain how you know.

**Tony’s vending machine matches the definition of a function. Each time he inputs A7; only potato chips come out.**

2. Explain why the other person’s vending machine is not a function.

**Maria’s vending machine is not a function since for one input (B4) there are two outputs (chocolate bar and mixed nuts).**

The **domain** of a function is the set of all inputs with outputs. The set of all outputs a function has is its **range**.

3. In the example of the vending machine, what would represent the domain and range?

**Domain:** The button combinations for each snack. **Educator Note:** Students may suggest money as an input too, which is correct.

**Range:** The snacks

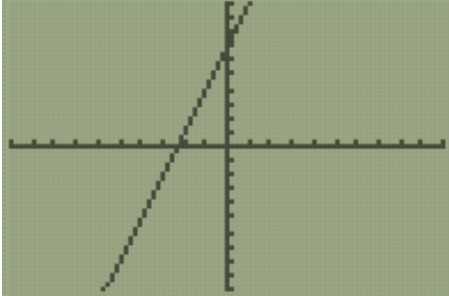
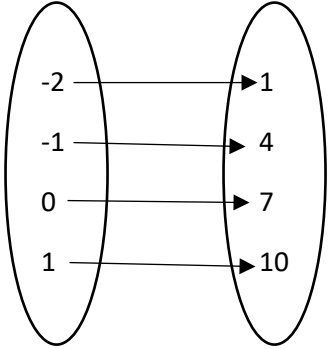
A function is usually some mathematical rule that tells you what to do with  $x$  in order to get  $y$ .

Consider the linear function

$$y = 3x + 7$$

This rule tells you to multiply each  $x$ -value by 3 then add 7 to get the  $y$ -value.

Here are other ways to represent  $y = 3x + 7$ .

<p>Graph</p> 	<p>Table</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 33%;">X</th> <th style="width: 33%;">Y<sub>1</sub></th> <th style="width: 33%;"></th> </tr> </thead> <tbody> <tr><td>-3</td><td>-2</td><td></td></tr> <tr><td>-2</td><td>1</td><td></td></tr> <tr><td>-1</td><td>4</td><td></td></tr> <tr><td>0</td><td>7</td><td></td></tr> <tr><td>1</td><td>10</td><td></td></tr> <tr><td>2</td><td>13</td><td></td></tr> <tr><td>3</td><td>16</td><td></td></tr> </tbody> </table> <p>X=0</p>	X	Y <sub>1</sub>		-3	-2		-2	1		-1	4		0	7		1	10		2	13		3	16		<p>Arrow Diagram (mapping)</p> 
X	Y <sub>1</sub>																									
-3	-2																									
-2	1																									
-1	4																									
0	7																									
1	10																									
2	13																									
3	16																									

Every  $x$ -value has exactly one  $y$ -value.

4. Solve the following Regents question.

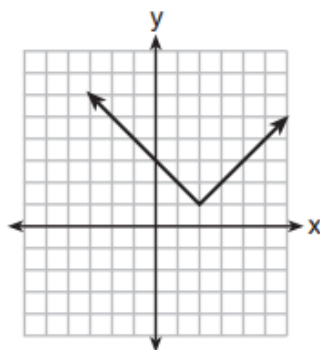
7 Which relation does *not* represent a function?

<b>x</b>	1	2	3	4	5	6
<b>y</b>	3.2	4	5.1	6	7.4	8.8

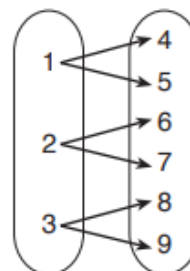
(1)

$$y = 3\sqrt{x+1} - 2$$

(3)



(2)



(4)

**Choice 4 is not a function since each input goes to two different outputs.**

An equivalent way to write this linear function is with function notation:  $f(x) = 3x + 7$

You say “**f of x**” when you see  $f(x)$  and it is another way of writing the variable  $y$ .

Function notation is used to input numbers for  $x$ .

For instance, consider  $f(8)$ . This means, “find the  $y$ -value when  $x$  equals 8.” For a simple linear function such as  $f(x) = 3x + 7$ , you may be able to find  $y$  in your head, or with the home screen of your calculator. Here is how to calculate  $f(8)$  *algebraically*.

$$f(8) = 3 \cdot 8 + 7$$

$$f(8) = 24 + 7$$

$$f(8) = 31$$

5. Determine the value of  $f(3)$ ,  $f(25)$ , and of  $f(-6)$  algebraically.

$$f(3) = 3(3) + 7 = 9 + 7 = 16$$

$$f(25) = 3(25) + 7 = 82$$

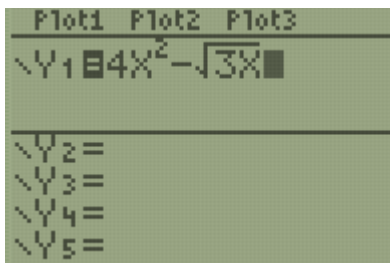
$$f(-6) = 3(-6) + 7 = -18 + 7 = -11$$

Linear functions like  $f(x) = 3x + 7$  are the easiest ones to work with.

## Using the TABLE

Consider a non-linear function  $g(x) = 4x^2 - \sqrt{3x}$ . Notice that this function has been named  $g(x)$  “ $g$  of  $x$ ”. We can use any letter to name a function, which is helpful when a problem involves more than one function. Let’s explore  $g(x)$  with the calculator.

Every function can be represented with a table or a graph.



All function rules can be put into **Y=**

The 2 on top of the  $x$  is an exponent. To get an exponent, **press the ^ key** and then the value of the exponent. Since 2 is a very common exponent, you can also press **the  $x^2$  key**. Notice that anything you type will stay in the exponent until you press the right arrow button  $\rightarrow$ .

Notice that this function involves the square root of  $3x$ . This symbol  $\sqrt{\quad}$  is called a radical. Find the radical symbol on your calculator by pressing **2<sup>nd</sup>** and  $x^2$ .

From the **Y=** menu, we can find specific values of  $g(x)$  by using tables or graphs.

Alejandro wants to find  $g(4)$  using a table.

X	Y <sub>1</sub>
1	2.2679
2	13.551
3	33
4	60.536
5	96.127
6	139.76
7	191.42

X=4

Press **2<sup>nd</sup>**, **GRAPH**

Use the up and down arrows until you get to  $x = 4$ .

$$g(4) = 60.536$$

Next, Alejandro wants to find  $g(528)$  using a table. He definitely doesn't want to press the down arrow that many times.

TABLE SETUP	
TblStart=	528
ΔTbl=	1
Indent:	Auto Ask
Depend:	Auto Ask

He presses **2<sup>nd</sup>**, **WINDOW** and changes **TblStart** to 528

$\Delta Tbl = 1$  means that the x-values will count by 1.

Don't worry about the rest.

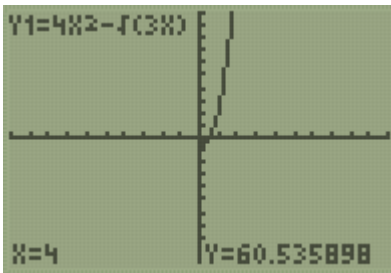
X	Y <sub>1</sub>
528	1115096.2005
529	1.12E6
530	1.12E6
531	1.13E6
532	1.13E6
533	1.14E6
534	1.14E6

Y<sub>1</sub>=1115096.2005

When Alejandro presses **2<sup>nd</sup>** **GRAPH**, his table starts with  $x = 528$ . The y-value is too large to display properly unless he highlights it using the arrow keys. This sometimes happens when  $x$  is very large.

Alejandro concludes that  $g(528) = 1115096.2005$

## Using the GRAPH



Tatiana is working with the same function,  $g(x) = 4x^2 - \sqrt{3x}$

Tatiana wants to find  $g(4)$  using a graph.

Press **GRAPH**. If your viewing window is different, press **ZOOM, 6** to get a standard window.

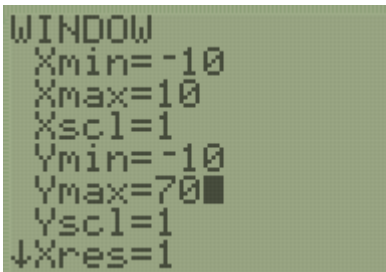
Press **TRACE, 4, ENTER**

Tatiana notices that when  $x = 4$ ,  $y = 60.535898$ .

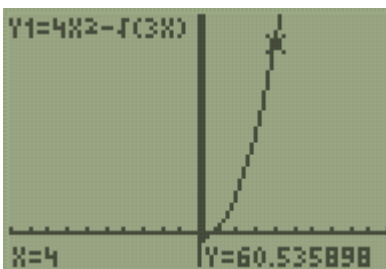
**Alejandro's and Tatiana's answers for  $g(4)$  are slightly different, but that's okay.**

Tatiana notices that her calculator tells her that  $g(4) = 60.535898$  at the bottom of her screen, but that she can't see the  $y$ -value 60.535898 on the graph of the function. She counts along the  $y$ -axis and notices that it only goes up to 10.

She decides to make her graph show more  $y$ -values.



She presses **WINDOW** and sets her **maximum  $y$ -value** to 70.



When she presses **GRAPH**, the  $y$ -axis goes up to 70.

When she presses **TRACE, 4, ENTER**, she can see the location of  $g(4)$  on the graph.

In general, make the **Xmax** and **Ymax** higher than the number you want, and make the **Xmin** and **Ymin** lower than the number you want. You never need to change the **Xscl** or **Yscl**, but if you do, they tell the calculator how far to space the marks on the  $x$ -axis and  $y$ -axis.

Try the following Regents questions. Try to use the graphing calculator tables and graphs, and what you know about functions, to answer them.

6.

The function  $g(x)$  is defined as  $g(x) = -2x^2 + 3x$ . The value of  $g(-3)$  is

(1)  $-27$

(3)  $27$

(2)  $-9$

(4)  $45$

**Choice 1**

7.

If  $k(x) = 2x^2 - 3\sqrt{x}$ , then  $k(9)$  is

(1)  $315$

(3)  $159$

(2)  $307$

(4)  $153$

**Choice 4**

8.

Marc bought a new laptop for \$1250. He kept track of the value of the laptop over the next three years, as shown in the table below.

Years After Purchase	Value in Dollars
1	1000
2	800
3	640

Which function can be used to determine the value of the laptop for  $x$  years after the purchase?

(1)  $f(x) = 1000(1.2)^x$

(3)  $f(x) = 1250(1.2)^x$

(2)  $f(x) = 1000(0.8)^x$

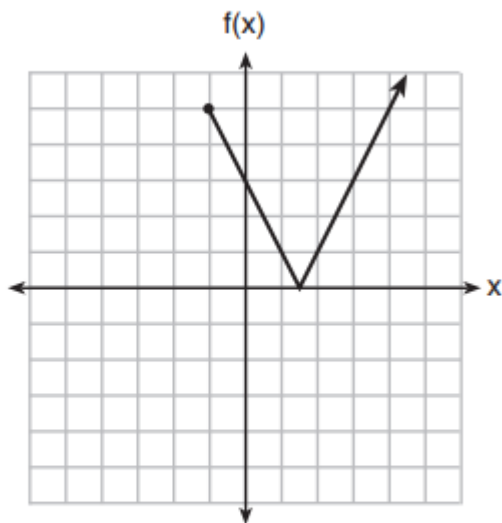
(4)  $f(x) = 1250(0.8)^x$

**Choice 2**

**Educator Note:** Try each choice in  $Y=$  and choose the answer with the same table

9.

The function  $f(x)$  is graphed below.



The domain of this function is

(1) all positive real numbers      (3)  $x \geq 0$

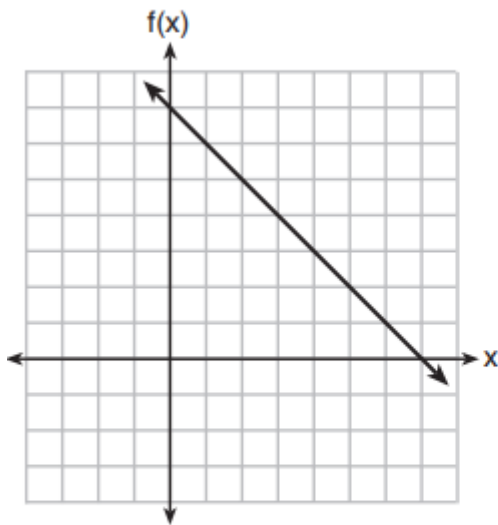
(2) all positive integers          (4)  $x \geq -1$

**Choice 4**

**Educator Note:** Notice that there are no  $y$ -values for  $x$ -values below  $-1$ .

10.

The functions  $f(x)$ ,  $q(x)$ , and  $p(x)$  are shown below.



$$q(x) = (x - 1)^2 - 6$$

$x$	$p(x)$
2	5
3	4
4	3
5	4
6	5

When the input is 4, which functions have the same output value?

- (1)  $f(x)$  and  $q(x)$ , only                      (3)  $q(x)$  and  $p(x)$ , only  
 (2)  $f(x)$  and  $p(x)$ , only                      (4)  $f(x)$ ,  $q(x)$ , and  $p(x)$

**Choice 4**

11.

Materials  $A$  and  $B$  decay over time. The function for the amount of material  $A$  is  $A(t) = 1000(0.5)^{2t}$  and for the amount of material  $B$  is  $B(t) = 1000(0.25)^t$ , where  $t$  represents time in days. On which day will the amounts of material be equal?

- (1) initial day, only                      (3) day 5, only  
 (2) day 2, only                              (4) every day

**Choice 4**



# Lesson 2 – Linear and Exponential Functions

Think about your own family. Everyone in your family has things about them that make them unique. But now think about what makes your family similar. Maybe it's the way you speak, or dress, or behave that makes the people in your family similar.

Just like you, certain functions belong to a family. In Algebra 1, we focus on four main families: linear, exponential, quadratic, and absolute value.

This lesson will aim to deepen your understanding of the characteristics of linear and exponential function “families.” We will also learn a technique, called **regression**, which will allow us to write a function rule using a set of coordinates.

## Linear Functions

The following four situations are examples of linear functions.

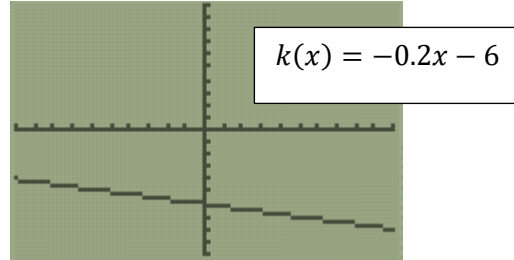
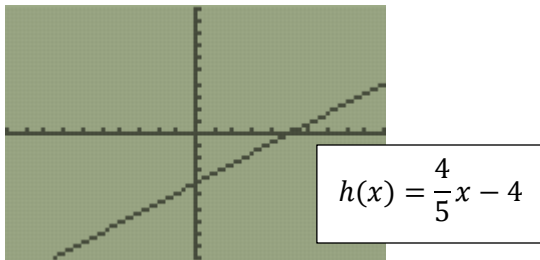
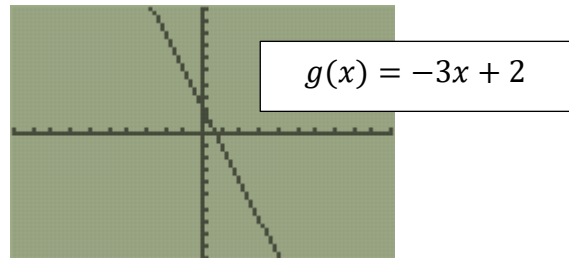
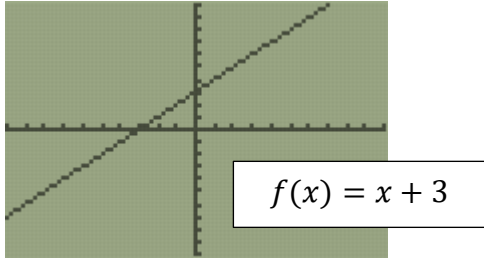
- Pedro earns \$12 per hour at his job.
- A plant's height increases 3 inches per week.
- As water empties from a bathtub, its volume decreases at a rate of 2 gallons per minute.
- A car's distance traveled is changing at a constant speed of 55 miles per hour.

Write all the ways these situations are similar and how they are different.

**Each of these situations involve something that increases or decreases by a constant amount. Pedro's job, the plant's height, and the car's distance are all increasing, while the volume of water is decreasing. Each situation starts with a different amount and changes by a different amount.**

***Educator Note:*** You may want to discuss the car's distance scenario in more detail as many students truly do not understand speed as a set amount of distance traveled each hour. Encourage students to represent each situation using a table and ask them probing questions about how a graph of each situation might look and why they think so.

The following graphs and rules are linear functions.



What is similar about the graphs of linear functions? What are some differences between the graphs?

**Each graph is a perfectly straight line. Two of the lines slope (point) upward from left to right while the other two slope downward.**

How are the linear function rules similar to one another? How are the linear function rules different?

**Each rule has  $x$  in it. Each rule has a number that is added to or subtracted from the  $x$ -term. Three out of the four rules show a number in front of  $x$ . When there is no number in front of  $x$  that is showing it is 1, so all four rules have a number that multiplies  $x$  and a constant that is added or subtracted.**

***Educator Note:*** Lead a discussion to help students connect that the lines slope down when you multiply  $x$  by a negative and slope up when  $x$  is multiplied by a positive value. The number added or subtracted to the  $x$ -term is the starting location on the  $y$ -axis. However, if students don't notice it yet, don't feel the need to force the ideas.

Use your calculator to fill in tables for  $f$ ,  $g$ ,  $h$ , and  $k$  from  $x = -5$  to  $x = 5$ .

$x$	$f(x)$ $= x + 3$
-5	<b>-2</b>
-4	<b>-1</b>
-3	<b>0</b>
-2	<b>1</b>
-1	<b>2</b>
0	<b>3</b>
1	<b>4</b>
2	<b>5</b>
3	<b>6</b>
4	<b>7</b>
5	<b>8</b>

$x$	$g(x)$ $= -3x + 2$
-5	<b>17</b>
-4	<b>14</b>
-3	<b>11</b>
-2	<b>8</b>
-1	<b>5</b>
0	<b>2</b>
1	<b>-1</b>
2	<b>-4</b>
3	<b>-7</b>
4	<b>-10</b>
5	<b>-13</b>

$x$	$h(x)$ $= \frac{4}{5}x - 4$
-5	<b>-8</b>
-4	<b><math>-\frac{36}{5}</math></b>
-3	<b><math>-\frac{32}{5}</math></b>
-2	<b><math>-\frac{28}{5}</math></b>
-1	<b><math>-\frac{24}{5}</math></b>
0	<b>-4</b>
1	<b><math>-\frac{16}{5}</math></b>
2	<b><math>-\frac{12}{5}</math></b>
3	<b><math>-\frac{8}{5}</math></b>
4	<b><math>-\frac{4}{5}</math></b>
5	<b>0</b>

$x$	$f(x)$ $= -0.2x - 6$
-5	<b>-5</b>
-4	<b>-5.2</b>
-3	<b>-5.4</b>
-2	<b>-5.6</b>
-1	<b>-5.8</b>
0	<b>-6</b>
1	<b>-6.2</b>
2	<b>-6.4</b>
3	<b>-6.6</b>
4	<b>-6.8</b>
5	<b>-7</b>

What similarities and differences do you notice about the tables of linear function rules?

**When  $x$  increases by 1, the  $y$  changes by the number that multiplies the  $x$  in the rule. This is the rate of change.**

**The  $y$ -value when  $x$  is zero is the constant amount that is added to or subtracted from  $x$ . This is the starting value and the  $y$ -intercept.**

**Educator Note:** Some students may state that  $f(x)$  is increasing, so you may point out that when a number is becoming “more negative” it is actually decreasing. Temperature is a good analogy. If students struggle to reason with fractions, you may point out that  $h(x) = \frac{4}{5}x - 4$  is equivalent to  $y = 0.8x - 4$ .

Every linear function rule can be written in the form

$$y = mx + b$$

The  $m$  and  $b$  represent numbers. Here are important ideas about them that you may have noticed from the similarities and differences work you just did.

- | $m$   | $b$  |
|---|--|
| <ul style="list-style-type: none"> <li>• Also called the <u>rate of change</u></li> <li>• It is the <u>slope</u> of the graph</li> <li>• How much the <math>y</math>-values increases or decreases by when the <math>x</math>-value increases by 1</li> <li>• When <math>m</math> is positive, the graph <u>increases</u> as <math>x</math> increases from left to right</li> <li>• When <math>m</math> is negative, the graph <u>decreases</u> as <math>x</math> increases from left to right</li> </ul> | <ul style="list-style-type: none"> <li>• The value of <math>y</math> when <math>x = 0</math></li> <li>• It is where the graph intersects the <math>y</math>-axis</li> <li>• The starting amount for linear situations</li> </ul> |

## Exponential Functions

Exponential functions are a type of nonlinear function, because their patterns do not result in straight lines. The following situations are examples of exponential functions.

- A. 500 bacteria double every hour.
- B. A car bought for \$25000 is worth half of its value each year.
- C. A savings account has \$1000 initially and gains 5% interest annually.
- D. A school with 900 students is decreasing its enrollment by 2% each year.

What do each of these situations have in common? How are they different?

**Each situation is changing by multiplying or dividing. Situations A and C are increasing and situations B and D are decreasing. Situations C and D are changing by a percent while A and B multiply or divide by 2. B and C are situations involving money while A and D model population change.**

Here are rules for each of the situations A through D.

$$A(x) = 500(2)^x$$

$$B(x) = 25000 \left(\frac{1}{2}\right)^x$$

$$C(x) = 1000(1.05)^x$$

$$D(x) = 900(0.98)^x$$

How are these exponential rules similar? How are they different?

**Each rule has an  $x$  as an exponent. Each rule shows a constant times another number that is raised to the power of  $x$ . Students may point out that one rule has a fraction in the parentheses, two rules have decimals in the parentheses, and one rule uses a whole number.**

What connections do you see between the situation described and its function rule?

**The starting amount is the number that appears outside of the parentheses. The amount that each situation multiplies by to get the next amount is the number that is in the parentheses.**

**Educator Note:** The multiplier for situations A and B are more obvious than the multiplier for C and D. You may wish to explore situations C and D more with additional situations that lead students to see that when a situation changes by a percent, the multiplier is  $1 + \frac{\%}{100}$  or  $1 - \frac{\%}{100}$ .

Here are tables for each of the rules.

$$A(x) = 500(2)^x$$

X	Y <sub>1</sub>
0	500
1	1000
2	2000
3	4000
4	8000
5	16000
6	32000

$$B(x) = 25000\left(\frac{1}{2}\right)^x$$

X	Y <sub>1</sub>
0	25000
1	12500
2	6250
3	3125
4	1562.5
5	781.25
6	390.63

$$C(x) = 1000(1.05)^x$$

X	Y <sub>1</sub>
0	1000
1	1050
2	1102.5
3	1157.6
4	1215.5
5	1276.3
6	1340.1

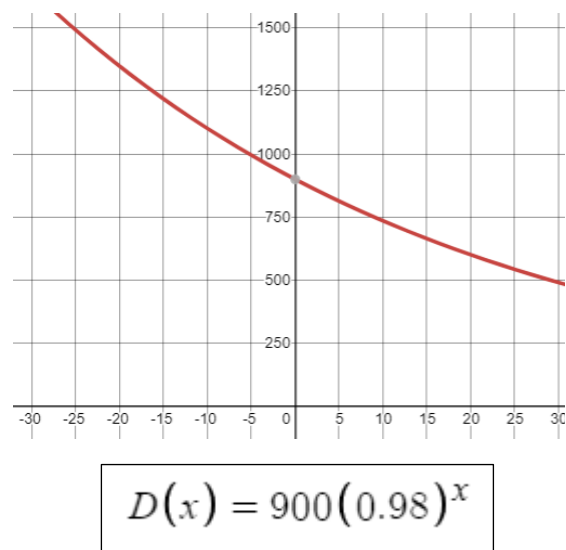
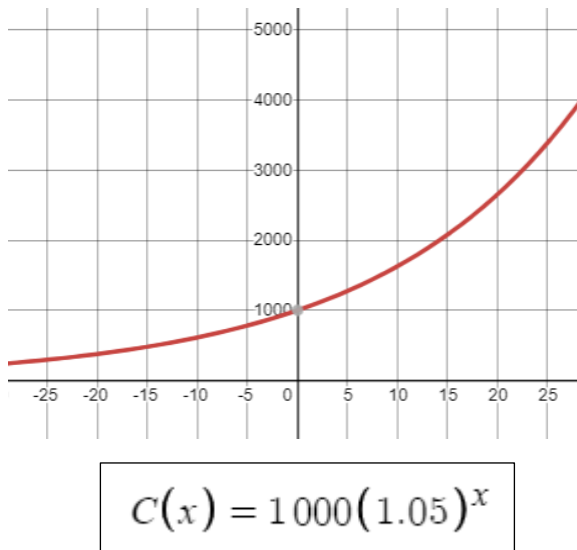
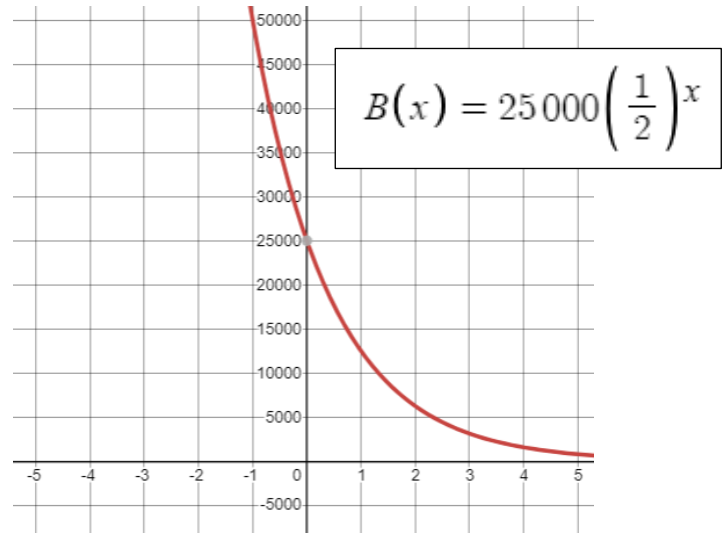
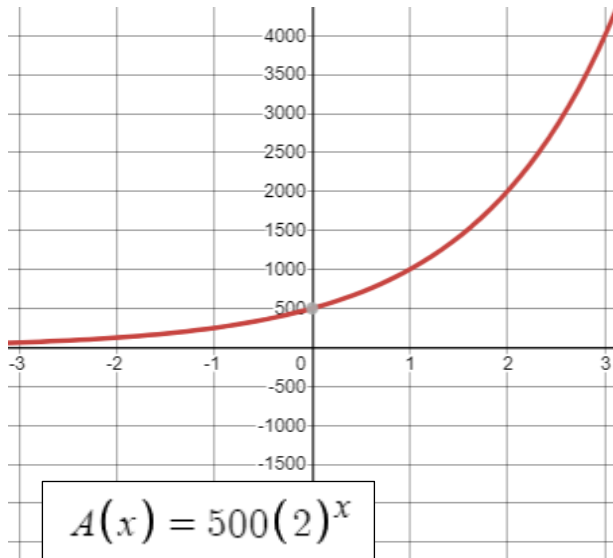
$$D(x) = 900(0.98)^x$$

X	Y <sub>1</sub>
0	900
1	882
2	864.36
3	847.07
4	830.13
5	813.53
6	797.26

What similarities and differences do you notice between the exponential patterns in the tables? What connections do you notice about the function rule and the table of values?

**Each table starts when x is 0 at the number that matches the value outside of the parentheses (without the exponent) in the function rule. As x increases by 1, the y-value multiplies by the amount inside the parentheses.**

Here are graphs of each exponential function.



What are some similarities and differences you notice about how each graph is shaped?

**Students may describe graphs as being L-shaped or J-shaped, which is fine. Two of the graphs are decreasing from left to right while the other two are increasing.**

How do the exponential graphs compare to the linear graphs?

**Unlike linear graphs that increase or decrease at a constant amount, exponential graphs are not perfectly straight and increase or decrease by different amounts as x increases. You may want to lead a discussion about how exponential graphs that increase do so at a faster and faster rate. Exponential graphs that decrease do so slower and slower as x increases.**

**Similar to linear graphs, exponential graphs always increase or decrease. They never change direction.**

What are some observations you can make about the how the exponential function rules and their graphs are related?

**The number outside the parentheses is the y-intercept. The graph will increase if the number in the parentheses is more than 1 and it will decrease if the number in parentheses is between 0 and 1.**

Every exponential function rule can be written in the form

$$y = a(b)^x$$

Where **a** and **b** are numbers. Here are some important ideas about how these numbers affect the behavior of tables and graphs of exponential function rules.

- | <b>a</b>   | <b>b</b>  |
|--|---|
| <ul style="list-style-type: none"> <li>• This is the initial (starting) value of the function</li> <li>• On a table, this is the y-value when <math>x = 0</math></li> <li>• On a graph, this is where the graph will intersect the y-axis (vertical axis)</li> </ul> | <ul style="list-style-type: none"> <li>• This is the <u>multiplier</u> of the function<br/>If the function is growing or shrinking by a percent, <math>b</math> will be equal to <math>1 + \frac{\%}{100}</math> or <math>1 - \frac{\%}{100}</math></li> <li>• To get the next y-value in the table, multiply the current y-value by <math>b</math></li> <li>• If <math>b</math> is greater than 1, the graph will be increasing from left to right</li> <li>• If <math>b</math> is between 0 and 1, the graph will be decreasing from left to right</li> </ul> |

Decide whether the rule belongs to the linear function family, the exponential function family, or neither. Use evidence from the calculator or your knowledge of mathematics to justify your answer.

A.  $y = 4 + 2x$

**Linear**

D.  $g(x) = |x + 1| - 3$

**Neither (absolute value)**

B.  $y = 2(x + 1)^2 - 5$

**Neither (Quadratic)**

E.  $k(x) = 0.5x - 6 + x$

**Linear**

C.  $f(x) = 3^x$

**Exponential**

F.  $p(t) = 5000(0.87)^x$

**Exponential**

Decide which situations below are best modeled by linear functions, exponential functions, or neither. Explain your reasoning.

- i. A bamboo plant grows at a constant rate of 3 inches per day.

**Linear since the rate is constantly adding 3**

- ii. An amusement park allows 50 people to enter every 30 minutes

**Linear since the rate constantly adds the same amount**

- iii. The value of a cell phone depreciates by 3.5% each year.

**Exponential since the value decreases by a percent**

- iv. A baseball tournament eliminates half of the teams after each round.

**Exponential since the number of teams divides by 2 each round (multiplies by 0.5).**

- v. A football is kicked into the air from an initial height of 4 feet. It reaches a maximum height of 82 feet before it returns to the ground.

**Neither since this situation involves something that increases and decreases.**

## Linear Regression

This technique will transform a set of data into a line of best fit.

For example, in January 2019, the Regents exam gave data on the height of a certain breed of dog based upon its mass (weight).

First, make a prediction. What do you think is the relationship between the height of a dog and its weight? Would a linear function make sense to model this relationship?

**Invite all discussion and discourse. Students may say in general as height increases so does the weight of a dog, while other students may disagree.**



Here is the question:

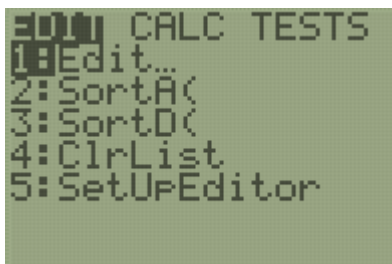
- 34 The data given in the table below show some of the results of a study comparing the height of a certain breed of dog, based upon its mass.

<b>Mass (kg)</b>	4.5	5	4	3.5	5.5	5	5	4	4	6	3.5	5.5
<b>Height (cm)</b>	41	40	35	38	43	44	37	39	42	44	31	30

Write the linear regression equation for these data, where  $x$  is the mass and  $y$  is the height. Round all values to the *nearest tenth*.

State the value of the correlation coefficient to the *nearest tenth*, and explain what it indicates.

The solution starts here. Follow along on your calculator as well.



Press **STAT**, **ENTER**

L1	L2	L3	3
4.5	41		
5	40		
4	35		
3.5	38		
5.5	43		
5	44		
5	37		

Type the **x**-values in **L1** and the **y**-values in **L2**

```

EDIT [0:1] TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7↓QuartReg
    
```

Press **STAT**, →

Since this problem asks for a linear regression, choose **4**

(Exponential regression is choice **0**)

```

LinReg(ax+b)
Xlist:L1
Ylist:L2
FreqList:
Store RegEQ:
Calculate
    
```

Highlight “Calculate” and press **ENTER**

```

LinReg
y=ax+b
a=1.917355372
b=29.79889807
r²=.1136307339
r=.3370915809
    
```

The first line,  $y = ax + b$ , is the function rule.

The next lines tell you the numbers to write in place of **a** and **b**. **x** and **y** will remain letters.

Ignore  $r^2$

“**r**” is called the **correlation coefficient**, and it is always between 0 and 1.

The closer **r** is to 1, the better fit this data is to the type of function rule you chose.

Notice that this question asked us to round the values of the function rule to the nearest tenth.

The linear regression equation is  $y = 1.9x + 29.8$

The correlation coefficient, to the nearest tenth, is **0.3**

Since 0.3 is not close to 1, this is a weak correlation.

Notice how the *r*-value 0.3 was closer to zero than one. This tells us that there is a weak relationship between the breed’s height and weight.

To better see this, do the following:

```

>Plot1 Plot2 Plot3
\Y1=
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
    
```

**Step 1:** When you press **Y=** be sure that “**Plot1**” is highlighted. If it isn’t, use the **arrow keys** to highlight it and press **ENTER**

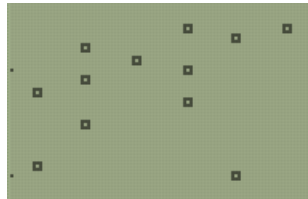
All **Y=** rules should be blank.

**Step 2:** Press **ZOOM**, and choose “**9: ZoomStat**” and then press **ENTER**

Describe the overall pattern of the data points. How do they compare to a linear function?

**The points generally go up from left to right, but they definitely don't resemble a perfectly straight line.**

**Educator Note:** Encourage students to identify which point might be the Chihuahua or the Great Dane and why they think that is.



**Next**, to see how well our function fits the data, type  $y = 1.9x + 29.8$  into **Y1** by pressing **Y=**. This is the line that best fits our data. You can see that the line traces the pattern in the data, but that the points are relatively spread out. This makes for a weak correlation.

When working with real world measurements, or data, it is rare that the measurements make perfectly linear patterns. However, data is often close to linear, which is why the correlation coefficient is important. It tells us how well our data fits a line.

The correlation coefficient,  $r$ , is either strong or weak depending on whether it is closer to zero or one.

Complete the following problems on your own.

- 36 The percentage of students scoring 85 or better on a mathematics final exam and an English final exam during a recent school year for seven schools is shown in the table below.

Percentage of Students Scoring 85 or Better	
Mathematics, $x$	English, $y$
27	46
12	28
13	45
10	34
30	56
45	67
20	42

Write the linear regression equation for these data, rounding all values to the *nearest hundredth*.

$$y = 0.96x + 23.95$$

State the correlation coefficient of the linear regression equation, to the *nearest hundredth*. Explain the meaning of this value in the context of these data.

$$r = 0.92$$

There is a strong positive correlation between the percentage of students who score an 85 or higher on the math and English exams.

- 35 The table below shows the number of hours ten students spent studying for a test and their scores.

<b>Hours Spent Studying (x)</b>	0	1	2	4	4	4	6	6	7	8
<b>Test Scores (y)</b>	35	40	46	65	67	70	82	88	82	95

Write the linear regression equation for this data set. Round all values to the *nearest hundredth*.

$$y = 7.79x + 34.27$$

State the correlation coefficient of this line, to the *nearest hundredth*.

$$r = 0.98$$



# Lesson 3 – Piecing it Together

You are becoming more familiar with linear and exponential functions. This lesson, we will introduce two more important families: **quadratic** and **absolute value** functions. Finally, we will learn about **piecewise** functions.

## Quadratic Functions

The third major function family, quadratic functions, is also **nonlinear**. Situations that are modeled with quadratic function rules many times involve gravity or motion. Here are a few examples:

- Jonathan kicks a football into the air. The height in feet at any given time  $t$  can be modeled by the function rule  $h(t) = -16t^2 + 60t + 3$
- A skydiver jumps from a height of 3,500 meters. The total distance fallen at any given time  $t$  can be modeled by the function rule  $d(t) = 3500 - 9.8t^2$
- A concert venue's revenue depends on the ticket price,  $x$ . The concert revenue can be modeled by the quadratic function rule  $R(x) = x(200 - 20x)$
- The path of a comet as it slingshots around planet Earth can be modeled by the quadratic function  $p(x) = (x - 3)^2 + 4$

What similarities and differences do you notice about these function rules?

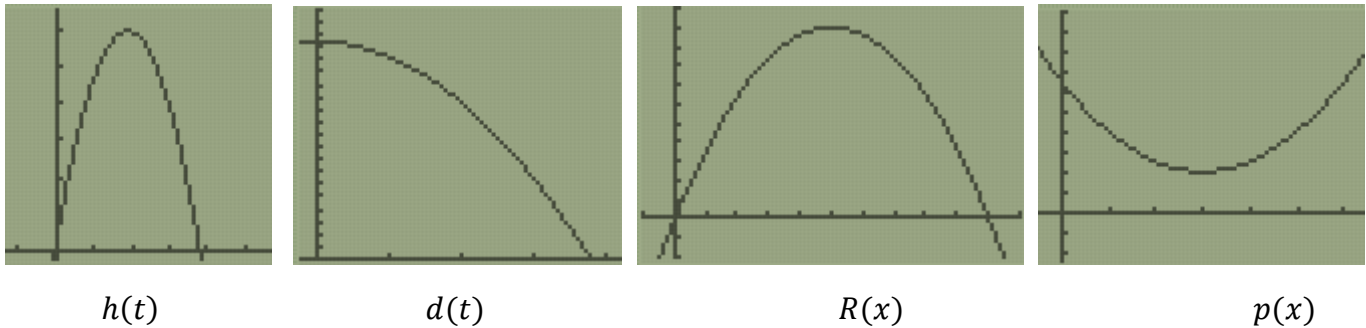
**Three rules have an exponent of 2. One rule does not.**

**Educator Note:** Students may point out that when you distribute the  $x$  in  $R(x)$  that it will in fact have an exponent of 2.

Based on the situations what shape do you predict quadratic graphs to have? Make a sketch.

**Students should draw upward or downward facing U-shapes (parabolas).**

Quadratic function rules appear in three major forms. We are going to learn how to convert between these forms later, but it makes it more difficult to recognize what exactly makes function rules quadratic. Did you notice how three rules involved an exponent of 2? Quadratic function rules do have  $x^2$  in them. It is less obvious that  $R(x) = x(200 - 20x)$  is quadratic, but it is. Let's take a look at each of their graphs.



Which graphs are similar? How are they similar? Which graph is different? How is it different?

**Students may point out that  $d$  is different because it only appears to decrease, or that  $p$  is different because it opens upward.**

**Students might say that  $h$ ,  $R$ , and  $p$  are all similar because they all increase and decrease.**

Karla answered the previous question by saying that  $d(t)$  is the graph that is different. She said,

**"I think  $d(t)$  is different because it decreases the whole time. Every other graph has one part that increases and another part that decreases."**

Robert and Karla exchange papers and read each other's responses. Robert disagrees with Karla and tells her,

**"If you graph it on the left,  $d(t)$  is similar to  $h$  and  $R$ . I think  $p$  is the different one because it goes up on both sides."**

Explain Robert's thinking and expand upon it in your own words. What evidence could Robert have shown Karla to help support his response?

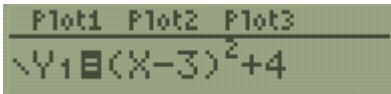
**Robert extended the graph on the left side, making it symmetrical to the part that is showing.**



Looking at a graph will always reveal whether a rule is quadratic. These U-shaped graphs are called **parabolas**. Notice that it is not obvious that  $d(t)$  is a parabola. We will discuss how to adjust the calculator **WINDOW** in greater detail in the future.

Notice that linear and exponential functions either always increase or always decrease from left to right. Parabolas change directions. This turning point is called the **vertex**. If the quadratic graph opens up (shaped like a cup), the vertex is the minimum of the graph. If the parabola opens down (shaped like a frown), the vertex is a maximum point.

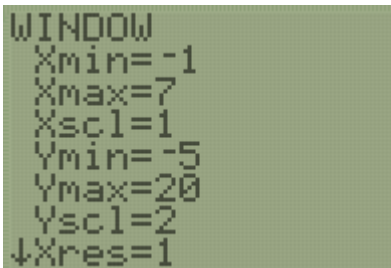
Suppose you want to find the minimum point of  $p(x)$ .



Press **Y=** and enter the function rule for **Y1**

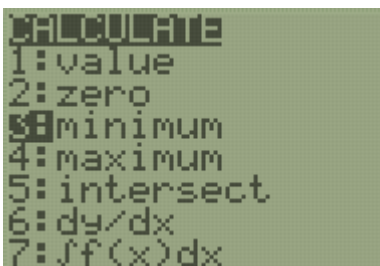
X	Y1
0	13
1	8
2	5
3	4
4	5
5	8
6	13

To get an idea of the correct window settings, press **2<sup>nd</sup>**, **GRAPH**. Decide on the lowest and highest values for x- and y-values.

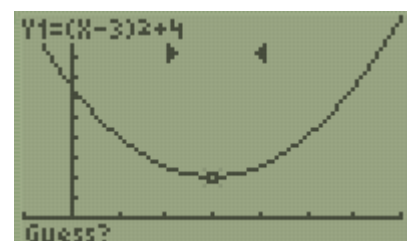
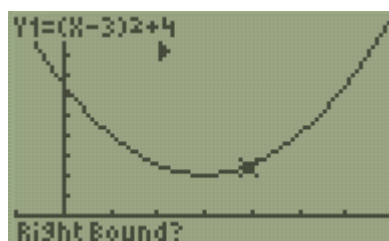
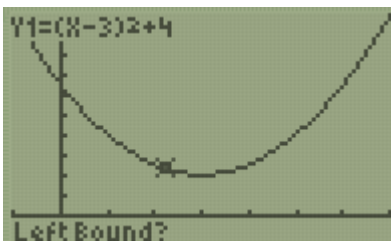


To follow along with this example, press **WINDOW** and enter the following settings.

Are these choices close to the ones you made based on looking at the table?



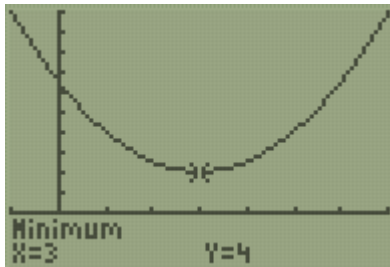
Press **2<sup>nd</sup>**, **TRACE**, and choose choice **3** for the minimum finder.



Move the cursor using the arrow keys until it is on the left side of the minimum, then press **ENTER**.

Move the cursor to the right side of the minimum and press **ENTER**

Finally, move the cursor to somewhere close to the minimum point and press **ENTER**.



The coordinates of the minimum point of  $p(x)$  are  $(3, 4)$ .

This process works nearly the same for finding a maximum.

Try to find the coordinates of the maximum point on  $h(t) = -16t^2 + 60t + 3$ . Note that when you put the rule into **Y=** you always use  $x$  as your variable.

```
WINDOW
Xmin=-1
Xmax=7
Xscl=1
Ymin=-5
Ymax=65
Yscl=10
↓Xres=1
```

These settings produce a nice viewing window for  $h$ . Again, we will discuss strategies for how to do this in the future.

Use these Window settings and write the coordinates of the maximum point of  $h(t) = -16t^2 + 60t + 3$ .

**(1.87, 59.25)**

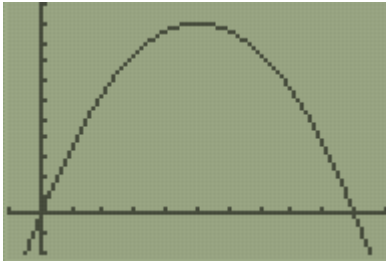
The Regents Exam also loves to ask about the **zeros** of quadratic functions. **Zeros** are  $x$ -values for which  $y = 0$ . Sometimes we can find zeros on a table.

Locate the zeros of  $R(x) = x(200 - 20x)$  using your table.

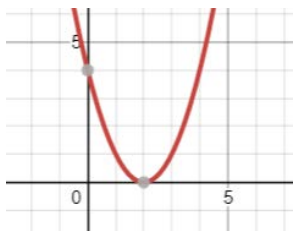
**The zeros occur when  $x$  is 0 and when  $x$  is 10.**

From this example, we see that quadratics can have two zeros.

The zeros of a function are the **x-intercepts** of its graph. Notice how the graph of  $R(x)$  intersects the x-axis at two locations,  $x = 0$  and  $x = 10$

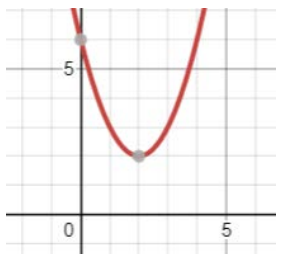


Sketch the graph of a quadratic function that has one zero.



The graph touches the x-axis only once

Sketch the graph of a quadratic function that has no zeros.



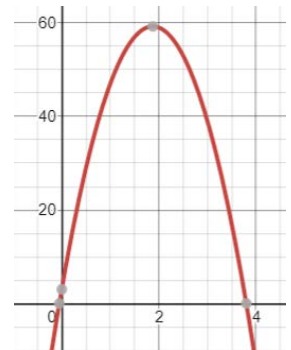
The graph does not intersect the x-axis

Is it possible for a quadratic function to have more than two zeros? Explain your reasoning.

Since a quadratic only changes direction one time, it is only possible for a quadratic function to increase through the x-axis then decrease through the x-axis a second time, or vice versa. Therefore, it is not possible for a quadratic to have more than two zeros.

Consider the function  $h(t) = -16t^2 + 60t + 3$ . View a graph of this function. How many zeros does  $h$  have?

Two zeros



Fill out the table of values for  $h(t)$  for integers  $x = -2$  through  $x = 5$

$x$	-2	-1	0	1	2	3	4	5
$h(x)$	-181	-73	3	47	59	39	-13	-97

It's not immediately obvious what exact  $x$ -values the zeros are. Between what values of  $x$  must the zeros be "hiding"? Explain how you can tell.

**The zeros are between -1 and 0 and also between 3 and 4. You can tell because the  $y$ -values change between positive and negative at these locations.**

This example shows that we can't rely on tables to find the zeros of every quadratic function.

To find the zeros on a graph, press **2<sup>nd</sup>**, **TRACE**, and choose option **2: zero**.

Then, follow the same process to locate a max or min.

Use the following **WINDOW** for a nice picture of the graph.

```

WINDOW
Xmin=-2
Xmax=6
Xscl=1
Ymin=-5
Ymax=70
Yscl=10
↓Xres=1

```

Try this process now to locate the zeros of  $h(t) = -16t^2 + 60t + 3$ . Note that they will be decimal values.

**$x = -0.049$  and  $x = 3.799$**

## **Absolute Value Functions**

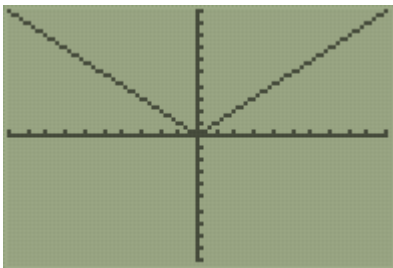
There is another family, in addition to linear, exponential, and quadratic, that is worth knowing – the absolute value function.

The **absolute value** of a number  $a$ , written as  $|a|$ , is that number's distance from 0. Since distance is always positive, the absolute value of a number is always positive.

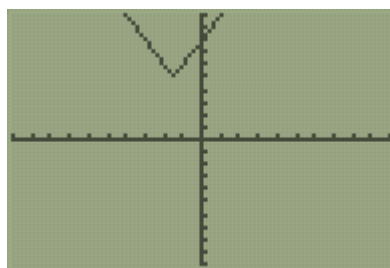
For example, the absolute value of negative four is four because negative four is four units away from zero on a number line. This is written as  $|-4| = 4$

Absolute value functions have similarities in the shape of their graphs.

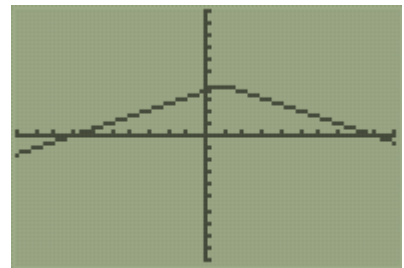
$$y = |x|$$



$$y = |2x + 3| + 5$$



$$y = -0.5|x - 1| + 4$$



Get the absolute value bars in the calculator by pressing **ALPHA, WINDOW, ENTER**.

What are some similarities between each absolute value function rule?

**Each rule has the absolute value bars (vertical lines) that are around the variable. Each rule is slightly different based on the numbers that are added, subtracted, or multiplied inside or outside of the absolute value bars.**

What similarities do you notice in each graph?

**Each graph is “V-shaped.” It is made of two linear sections and has either a minimum or maximum point.**

What similarities and differences do you notice between absolute value functions and quadratic function graphs? Rules?

**Absolute value and quadratic functions both increase and decrease and have a maximum or minimum point. Absolute value functions change by a constant amount on either side of the min or max whereas quadratic functions do not. Their rules are mostly different as quadratics involve squaring the variable and absolute value functions have their own unique notation.**

## Domain and Graphing

In your own words, explain the idea of a function's domain and range.

**Domain is the set of inputs or x-values for a function.**

**Range is the set of outputs or y-values for a function.**

For many situations, there are only certain input values that make sense. For example, consider the example from the beginning of the lesson involving revenue:

- A concert venue's revenue depends on the ticket price,  $x$ . The concert revenue can be modeled by the quadratic function rule  $R(x) = x(200 - 20x)$

What does  $x$  represent in this situation?

**$x$  is the ticket price**

What is one value of  $x$  that does not make sense for this situation?

**Zero or negative values.**

Select the inequality that best represents the domain for this situation. Explain your reasoning for your choice.

$$\{x|x = 5\}$$

$$\{x|x > 0\}$$

$$\{x|x \geq 0\}$$

$$\{x|x < 0\}$$

**$\{x|x > 0\}$  means "x-values that are greater than zero."**

Tyrece is exploring  $R(x) = x(200 - 20x)$  using a table of values on his calculator. He notices a pattern and writes it down.

*As the ticket price increases, the money or revenue increases for a little while, then it decreases and becomes negative.*

Do you agree with the pattern Tyrece noticed?

**Students should view  $R(x)$  in the table and agree.**

Explain why it makes sense in the context of the problem that as ticket price increases, the revenue will eventually decrease.

**As ticket price increases, fewer people will purchase tickets and the concert won't make as much money.**

Explain why it makes sense in the context of the problem that revenue will eventually be negative.

**Eventually the concert venue will not make enough money to cover its costs and will end up owing money to its employees, for its bills, etc.**

Tyrece wants to consider the domain of ticket prices for which revenue is positive. Using words, which ticket prices give positive revenues?

**All ticket prices between \$0 and \$10**

Which inequality below represents the domain you described? Explain your reasoning.

$$\{0 < x < 10\}$$

$$\{x < 10\}$$

$$\{0 > x > 10\}$$

**$\{0 < x < 10\}$**

**Literally, 0 is less than  $x$ , which is less than 10. This means all the  $x$ 's between 0 and 10.**

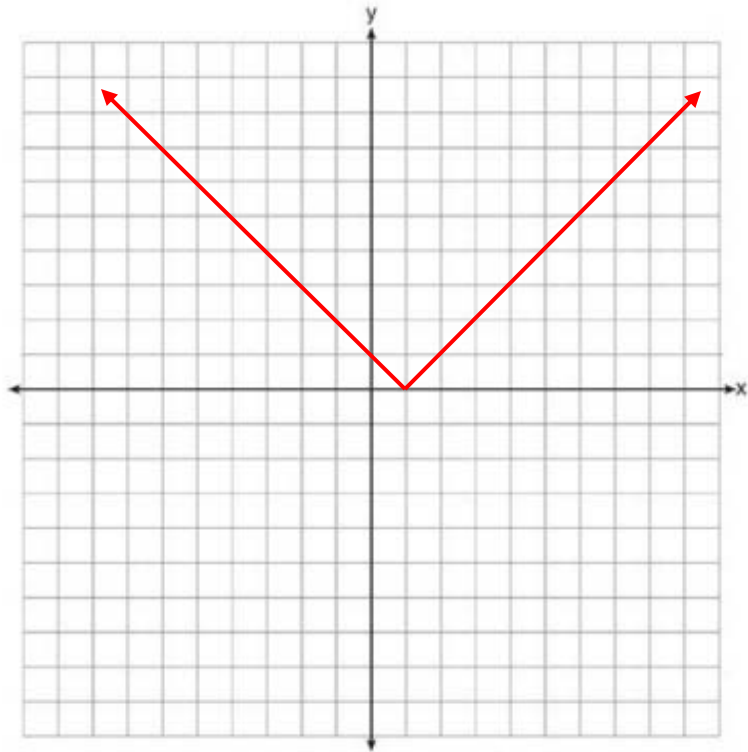
Many students struggle to interpret these types of inequalities. To help you, fill in the chart below:

Inequality	Literally	In my own words
$0 < x < 10$	Zero is less than x which is less than ten	All the x-values between zero and ten, not including zero or ten
$2 \leq x < 5$	2 is less than or equal to x which is less than 5	All the x's between 2 and 5, including 2 but not including 5
$-1 < x \leq 2$	-1 is less than x which is less than or equal to 2	All the numbers between -1 and 2 not including -1 and including 2
$x > 10$	x is greater than 10	All the numbers more than 10 but not 10 itself
$4 < x \leq 7$	4 is less than x which is less than or equal to 7	All x-values between 4 and 7, not including 4 but including 7
$x < 8$	x is less than 8	All x-values smaller than 8



Complete the following Regents question.

On the set of axes below, graph the function  $y = |x + 1|$ .



State the range of the function. State the domain over which the function is increasing.

**Range:  $\{y \geq 0\}$**

**The function is increasing for the domain  $\{x > -1\}$**

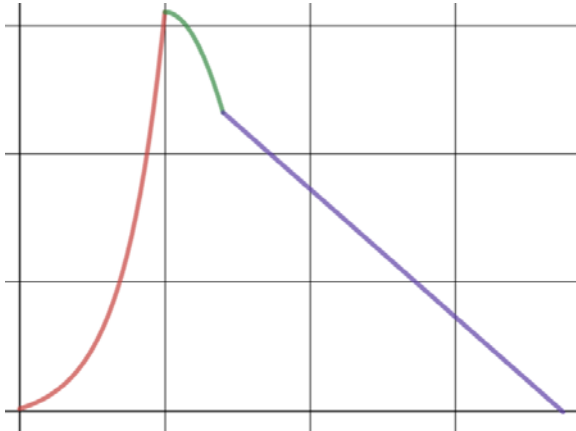
## Piecewise Functions

Brendalee receives a model rocket kit for her birthday. The rocket has a GPS signal that allows her to track its distance from the ground using a phone app.

- For the first five seconds, the engine causes the rocket's distance from the ground in meters to increase according to the exponential function  $y = 5(2)^t - 4.5$ .
- When the fuel runs out, the rocket free falls toward the ground for two seconds. Its distance from the ground decreases according to the quadratic function  $y = -9.8(t - 5)^2 + 155.5$
- Finally, the parachute opens, and the rocket's distance from the ground decreases at a constant rate of  $-10$  meters per second according to the linear function  $y = 186.3 - 10t$  until the rocket hits the ground.



Let  $D(t)$  represent the distance from the ground of the rocket for the entire time between launch to when it hits the ground. Sketch what you think the graph of  $D(t)$  looks like.



**Encourage students to identify whether each function rule is quadratic, exponential, or linear. The goal is not for them to make an accurate graph of individual points, but to think about a general graph shape. This is difficult, so it's okay if they are not able to visualize this graph shape right away.**

During which times will Brendalee's rocket follow the exponential function rule? The quadratic function rule? The linear function rule? Write inequalities to represent each of these intervals.

**From 0 to 5 seconds,  $0 < t < 5$ , the rocket will follow the exponential rule  $y = 5(2)^t - 4.5$**

**From 5 to 7 seconds,  $5 < t < 7$ , the rocket will follow the quadratic rule  $y = -9.8(x - 5)^2 + 155.5$**

**From times more than 7 seconds,  $t > 7$ , the rocket will follow the linear rule  $y = 186.3 - 10x$**

Brendalee uses a **piecewise function** to define  $D(t)$ .

$$D(t) = \begin{cases} 5(2)^t - 4.5 & 0 \leq t \leq 5 \\ -9.8(t - 5)^2 + 155.5 & 5 < t \leq 7 \\ 186.3 - 10t & 7 < t < 18.6 \end{cases}$$

Explain what this notation means in your own words.

**The function rule is on the left and when to use it is on the right. Use the top rule when  $t$  is between 0 and 5 including 0 and 5. Use the middle rule when  $t$  is between 5 and 7. Use the bottom rule when  $t$  is between 7 and 18.6**

According to the piecewise function Brendalee wrote, which of the three function rules should be applied when  $t = 2$ ? What about when  $t = 6$ ? How do you know?

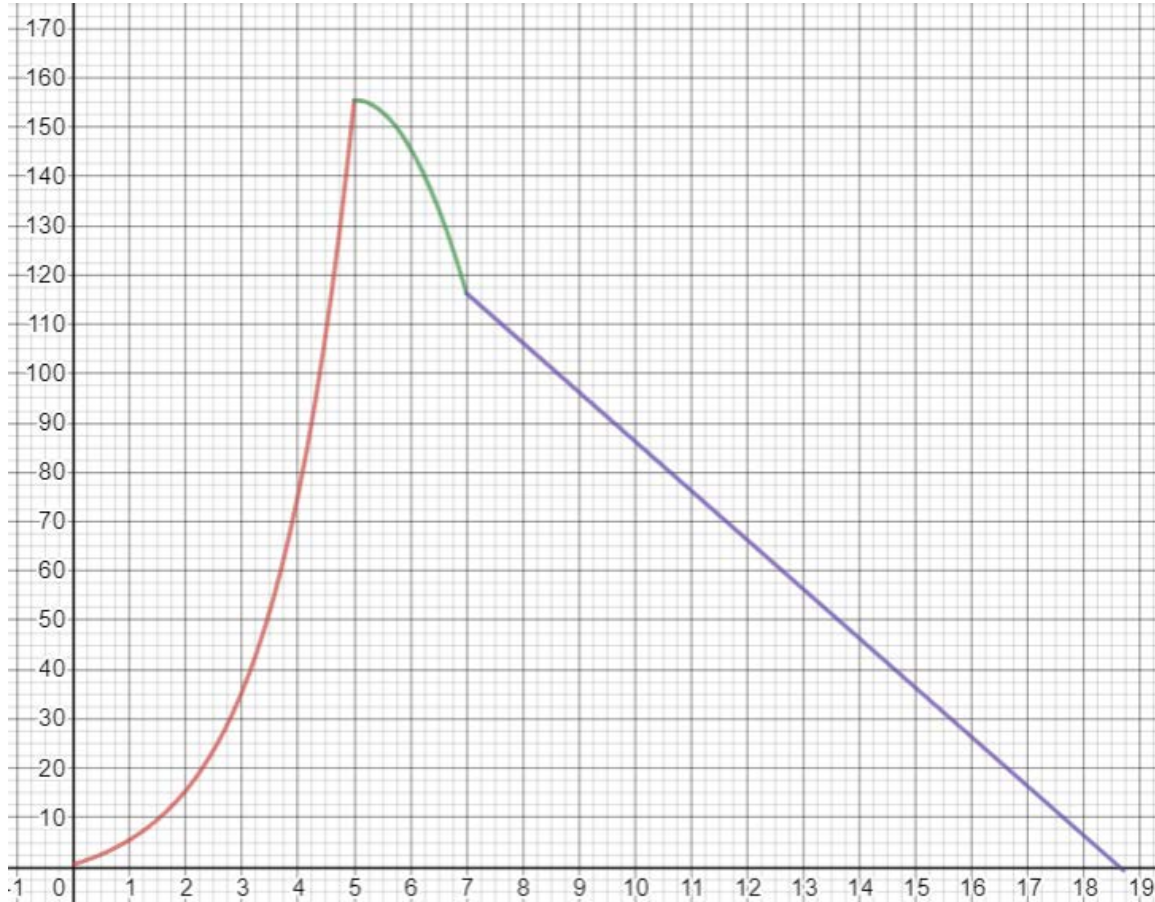
**When  $t = 2$ , Brendalee should use the top rule  $5(2)^t - 4.5$ .**

**When  $t = 6$ , she should use the middle rule,  $-9.8(t - 5)^2 + 155.5$**

What is the domain of  $D(t)$ ?

$$\mathbf{0 \leq t \leq 18.6}$$

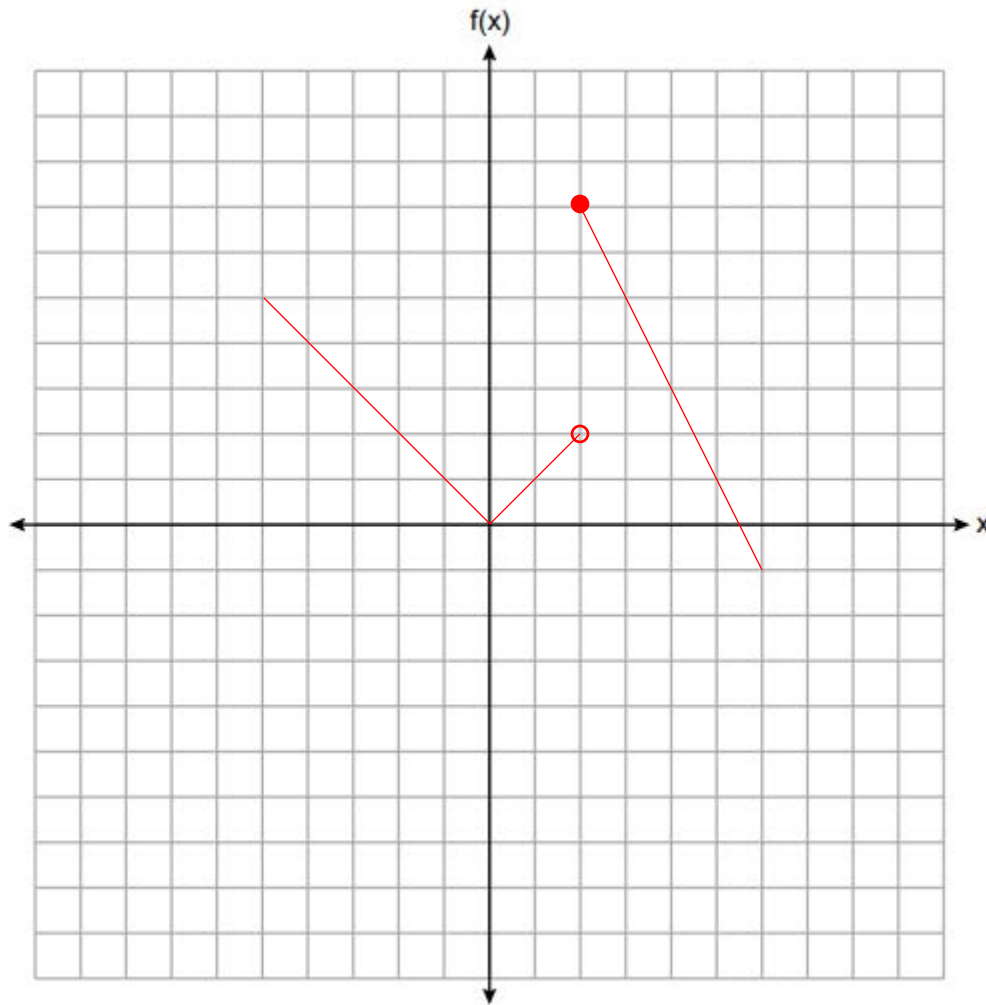
Make a graph of  $D(t)$  below.



Did you notice that each rule matched up to form one continuous graph? Not all piecewise functions do this.

**27** Graph the following piecewise function on the set of axes below.

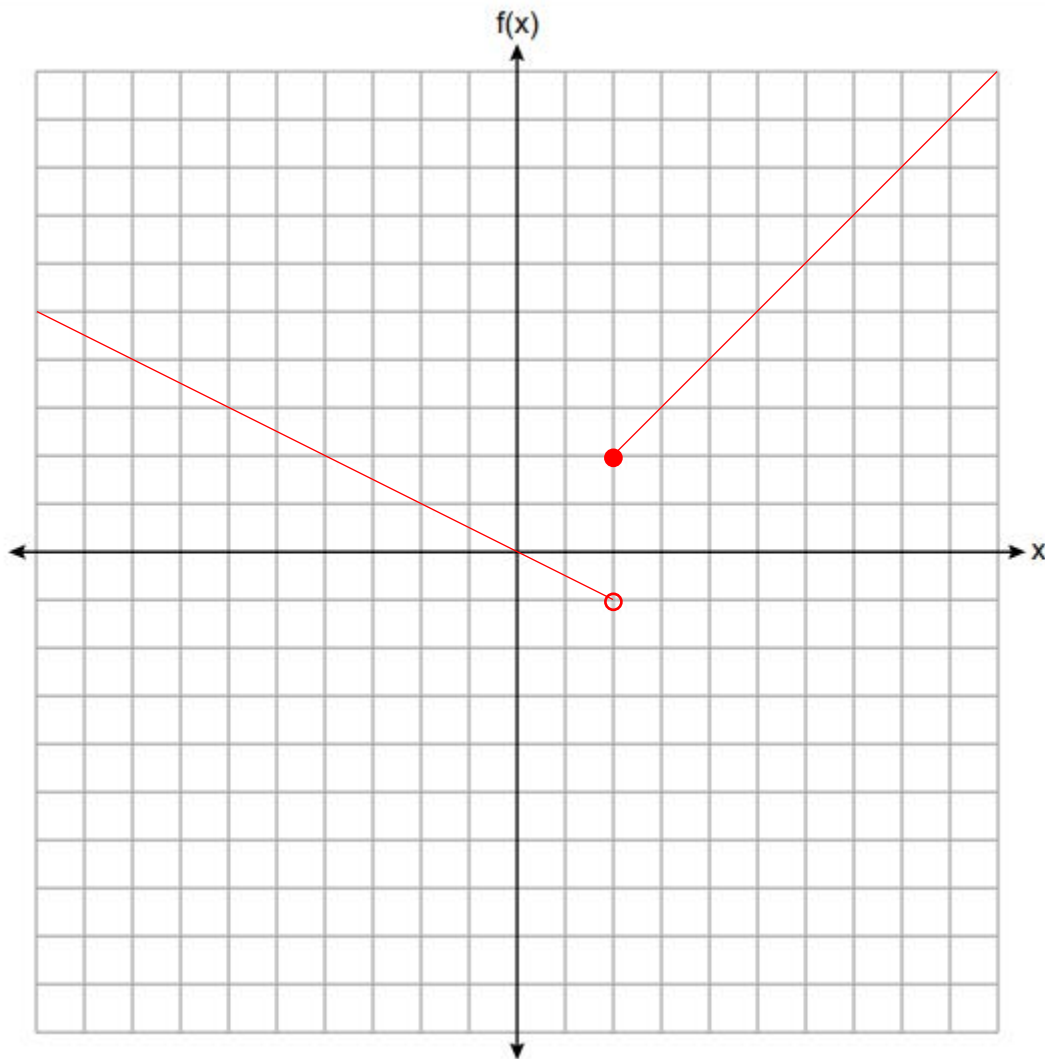
$$f(x) = \begin{cases} |x|, & -5 \leq x < 2 \\ -2x + 10, & 2 \leq x \leq 6 \end{cases}$$



**Educator Note:** Notice that when graphing “ $-5 \leq x < 2$ ” there is an open circle for point  $(2, 2)$  because  $x$  is less than 2, but not equal to 2.

32 On the set of axes below, graph the piecewise function:

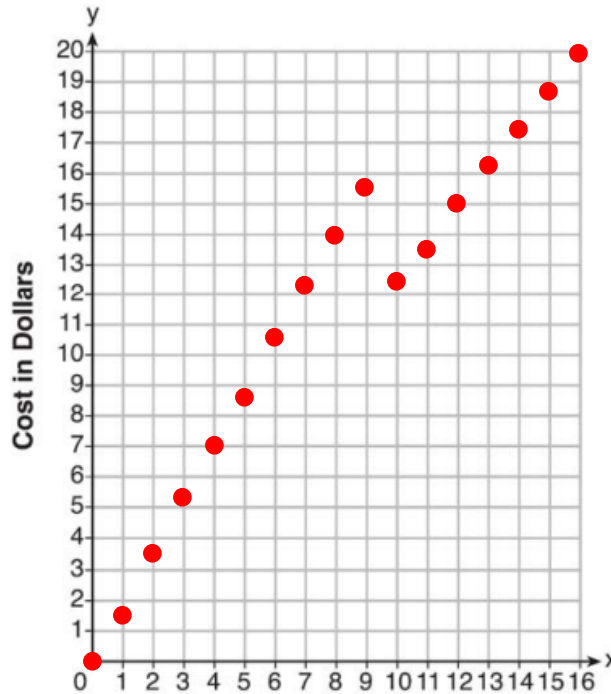
$$f(x) = \begin{cases} -\frac{1}{2}x, & x < 2 \\ x, & x \geq 2 \end{cases}$$



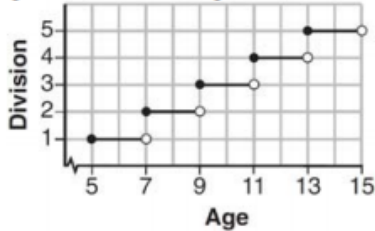
At an office supply store, if a customer purchases fewer than 10 pencils, the cost of each pencil is \$1.75. If a customer purchases 10 or more pencils, the cost of each pencil is \$1.25. Let  $c$  be a function for which  $c(x)$  is the cost of purchasing  $x$  pencils, where  $x$  is a whole number.

$$c(x) = \begin{cases} 1.75x, & \text{if } 0 \leq x \leq 9 \\ 1.25x, & \text{if } x \geq 10 \end{cases}$$

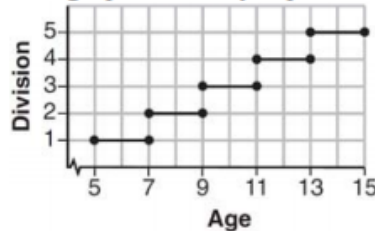
Create a graph of  $c$  on the axes below.



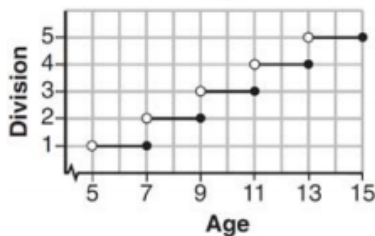
Morgan can start wrestling at age 5 in Division 1. He remains in that division until his next odd birthday when he is required to move up to the next division level. Which graph correctly represents this information?



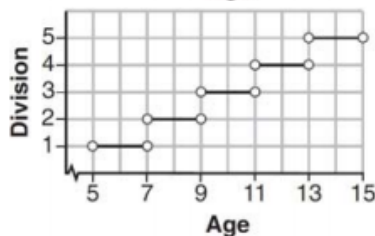
a.



c.



b.



d.

**Choice a**

## Lesson 4 – Solving Linear Equations

For the past three lessons, we learned about functions. A function is a rule that tells you what to do with an  $x$  in order to get a  $y$ -value. But how are functions different from **equations**?

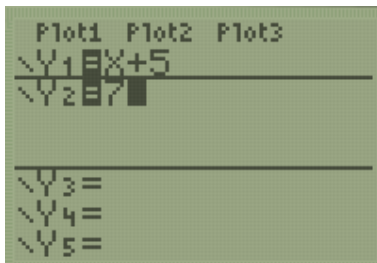
When told to solve an equation for a variable, such as solving  $3x + 2 = x - 1$ , we are looking for a single value (sometimes more) that will makes both sides of the equation have the same  $y$ -value.

Most students solve *algebraically*. They do operations on both sides of the equation until the variable is by itself and a number is on the other side of the equal sign. However, all equations can be solved using graphs and tables as well. This lesson, we will practice solving linear equations.

### Solving (mostly) linear equations

Consider how to solve the equation  $x + 5 = 7$ .

#### Method 1 – Using a table



Press **Y=**, enter one side of the = sign as Y1 and the other side as Y2

X	Y1	Y2
-2	3	7
-1	4	7
0	5	7
1	6	7
2	7	7
3	8	7
4	9	7

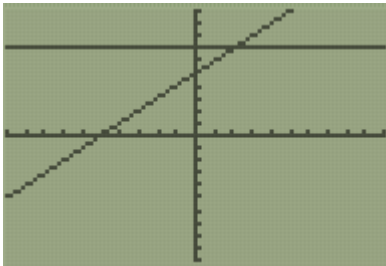
X=4

Find the  $x$ -value where **Y1** equals **Y2**

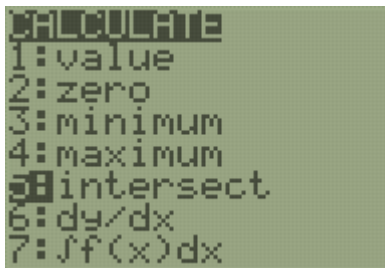
Notice that  $Y1 = Y2$  when  $x = 2$ . **The  $x$ -value is the solution to the equation.**

**Method 2 – Using a graph**

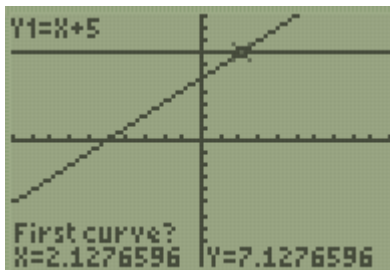
Do the same first step as before: Press **Y=**, enter one side of the = sign as Y1 and the other side as Y2.



Press **GRAPH**. The solution is the point where the graphs intersect.

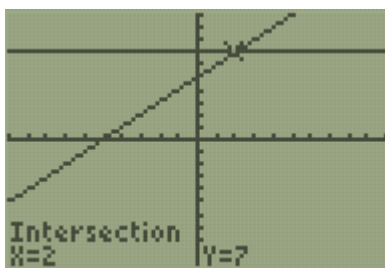


Press **2<sup>nd</sup>**, **TRACE**, use the **down arrow** to choose choice “**5: intersect**,” and press **ENTER**



Use the left or right arrow to move the blinker close to the desired point.

Then press **ENTER, ENTER, ENTER**.

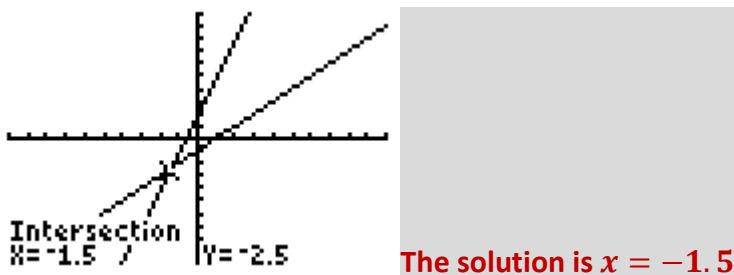


The graph intersects at the point **(2,7)**. The solution to the equation is the **x-value 2**.

**We need both the table method and the graph method to solve equations.** The table method is fast and simple, but if the solution to the equation is a decimal or a fraction, then the solution will not show up in the standard table. The graphing method will always work, but it is a little more complicated.



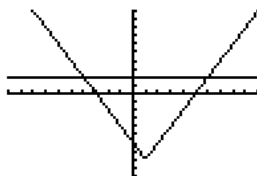
1. Solve the equation  $3x + 2 = x - 1$  using a table or graph. Show evidence of your method to support your answer.



Note that the solution is not obvious from the table.

You also have the tools to solve equations involving the other nonlinear function families using graphs or tables. **Solve** the following equations using graphs or tables in your calculator. Some equations have more than one solution. **One equation has no solution!**

2.  $2|x - 1| - 8 = 2$

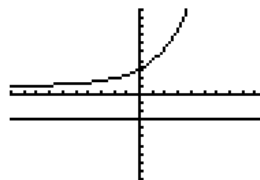


X	Y <sub>1</sub>	Y <sub>2</sub>
-4	2	2
6	2	2

$x = -4$  and  $x = 6$

To get absolute value bars, you can press ALPHA, WINDOW, ENTER

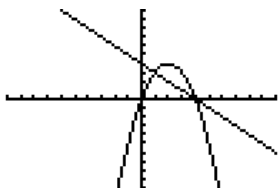
4.  $2(1.5)^x + 1 = -3$



no solution

(The graphs do not intersect.)

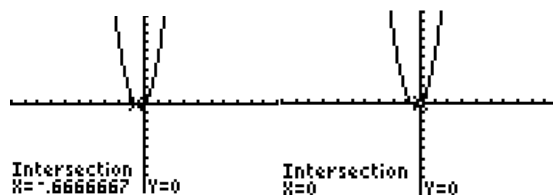
3.  $-x^2 + 4x = 4 - x$



X	Y <sub>1</sub>	Y <sub>2</sub>
1	3	3
4	0	0

$x = 1$  and  $x = 4$

5.  $3x^2 + 2x = 0$



$x = -0.\overline{6}$  and  $x = 0$

6. Summarize how to determine a solution to an equation using a graph or table. Explain how you can tell when an equation has no solution.

**On a graph, the solutions are the x-values where the graphs intersect. On a table, the solutions are the x-values that have matching y-values in Y1 and Y2.**

**There is no solutions if the graphs do not intersect.**

### **Method 3 – Solving linear equations algebraically**

Marcos solved the equation  $7x - 3(x + 1) = 2(x + 4)$  algebraically below.

- 7. Explain each step of his process.**

$$7x - 3(x + 1) = 2(x + 4)$$

**Distributive Property**

$$7x - 3x - 3 = 2x + 8$$

**Combine like terms ( $7x - 3x = 4x$ )**

$$4x - 3 = 2x + 8$$

**Subtract  $2x$  from both sides.**

$$2x - 3 = 11$$

**Add 3 to both sides.**

$$2x = 14$$

**Divide both sides by 2. (Multiply both sides by  $\frac{1}{2}$ )**

$$x = 7$$

8. Show how Marcos could check that his answer is correct using a graphing calculator.

**Students should that the graphs of  $Y1 = 7x - 3(x + 1)$  and  $Y2 = 2(x + 4)$  intersect when  $x = 7$  or that the y-values are both the same number (22) at  $x = 7$  on a table.**

**Students may be tempted to enter Y1 and Y2 based on steps further down in their algebraic solution. This a choice that they have, but you should have a conversation with them that since it is possible they made a mistake in the solution process, choosing the original equation may be the best choice when checking.**

9. Solve each equation algebraically. Check that your answer is correct using a graphing calculator.

a.  $4x + 10 = 2x + 7$

c.  $12 - 2(x + 5) = 4$

$x = -1.5$

$x = -1$

b.  $2(5x - 3) = 3(4x + 1)$

d.  $17 - 3(3x - 5) = 2 + x$

$x = -4.5$

$x = 3$

10. Write a general summary explaining how to solve linear equations algebraically.

**Answers will vary.**

**In general:**

**Do the distributive property to eliminate parentheses, then combine like terms. Next, work to get the variables on one side of the equal sign and constant terms on the other side by performing opposite operations on each side of the equation.**

**If students are already familiar with the acronym PEMDAS, they might benefit from understanding that solving equations is the reversal of this process.**

You are also expected to solve linear equations algebraically that contain fractions.

11. Explain each step of the process to solve  $6 - \frac{2}{3}(x + 5) = 4x$ .

$$6 - \frac{2}{3}(x + 5) = 4x$$

**Distributive property**

$$6 - \frac{2}{3}x - \frac{10}{3} = 4x$$

**Combine like terms**  $\left(6 - \frac{10}{3} = \frac{8}{3}\right)$

$$\frac{8}{3} - \frac{2}{3}x = 4x$$

**Add  $\frac{2}{3}x$  to both sides.**

$$\frac{8}{3} = \frac{14}{3}x$$

**Divide by  $\frac{14}{3}$  on both sides (or multiply by  $\frac{3}{14}$  on both sides)**

$$x = \frac{4}{7} \text{ or } .5714285714$$

**Operations with fractions are often a weakness of Algebra I students. Emphasize the idea that when solving equations, fractions behaves just as whole numbers, and that the calculator can perform operations with them. To type a fraction, press ALPHA, Y=, ENTER. For example, while the calculator cannot do  $x + \frac{2}{3}x$ , a student can type  $4 + \frac{2}{3}$  in the home screen.**

12. Show how to check that the answer is correct using the graphing calculator.

**Since the answer is a fraction, the solution is easier to find with a graph. Note that the intersection of the graph occurs at  $x = .57142857$ . Students can see this is equivalent to  $\frac{4}{7}$  if they type  $4 \div 7$  [ENTER] into their home screen.**

13. Solve each linear equation for the variable algebraically. Check that your answers are correct using a graphing calculator.

a.  $-\frac{2}{3}(x + 12) + \frac{2}{3}x = -\frac{5}{4}x + 2$

$$x = 8$$

b.  $\frac{3}{4}x - 6 = \frac{1}{2}x - 9$

$$x = -12$$

c.  $\frac{1}{6}y + 8 = 10 - \frac{2}{3}y$

$$x = 2.4 \text{ or } x = \frac{12}{5}$$



# Lesson 5 – Systems of Equations & Inequalities

In the last lesson, you practiced solving a single linear equation. In this lesson, we will focus on **systems of two equations**. A **system of equations** are two or more equations. The **solution** to a system of equations are the numbers for each variable that make all equations true at once.

This lesson, we will go over three methods to solve a system of equations, then practice solving systems of inequalities graphically.

**A group of students were given the following situation and asked to solve:**

A baseball team is planning a special promotion at its first game. Fans who arrive early will get a team athletic bag or cap, as long as supplies last. The promotion manager from the team can buy athletic bags for \$9 each and caps for \$5 each. The total budget for buying bags and caps is \$25,500. The team plans to give a bag or cap, but not both, to the first 3,500 fans. The promotion manager wants to know: *How many caps and bags should be given away?*

1. When given a problem in context, the first step is to always translate it into math equations.
  - a. All students first wrote two equations: one for the total cost of bags and caps, and one for the total number of bags and caps. Fill in the coefficients below.

$$9b + 5c = \$25,500$$

$$1b + 1c = 3,500$$

- b. What is the meaning of the variables  $b$  and  $c$ ?

**$b$  is the number of bags and  $c$  is the number of caps.**

**Including the language “number of” is important to emphasize that the variable stands for how many objects there are, and not the actual objects themselves.**

2. Tyrece chose to solve for  $b$  and  $c$  using the **substitution method**.
  - a. Follow Tyrece’s work and explain his process in your own words.

$$b + c = 3500$$

**Subtract  $c$**

$$b = 3500 - c$$

$$9b + 5c = 25500$$

**Rewrite the first equation**

$$9(3500 - c) + 5c = 25500$$

**Substitute for  $b$**

$$31500 - 9c + 5c = 25500$$

**Distribute**

$$31500 - 4c = 25500$$

**Combine  $-9c + 5c$**

$$6000 = 4c$$

**Add  $4c$  to both sides and subtract 25,500 from both sides.**

$$c = 1500$$

**Divide by 4**

$$b + c = 3500$$

**Rewrite the second original equation**

$$b + 1500 = 3500$$

**Substitute  $c = 1500$**

$$b = 2000$$

**Subtract 1500 from both sides.**

- b. Write a sentence to explain the meaning of Tyrece's answer.

**The manager should give away 2,000 bags and 1,500 caps**

- c. Another student, Clair, also used the substitution method to solve this problem. However, her first step was to write  $c = 3500 - b$ . Is she also correct? Use the substitution method to show that Clair will reach the same answer as Tyrece.

**She is correct.**

$$9b + 5c = 25500$$

$$9b + 5(3500 - b) = 25500$$

$$9b + 17500 - 5b = 25500$$

$$4b + 17500 = 25500$$

$$4b = 8000$$

$$b = 2000$$

$$b + c = 3500$$

$$2000 + c = 3500$$

$$c = 1500$$



4. Eddie and Monica used a different method, the **elimination method**, to solve the same system of equations. Eddie’s work is shown below. Follow Eddie’s work and discuss his process.

$$\begin{array}{l}
 9b + 5c = 25500 \\
 b + c = 3500
 \end{array}
 \xrightarrow{\text{Multiply the second equation by -9}}
 \begin{array}{l}
 9b + 5c = 25500 \\
 -9b - 9c = -31500
 \end{array}$$

Add these equations together

$$\begin{array}{l}
 -4c = -6000 \\
 c = 1500
 \end{array}$$

**Divide by -4**

**Substitute into  $b + c = 3500$**

$$\begin{array}{l}
 b + 1500 = 3500 \\
 b = 2000
 \end{array}$$

**Subtract 1500**

- a. Explain why you think Eddie chose to multiply the second equation by **-9**.

By doing this, the b-terms in each equation had opposite coefficients. This causes the b-terms to “eliminate” when the equations are added together.

- b. Monica looks at Eddie’s work and says, “I got the same answer, but I eliminated the c’s first.” Show the work that Monica must have used to get the same answer as Eddie.

$$\begin{array}{l}
 9b + 5c = 25500 \\
 b + c = 3500
 \end{array}
 \xrightarrow{\text{(Multiply 2<sup>nd</sup> eq. by -5)}}
 \begin{array}{l}
 9b + 5c = 25500 \\
 -5b - 5c = -17500
 \end{array}$$

Add these equations together

$$\begin{array}{l}
 4b = 8000 \\
 b = 2000
 \end{array}$$

**Divide by 4**

**Substitute into  $b + c = 3500$**

$$\begin{array}{l}
 2000 + c = 3500 \\
 c = 1500
 \end{array}$$

**Subtract 2000**

- c. Eddie and Monica are trying to solve the following systems of equations using **elimination**, but they are stuck on what to multiply by. Their teacher gives them a hint, “**Sometimes you have to multiply each equation by a different number.**”

$$-2a + 6b = 6$$

$$-7a + 8b = -5$$

- i. Eddie looks at the **a** coefficients and thinks aloud to his partner, “**What if we multiplied the first equation by negative 7 and the second equation by positive 2?**”  
What new system of equations will result from following Eddie’s idea? Will this first step be productive in finding the correct answer? Explain.

$$14a - 42b = -42$$

$$-14a + 16b = -10$$

**This is a productive first step since the coefficients of the a-terms are now opposites.**

- ii. Monica’s idea is slightly different. She wants to eliminate the **b**’s. What should she multiply by in order to help eliminate the **b**’s?

**Multiply the first equation by negative 8 and the second equation by positive 6.**

**OR**

**Multiply the first equation by positive 8 and the second equation by negative 6.**

- iii. Use either Eddie or Monica’s first step and solve for **a** and **b** using the elimination method.

$$a = 3, b = 2$$

5. Rita decides to solve the original system of equation **graphically**.

$$9b + 5c = 25500 \quad b + c = 3500$$

- a. Follow Rita’s work and explain her process.

$$9b + 5c = 25500$$

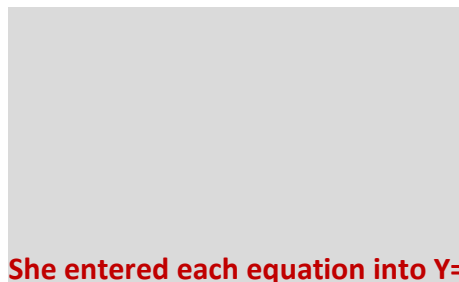
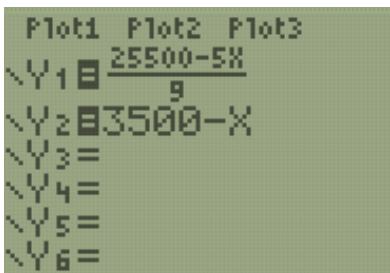
$$b + c = 3500$$

$$9b = 25500 - 5c$$

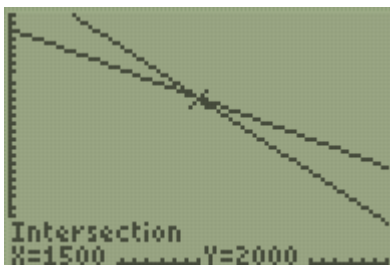
$$b = 3500 - c$$

$$b = \frac{25500 - 5c}{9}$$

**Rita solved each equation for  $b$**



**She entered each equation into Y=**



**$c = 1500$  and  $b = 2000$**

- b. Based on her calculator work, explain how Rita knew that  $c$  equaled 1500 and not the other way around.

**Since she solved each equation for  $b$  in terms of  $c$ , the  $b$ -variable behaves as  $y$  and the  $c$ -variable is  $x$ .**

6. After reviewing the **substitution**, **elimination**, and **graphing** method, identify one advantage and one disadvantage of each method. When might it make the most sense to use each method?

**Responses to this will vary.**

**Students do not need to be masters of all three methods, although they must be able to perform the substitution or elimination method if a Regents Exam question asks for an algebraic solution. In my experience, students tend to prefer the elimination method, but the substitution method can sometimes be faster if one of the equations is already isolated for one of the variables. If both equations are already isolated for  $y$ , then graphing is a natural choice.**

7. Solve each problem involving a system of two linear equations algebraically by using either substitution or elimination.

a. 
$$\begin{aligned} 2x - 6y &= 28 \\ -x - y &= 14 \end{aligned}$$

$x = -7, y = -7$

c. 
$$\begin{aligned} -5x - 14y &= -9 \\ -10x - 7y &= 3 \end{aligned}$$

$x = -1, y = 1$

b. 
$$\begin{aligned} -2x + 2y &= -16 \\ -3x - 2y &= 11 \end{aligned}$$

$x = 1, y = -7$

d. 
$$\begin{aligned} y &= 4x - 5 \\ -x - 8y &= 7 \end{aligned}$$

$x = 1, y = -1$

8. To participate in a school trip, Kim had to earn \$85 in one week. Kim could earn \$8 per hour babysitting and \$15 dollars per hour for yard work. Kim’s parents limit work time to 8 hours per week. How many hours should Kim work at each job in order to meet her income goal and work exactly eight hours?

$$y = 3, b = 5$$

9. Solve the following Regents question from the June 2019 exam.

When visiting friends in a state that has no sales tax, two families went to a fast-food restaurant for lunch. The Browns bought 4 cheeseburgers and 3 medium fries for \$16.53. The Greens bought 5 cheeseburgers and 4 medium fries for \$21.11.

Using  $c$  for the cost of a cheeseburger and  $f$  for the cost of medium fries, write a system of equations that models this situation.

$$4c + 3f = 16.53$$

$$5c + 4f = 21.11$$

The Greens said that since their bill was \$21.11, each cheeseburger must cost \$2.49 and each order of medium fries must cost \$2.87 each. Are they correct? Justify your answer.

$$\text{No, } 5(2.49) + 4(2.87) = 23.93$$

Using your equations, algebraically determine both the cost of one cheeseburger and the cost of one order of medium fries.

**This question requires an algebraic solution, so only the use of substitution or elimination can receive full credit.**

$$c = 2.79 \text{ and } f = 1.79$$

10. Solve the following systems using graphing.

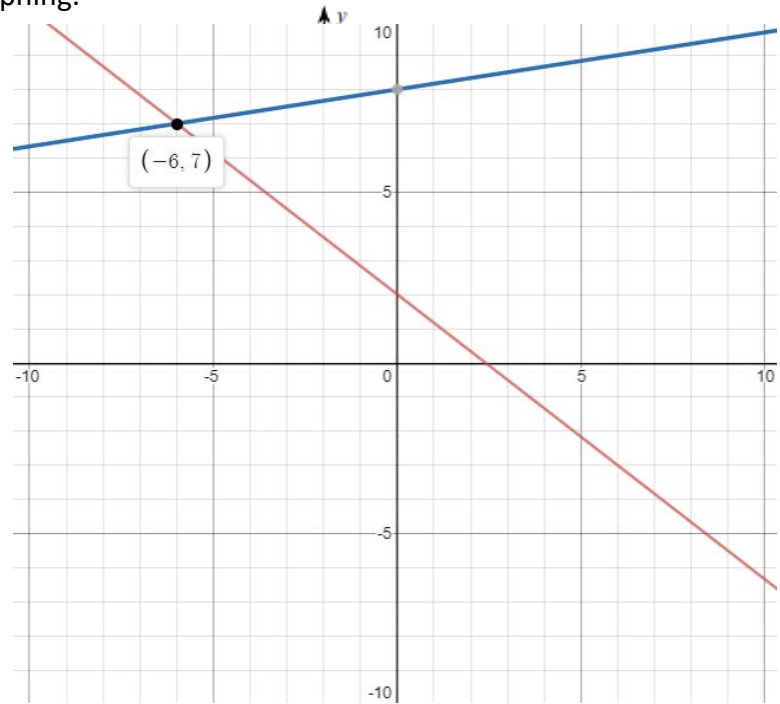
a.

$$y = -\frac{5}{6}x + 2$$

$$y = \frac{1}{6}x + 8$$

Students should use the table to plot exact coordinates. They may skip points that do not have whole number coordinates.

$x = -6, y = 7$

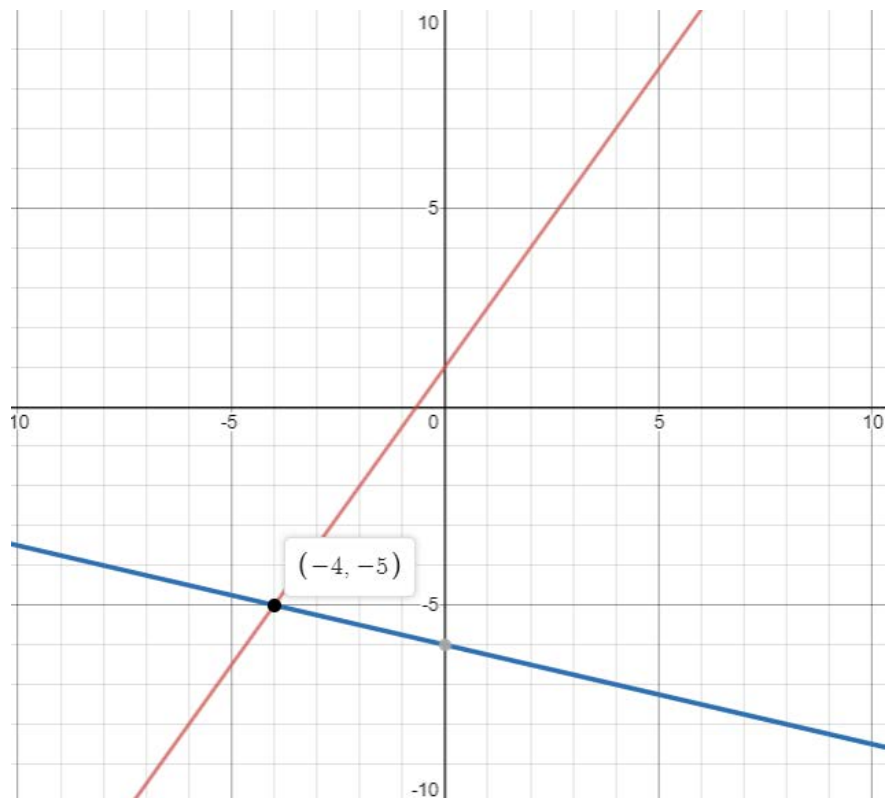


b.

$$y = \frac{3}{2}x + 1$$

$$y = -\frac{1}{4}x - 6$$

$x = -4, y = -5$



**Jess was reading the following part from problem 8:**

To participate in a school trip, Kim had to earn \$85 in one week. Kim could earn \$8 per hour babysitting and \$15 dollars per hour for yard work. Kim’s parents limit work time to less than 8 hours per week.

11. Jess thought aloud to her partner, “Kim wants to earn at least \$85, but she could make more than that. Also, her parents limit her work to 8 hours, but she could work less than that if she wanted.”
- a. Assuming Kim makes \$8 babysitting and \$15 per hour for yard work, determine three different ways she could work to earn at least \$85 and also work less than 8 hours.

**She could do yardwork only for six hours and earn \$90**

**She could do yardwork for five hours and babysit for two hours and earn \$91**

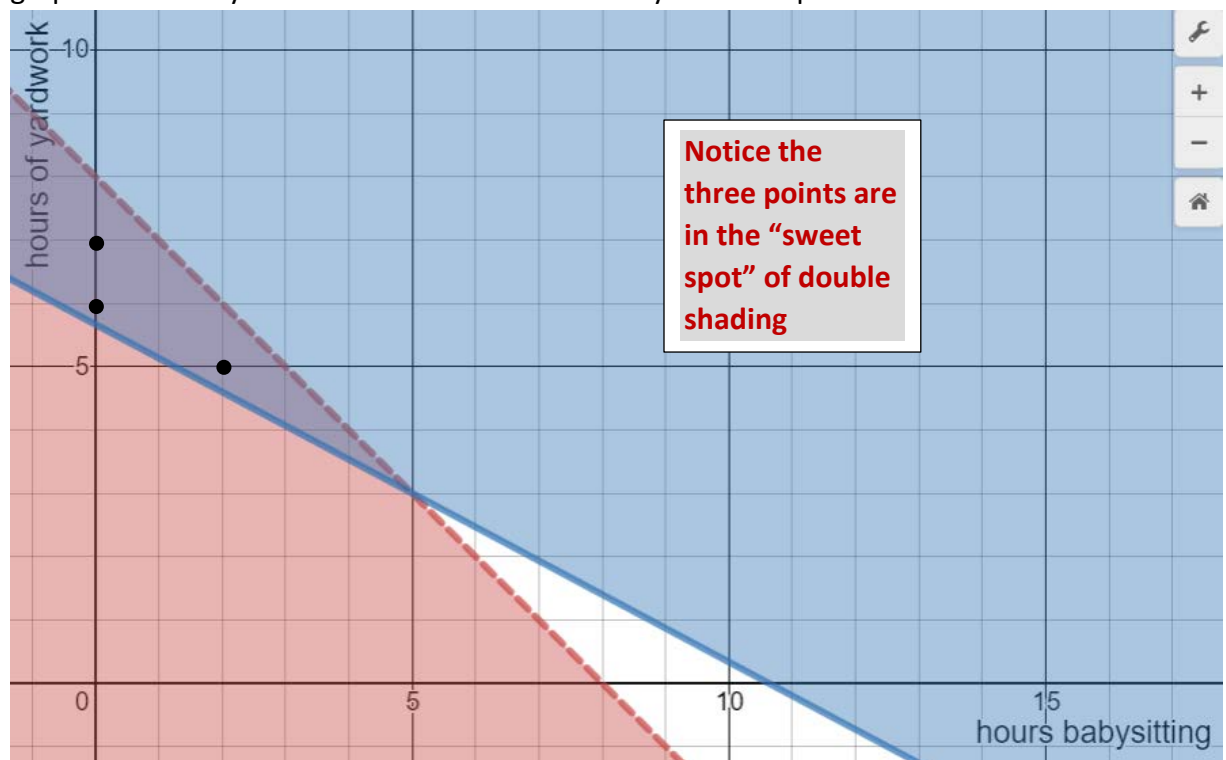
**She could do yardwork for seven hours and earn \$105**

- b. The system of equations from problem 8 is written below, but the equal signs have been removed. Replace them with the proper inequality symbol  $<$ ,  $>$ ,  $\leq$ ,  $\geq$ .

$$b + y < 8$$

$$8b + 15y \geq 85$$

- c. In Algebra 1, we solve **systems of inequalities** graphically. The graph to this system of inequalities is below. On the graph, plot your three answers to part a as points on the graph. What do you notice about the location of your three points?



- d. One of the inequality graphs uses a dotted line and the other uses a solid line. Why do you think this is?

**The dotted line indicates  $<$  or  $>$ , but not equal to.**

**The solid line is for  $\geq$  or  $\leq$**

- e. The dotted-line-inequality graph is shaded below the dotted line and the solid-line-inequality graph is shaded above the solid line. What features about the algebraic inequalities from part b explain why this is?

**For  $<$  or  $\leq$ , shade below the line**

**For  $>$  or  $\geq$ , shade above the line**

- f. The student who produced the graph in part c, Ernest, explained his process. “First, I solved each inequality for  $y$  just like I would solve a normal equation. Then I typed those into  $Y=$  and looked at the table to plot the points of each line graph. Then I used the inequality symbols to decide which line had to be dotted or solid, and which way to shade.”

- i. Show the algebraic work Ernest might have used to solve each inequality for  $y$ .

$$b + y < 8$$

$$8b + 15y \geq 85$$

$$y < 8 - b$$

$$15y \geq 85 - 8b$$

$$y \geq \frac{85-8b}{15}$$

- ii. What did Ernest mean when he wrote, “I used the inequality symbols to decide which line had to be dotted or solid, and which way to shade”?

**Since the first inequality is  $<$ , he used a dotted line and shaded below it**

**Since the second inequality is  $\geq$ , he used a solid line shaded above.**

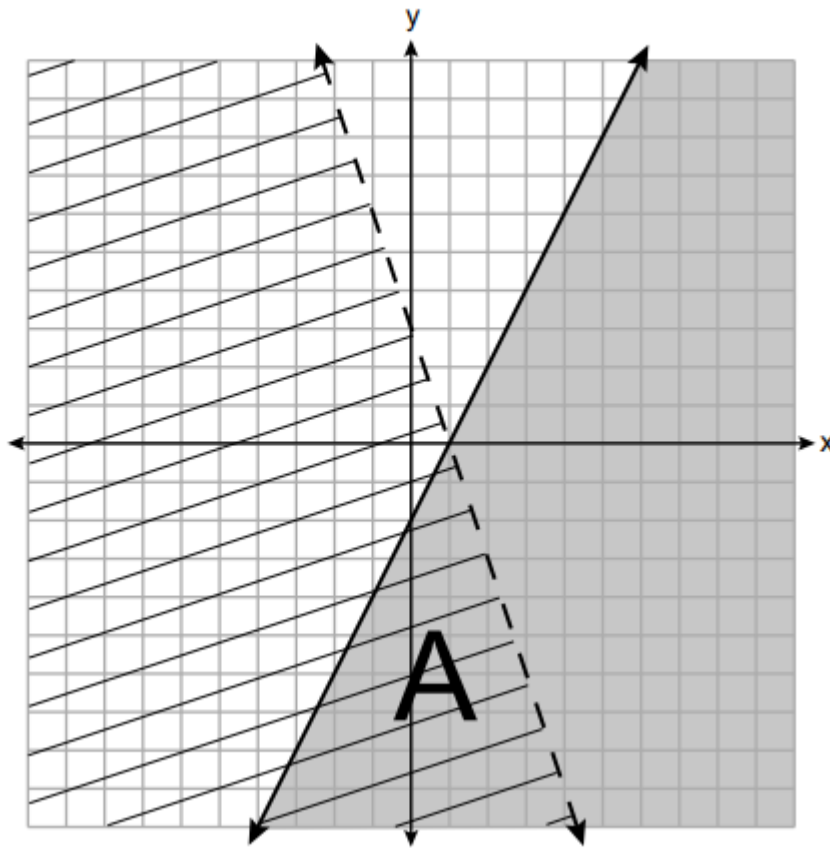


- g. The double-shaded region on the graph is called the **solution set**. Explain what the solution set of this graph represents in the context of Kim’s school trip situation.

**The solution set represents all the combinations of hours spent doing yard work and babysitting that would satisfy both constraints – working fewer than 8 hours as well as earning at least \$85.**

12. Solve the following question from the June 2019 Regents exam.

A system of inequalities is graphed on the set of axes below.



State the system of inequalities represented by the graph.

$$y \leq 2x - 2$$

$$y < -3x + 3$$

**One trick to generating equations is to use linear regression. Have students pick two point on the line and use regression.**

**It’s also interesting to view how partial credit can be allocated. Visit <http://www.jmap.org/JMAPRegentsExamArchives/ALGEBRAIEXAMS/0619ExamAl.pdf> and look at the anchor papers for this question starting on page 60.**

State what region A represents.

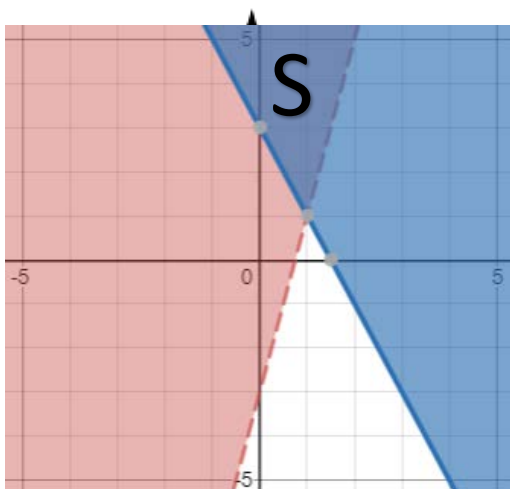
**Region A is the solution set**

State what the entire gray region represents.

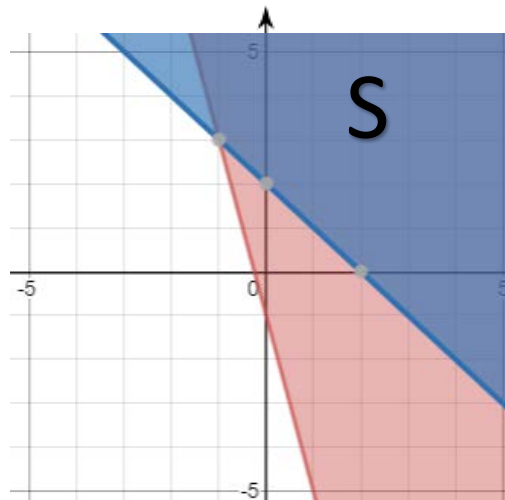
**This is the solution to  $y \leq 2x - 2$**

13. Sketch the solution to each system of inequalities. Label each solution set with the letter S.

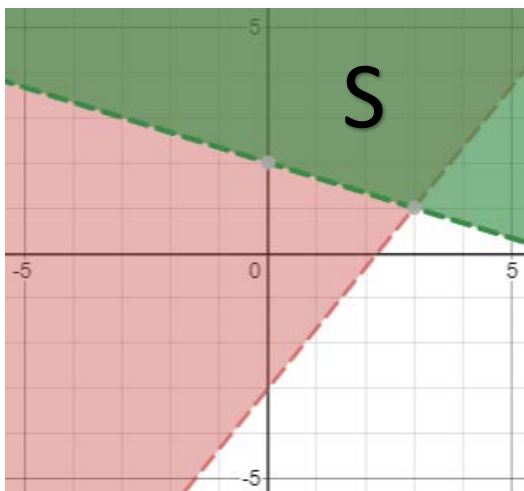
a.  $y > 4x - 3$   
 $y \geq -2x + 3$



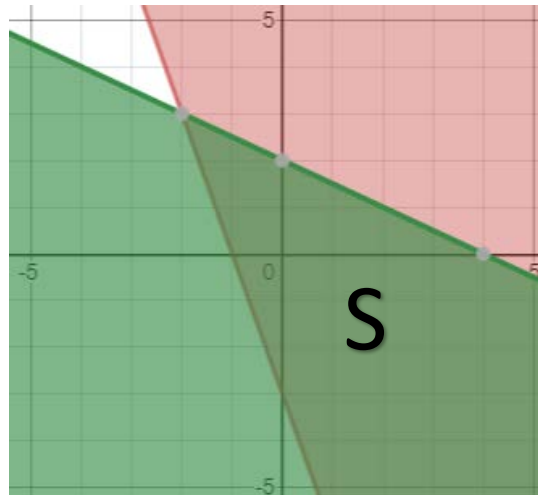
c.  $x + y \geq 2$   
 $4x + y \geq -1$



b.  $4x - 3y < 9$   
 $x + 3y > 6$



d.  $3x + y \geq -3$   
 $x + 2y \leq 4$

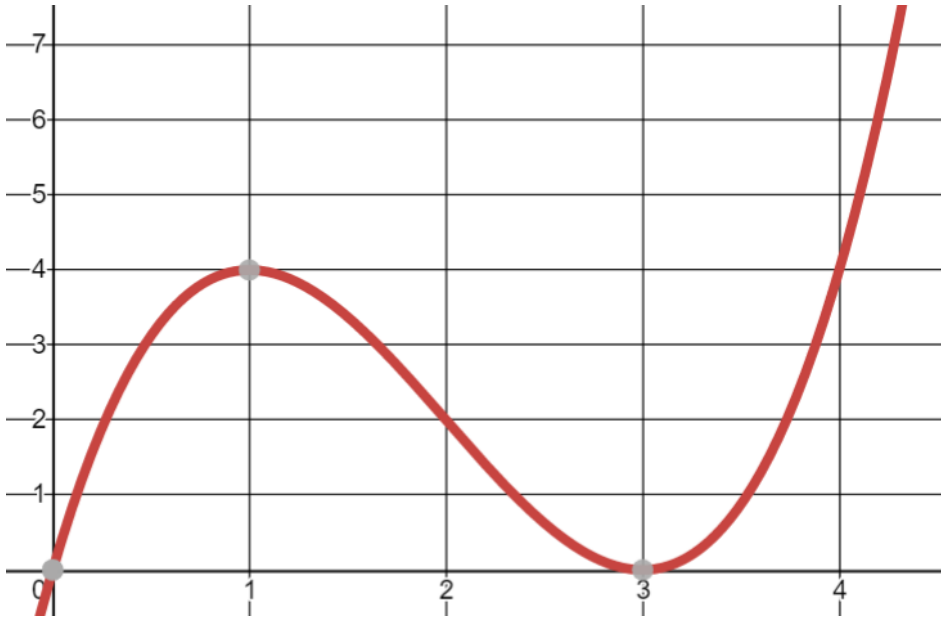


# Lesson 6 – An Overview of Polynomials

Francisca is an engineer who designs roller coasters. She is designing two sections of a new roller coaster for Six Flags theme park. Using **regression**, she generates two **polynomials** to model her track designs.

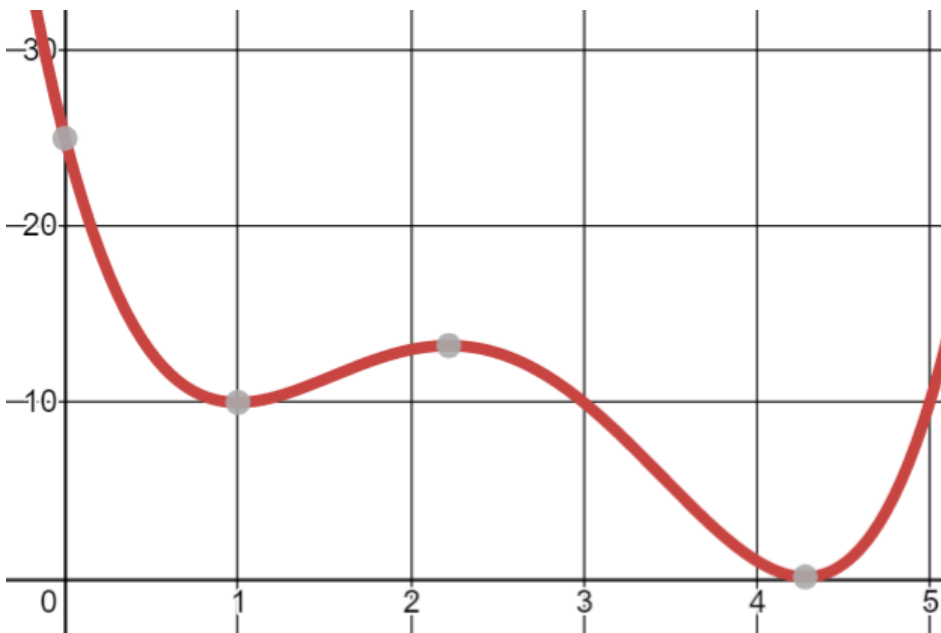
## Section 1

$$f(x) = x^3 - 6x^2 + 9x$$



## Section 2

$$g(x) = x^4 - 10x^3 + 32x^2 - 38x + 25$$



**Polynomials** are a very important family of functions that include two types of functions you have studied so far. All linear and quadratic functions are part of the polynomial function family. Polynomial rules can be written in **standard** or sometimes in **factored** form. Both  $f(x)$  and  $g(x)$  are standard form. Consider all the following examples written in standard form.

$$f(x) = x^3 - 6x^2 + 9x \quad \text{Degree 3, l.c. is 1}$$

$$g(x) = 4x^4 - 10x^3 + 32x^2 - 38x + 25 \quad \text{Degree 4, l.c. is 4}$$

$$h(x) = -2x^2 - 7x + 15 \quad \text{Degree 2, l.c. is -2}$$

$$k(x) = 5x + 1 \quad \text{Degree 1, l.c. is 5}$$

$$m(x) = x^6 + 4x^3 - 7x \quad \text{Degree 6, l.c. is 1}$$

$$n(t) = 16t^8 - 4 \quad \text{Degree 8, l.c. is 16}$$

- 1) What do you believe are the important characteristics of a standard form polynomial?

Students should notice that all polynomials have whole number positive exponents on the variable and multiple terms separated by addition and subtraction. Note that linear functions, such as  $k(x)$  have an invisible exponent of 1 on the  $x$ .

- 2) You may have noticed that polynomials all have exponents on the variable. Even  $k(x)$  has an exponent of 1 on the  $x$  that is usually not written. A polynomial's **degree** is the value of its highest exponent. Next to each polynomial function above, write its degree next to its function rule. For example, you can write "**degree 3**" or "**deg 3**" next to  $f(x)$ .

The **highest degree term** is the term that has the biggest exponent on the variable. Standard form polynomials are always written from the highest degree term to the lowest degree term. A term with no variable is called a **constant**.

- 3) A coefficient is the constant that multiplies the variable. The coefficient of  $-6x^5$  is  $-6$ . The coefficient of the highest degree term is called the **leading coefficient**. Write leading coefficient for each standard form polynomial above. For example, on page 1, in Section 2, the leading coefficient of  $g(x)$  is  $4$  so you could write "**l.c. is 4**" next to the rule for  $g(x)$ .
- 4) Rewrite each polynomial below in standard form. Then state its degree and the value of its leading coefficient.

a.  $4x^3 + 5x^5 - 4 - 2x^2$

$$5x^5 + 4x^3 - 2x^2 - 4, \text{ degree 5, l.c. is 5}$$

b.  $12 - 3x^2 + 7x - 3x^4$

$$-3x^4 - 3x^2 + 7x + 12, \text{ degree is 4, l.c. is -3}$$

Here are a selection of polynomials written in **factored form**.

$$a(x) = (x + 2)(x - 7)$$

$$b(x) = x(x + 3)(x + 5)$$

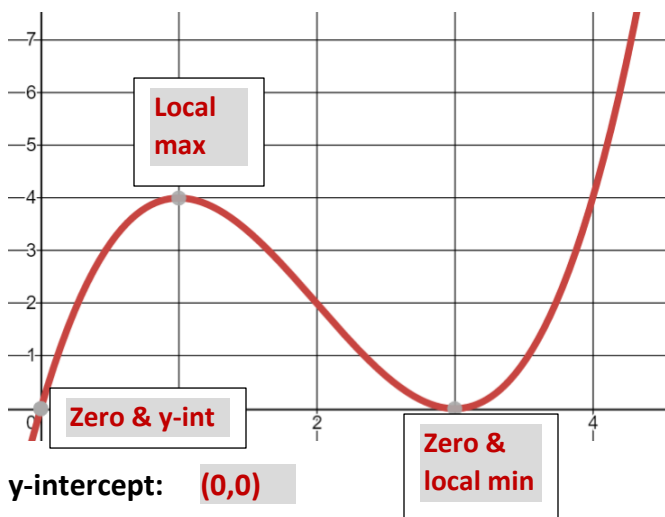
$$c(x) = (x - 4)(x + 2)(x - 1)$$

What do you think are the important characteristics of a factored form polynomial?

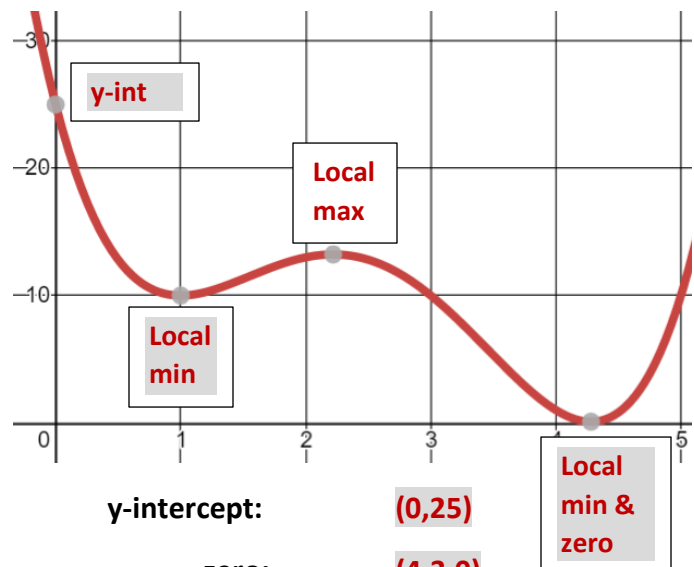
**Factored form are linear functions, usually in parentheses, that are separated by multiplication.**

Polynomials have some important features. Recall the vertex of a quadratic function was its turning point. Polynomial graphs can have many turning points. When a turning point makes a little “hill” it is called a **local maximum**. When a turning point makes a “valley” we call it a **local minimum**. Similarly, polynomials also have a **y-intercept**, and their **x-intercepts** are called **zeros** or **roots**.

- 5) Label Francisca’s roller coaster track designs’ local minimum(s), local maximum(s), roots, and zeros below and write their coordinates.



y-intercept: **(0,0)**  
 zeros: **(0,0) and (3,0)**  
 local maximum(s): **(1,4)**  
 local minimum(s): **(3,0)**



y-intercept: **(0,25)**  
 zero: **(4.3,0)**  
 local minimum(s): **(1,10) and (4.3,0)**  
 local maximum(s): **(2.2, 13)**

(Okay to estimate)



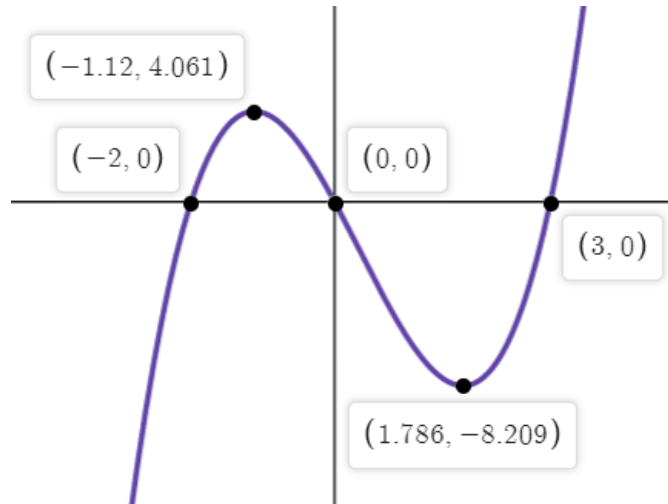
b.  $g(x) = x(x + 2)(x - 3)$

=  $x^3 - x^2 - 6x$

**Degree:** 3

**Leading Coefficient:** 1

**Sketch:**



**y-intercept:** (0,0)

**zero(s):** (-2,0), (0,0), and (3,0)

**Local maximum(s):** (-1.12,4.061)

**Local minimum(s):** (1.786,-8.209)

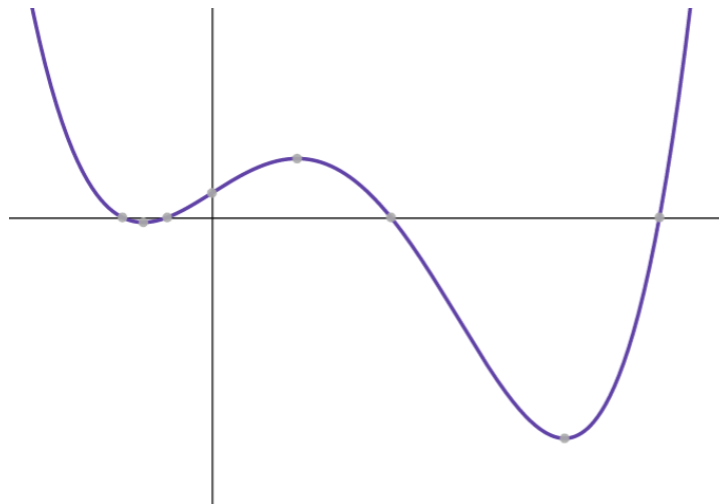
c.  $k(x) = 2x^4 - 11x^3 + 23x + 10$

=  $(x + 1)(2x + 1)(x - 5)(x - 2)$

**Degree:** 4

**Leading Coefficient:** 2

**Sketch:**



**y-intercept:** (0,10)

**zero(s):** (-1,0), (-0.5,0), (2,0), (5,0)

**Local maximum(s):** (0.952, 24.048)

**Local minimum(s):** (3.94, -90.209)

- 8) Jared and Karen were working together on problem 7. Jared notices “you can find the zeros if you look at the factored form. Just switch the sign in the middle and it tells you.” Karen looked at her work and noticed “that only sometimes work. Look at part c. When I checked with the graph the zeros were -1, 5, 2, but also -0.5.”
- a. Do you believe that the factored form  $k(x) = (x + 1)(2x + 1)(x - 5)(x - 2)$  can be helpful to determine that the zeros are -1, 5, 2, and -0.5? Which factor helps to find each zero?

**-1 goes with (x+1), 5 goes with (x-5), 2 goes with (x-2), and -0.5 goes with (2x+1)**

- b. Substitute each number into the factor that you believe it belongs with. What number do you get? Why do you think these numbers are called zeros?

**You get zero. These x-values are called zeros because the function’s y-value is zero when you plug in each of these x-values.**

- c. Karen had an idea. “We are looking for the  $x$  that makes each factor zero. What if I make each factor an equation that equals zero and solve for  $x$ .” Verify that this strategy works for each factor of  $k(x)$ .

**$-1 + 1 = 0$ ,  $2(0.5) + 1 = 0$ ,  $5 - 5 = 0$ ,  $2 - 2 = 0$**

- 9) Analyze your results from problem seven. Identify which form is best for finding a polynomial’s degree, leading coefficient,  $x$ -intercepts,  $y$ -intercept, and coordinates of its local maximum and/or minimum point(s). Briefly explain how you can use that form to identify each important characteristic.

**In general, standard form is best for determining the degree, leading coefficient, and  $y$ -intercept. The  $y$ -intercept is the value of the constant term. The factored form is best for finding the zeros/roots. To find them, set each factor equal to zero and solve for  $x$ . The graph is the best way to locate the local max and min points. To find them when viewing a graph, press 2<sup>nd</sup> TRACE, select minimum or maximum, then choose a point to the left, right, and near the max or min point.**

- 10) How can you use the calculator table or graph to find a polynomial’s zeros and  $y$ -intercept?

**In the table, the zeros are the  $x$ -values where  $y$  is zero. On a graph, the zeros are the  $x$ -intercepts.**

**The  $y$ -intercept is the  $y$ -value where  $x$  is zero on the table. On a graph, it is the location where the graph intersects the  $y$ -axis.**







8.

The quadratic functions  $r(x)$  and  $q(x)$  are given below.

$x$	$r(x)$
-4	-12
-3	-15
-2	-16
-1	-15
0	-12
1	7

$$q(x) = x^2 + 2x - 8$$

The function with the *smallest* minimum value is

- (1)  $q(x)$ , and the value is  $-9$       (3)  $r(x)$ , and the value is  $-16$   
(2)  $q(x)$ , and the value is  $-1$       (4)  $r(x)$ , and the value is  $-2$

**Choice 3**