Lesson 1 – Functions

Function Definition and Representations

A **function** is one of the most important concepts in Algebra. Consider the following examples.

Tony and Maria attend different schools that each have a vending machine in the cafeteria.

- Tony's favorite snack, potato chips, are in the location labeled A7. Each time he pays and inputs A7, potato chips come out.
- Maria's favorite snack, chocolate bars, are in the location labeled B4. Each time she pays and inputs B4, the vending machine drops a chocolate bar, but also mixed nuts.

Something is a **function** if every x-value (or input) in the domain is assigned to only one y-value (or output) in the range.

1. Based on this definition, which person's vending machine would be an example of something that is a function? Explain how you know.

2. Explain why the other person's vending machine is not a function.

The **domain** of a function is the set of all inputs with outputs. The set of all outputs a function has is its **range**.

3. In the example of the vending machine, what would represent the domain and range?

A function is usually some mathematical <u>rule</u> that tells you what to do with x in order to get y.

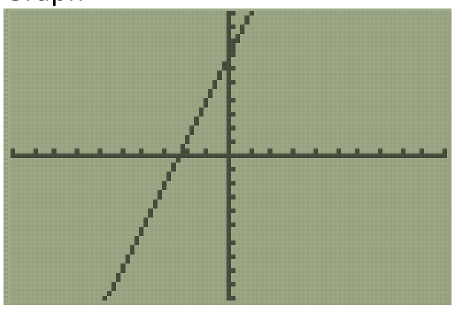
Consider the <u>linear</u> function

$$y = 3x + 7$$

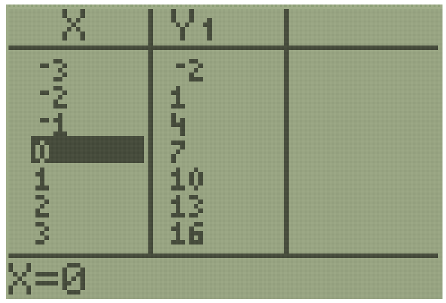
This rule tells you to multiply each x-value by 3 then add 7 to get the y-value.

Here are other ways to represent $y = 3x + 7.$

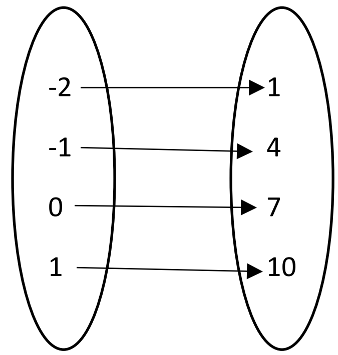
Graph



Table



Arrow Diagram (mapping)



Every x-value has exactly one y-value.

4. Solve the following Regents question.

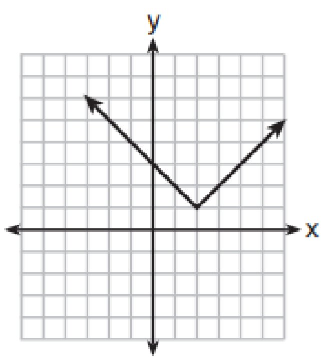
7 Which relation does *not* represent a function?

x	1	2	3	4	5	6
y	3.2	4	5.1	6	7.4	8.8

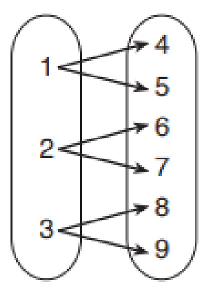
(1)

$y = 3\sqrt{x+1} - 2$

(3)



(2)



(4)

An equivalent way to write this linear function is with <u>function notation</u>: $\boldsymbol{f(x) = 3x + 7}$

You say **"f of x"** when you see $\boldsymbol{f(x)}$ and it is another way of writing the variable $\boldsymbol{y}$.

Function notation is used to input numbers for **x**.

For instance, consider $\boldsymbol{f(8)}$**.** This means "find the y-value when x equals 8." For a simple linear function such as $\boldsymbol{f(x) = 3x + 7}$, you may be able to find y in your head, or with the home screen of your calculator. Here is how to calculate $\boldsymbol{f(8)}$ *algebraically*.

$$\boldsymbol{f(8) = 3 \cdot 8 + 7}$$

$$\boldsymbol{f(8) = 24 + 7}$$

$$\boldsymbol{f(8) = 31}$$

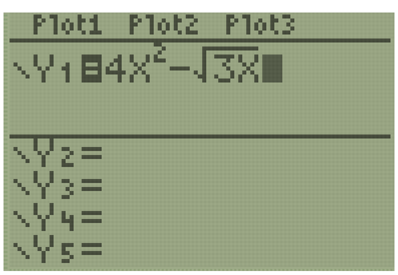
5. Determine the value of $\boldsymbol{f(3)}$**,** $\boldsymbol{f(25)}$, and of $\boldsymbol{f(-6)}$ algebraically.

Linear functions like $\boldsymbol{f(x) = 3x + 7}$ are the easiest ones to work with.

<u>Using the TABLE</u>

Consider a non-linear function $\boldsymbol{g(x) = 4x^2 - \sqrt{3x}}$**.** Notice that this function has been named $g(x)$ "g of x". We can use any letter to name a function, which is helpful when a problem involves more than one function. Let's explore $g(x)$ with the calculator.

Every function can be represented with a <u>table</u> or a <u>graph</u>.



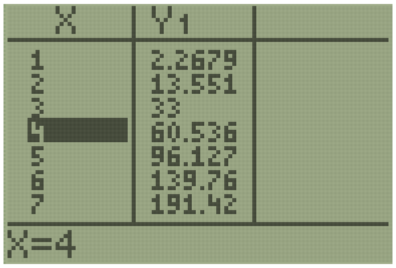
All function rules can be put into **Y=**

The 2 on top of the x is an <u>exponent</u>. To get an exponent, **press the ^ key** and then the value of the exponent. Since 2 is a very common exponent, you can also press **the** $\boldsymbol{x^2}$ **key**. Notice that anything you type will stay in the exponent until you press the right arrow button →.

Notice that this function involves the <u>square root</u> of 3x. This symbol $\sqrt{}$ is called a <u>radical</u>. Find the radical symbol on your calculator by pressing **2nd** and $\boldsymbol{x^2}$.

From the **Y=** menu, we can find specific values of $g(x)$ by using tables or graphs.

Alejandro wants to find $\boldsymbol{g(4)}$ using a table.

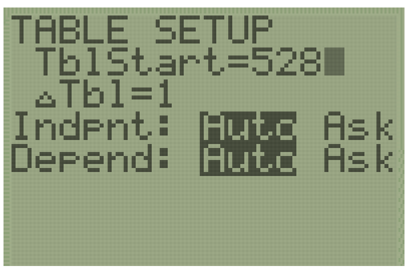


Press **2^nd^, GRAPH**

Use the up and down arrows until you get to x = 4.

$$g(4) = 60.536$$

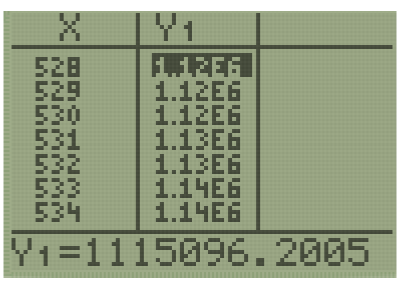
Next, Alejandro wants to find $\boldsymbol{g(528)}$ using a table. He definitely doesn't want to press the down arrow that many times.



He presses **2^nd^, WINDOW** and changes **TblStart** to 528

$\boldsymbol{\Delta Tbl = 1}$ means that the x-values will count by 1.

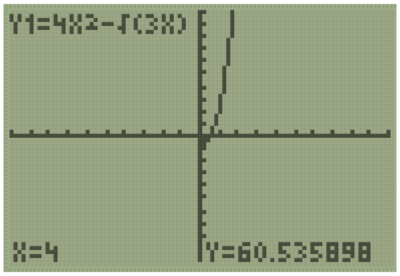
Don't worry about the rest.



When Alejandro presses **2^nd^ GRAPH**, his table starts with **x = 528**. The y-value is too large to display properly unless he highlights it using the arrow keys. This sometimes happens when x is very large.

Alejandro concludes that $g(528) = 1115096.2005$

Using the GRAPH



Tatiana is working with the same function, $\boldsymbol{g(x) = 4x^2 - \sqrt{3x}}$

Tatiana wants to find $\boldsymbol{g(4)}$ using a graph.

Press **GRAPH**. If your viewing window is different, press **ZOOM, 6** to get a standard window.

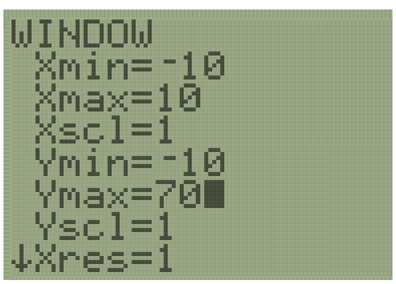
Press **TRACE**, **4**, **ENTER**

Tatiana notices that when x = 4, y = 60.535898.

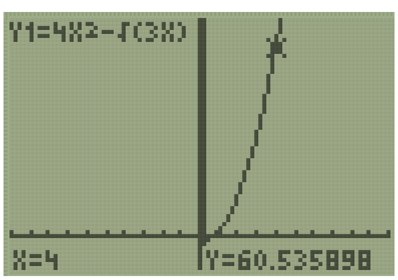
Alejandro's and Tatiana's answers for $g(4)$ are slightly different, but that's okay.

Tatiana notices that her calculator tells her that $g(4) = 60.535898$ at the bottom of her screen, but that she can't see the y-value 60.535898 on the graph of the function. She counts along the y-axis and notices that it only goes up to 10.

She decides to make her graph show more y-values.



She presses **WINDOW** and sets her **maximum y-value** to 70.



When she presses **GRAPH**, the y-axis goes up to 70.

When she presses **TRACE**, **4**, **ENTER**, she can see the location of $g(4)$ on the graph.

In general, make the **Xmax** and **Ymax** higher than the number you want, and make the **Xmin** and **Ymin** lower than the number you want. You never need to change the **Xscl** or **Yscl**, but if you do, they tell the calculator how far to space the marks on the x-axis and y-axis.

Try the following Regents questions. Try to use the graphing calculator tables and graphs, and what you know about functions, to answer them.

6.

The function $g(x)$ is defined as $g(x) = -2x^2 + 3x$. The value of $g(-3)$ is

(1) -27 (3) 27

(2) -9 (4) 45

7.

If $k(x) = 2x^2 - 3\sqrt{x}$, then $k(9)$ is

(1) 315 (3) 159

(2) 307 (4) 153

8.

Marc bought a new laptop for \$1250. He kept track of the value of the laptop over the next three years, as shown in the table below.

Years After Purchase	Value in Dollars
1	1000
2	800
3	640

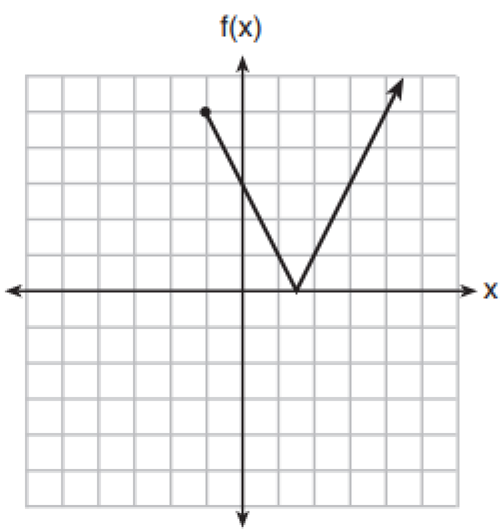
Which function can be used to determine the value of the laptop for x years after the purchase?

(1) $f(x) = 1000(1.2)^x$
(2) $f(x) = 1000(0.8)^x$
(3) $f(x) = 1250(1.2)^x$
(4) $f(x) = 1250(0.8)^x$

Try each choice in Y= and choose the answer with the same table

9.

The function $f(x)$ is graphed below.

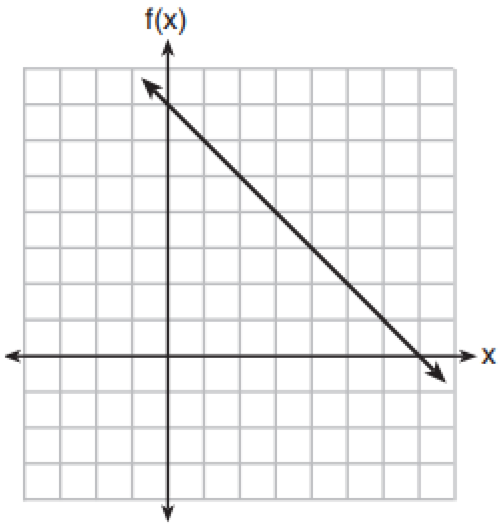


The domain of this function is

(1) all positive real numbers
(2) all positive integers
(3) $x \geq 0$
(4) $x \geq -1$

10.

The functions $f(x)$, $q(x)$, and $p(x)$ are shown below.



$$q(x) = (x - 1)^2 - 6$$

x	p(x)
2	5
3	4
4	3
5	4
6	5

When the input is 4, which functions have the same output value?

(1) $f(x)$ and $q(x)$, only
(2) $f(x)$ and $p(x)$, only
(3) $q(x)$ and $p(x)$, only
(4) $f(x)$, $q(x)$, and $p(x)$

11.

Materials A and B decay over time. The function for the amount of material A is $A(t) = 1000(0.5)^{2t}$ and for the amount of material B is $B(t) = 1000(0.25)^t$, where t represents time in days. On which day will the amounts of material be equal?

(1) initial day, only
(2) day 2, only
(3) day 5, only
(4) every day

Lesson 2 – Linear and Exponential Functions

Think about your own family. Everyone in your family has things about them that make them unique. But now think about what makes your family similar. Maybe it's the way you speak, or dress, or behave that makes the people in your family similar.

Just like you, certain functions belong to a family. In Algebra 1, we focus on four main families: linear, exponential, quadratic, and absolute value.

This lesson will aim to deepen your understanding of the characteristics of linear and exponential function "families." We will also learn a technique, called **regression**, which will allow us to write a function rule using a set of coordinates.

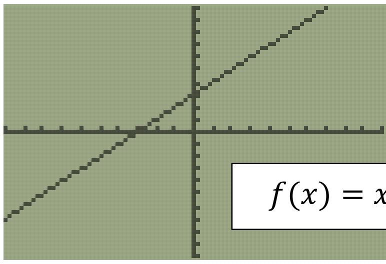
Linear Functions

The following four situations are examples of linear functions.

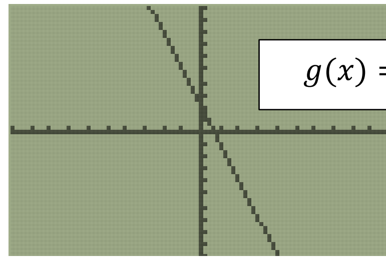
- Pedro earns $12 per hour at his job.
- A plant's height increases 3 inches per week.
- As water empties from a bathtub, its volume decreases at a rate of 2 gallons per minute.
- A car's distance traveled is changing at a constant speed of 55 miles per hour.

Write all the ways these situations are similar and how they are different.

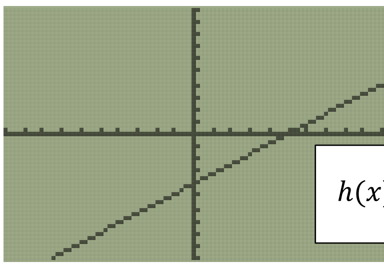
The following graphs and rules are linear functions.



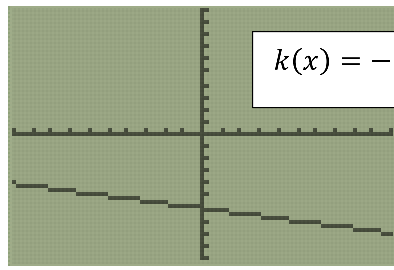
$f(x) = x + 3$



$g(x) = -3x + 2$



$h(x) = \frac{4}{5}x - 4$



$k(x) = -0.2x - 6$

What is similar about the graphs of linear functions? What are some differences between the graphs?

How are the linear function rules similar to one another? How are the linear function rules different?

Use your calculator the fill in tables for *f*, *g*, *h*, and *k* from $x = -5$ to $x = 5$.

x	$f(x) = x + 3$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

x	$g(x) = -3x + 2$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

x	$h(x) = \frac{4}{5}x - 4$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

x	$f(x) = -0.2x - 6$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

What similarities and differences do you notice about the tables of linear function rules?

Every linear function rule can be written in the form

$$\boldsymbol{y = mx + b}$$

The ***m*** and ***b*** represent numbers. Here are important ideas about them that you may have noticed from the similarities and differences work you just did.

m

- Also called the rate of change
- It is the slope of the graph
- How much the y-values increases or decreases by when the x-value increases by 1
- When m is positive, the graph increases as x increases from left to right
- When m is negative, the graph decreases as x increases from left to right

b

- The value of y when x = 0
- It is where the graph intersects the y-axis
- The starting amount for linear situations

Exponential Functions

Exponential functions are a type of nonlinear function, because their patterns do not result in straight lines. The following situations are examples of exponential functions.

A. 500 bacteria double every hour.
B. A car bought for $25000 is worth half of its value each year.
C. A savings account has $1000 initially and gains 5% interest annually.
D. A school with 900 students is decreasing its enrollment by 2% each year.

What do each of these situations have in common? How are they different?

Here are rules for each of the situations A through D.

$$A(x) = 500(2)^x$$

$$B(x) = 25000\left(\frac{1}{2}\right)^x$$

$$C(x) = 1000(1.05)^x$$

$$D(x) = 900(0.98)^x$$

How are these exponential rules similar? How are they different?

What connections do you see between the situation described and its function rule?

Here are tables for each of the rules.

$A(x) = 500(2)^x$

X	Y1
0	500
1	1000
2	2000
3	4000
4	8000
5	16000
6	32000

$B(x) = 25000\left(\frac{1}{2}\right)^x$

X	Y1
0	25000
1	12500
2	6250
3	3125
4	1562.5
5	781.25
6	390.63

$C(x) = 1000(1.05)^x$

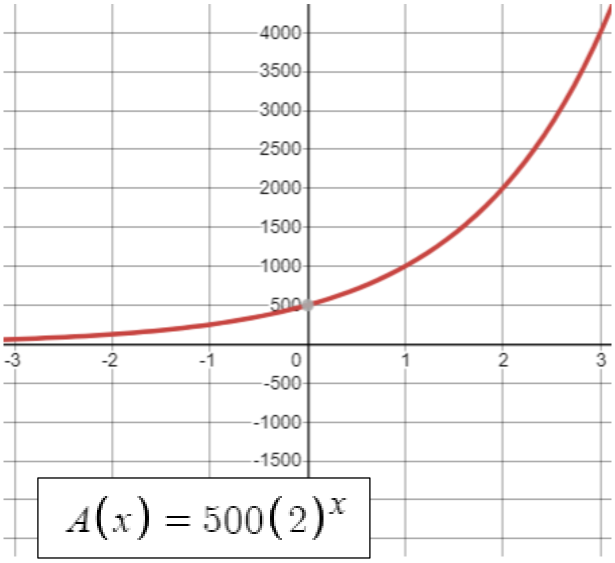
X	Y1
0	1000
1	1050
2	1102.5
3	1157.6
4	1215.5
5	1276.3
6	1340.1

$D(x) = 900(0.98)^x$

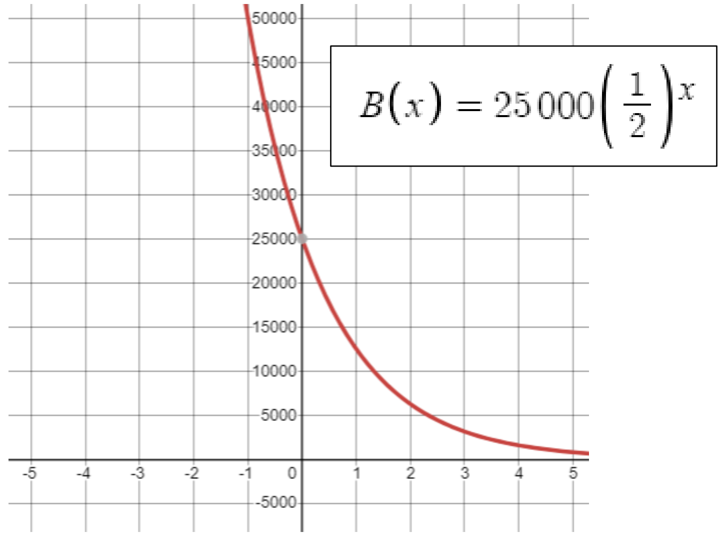
X	Y1
0	900
1	882
2	864.36
3	847.07
4	830.13
5	813.53
6	797.26

What similarities and differences do you notice between the exponential patterns in the tables? What connections do you notice about the function rule and the table of values?

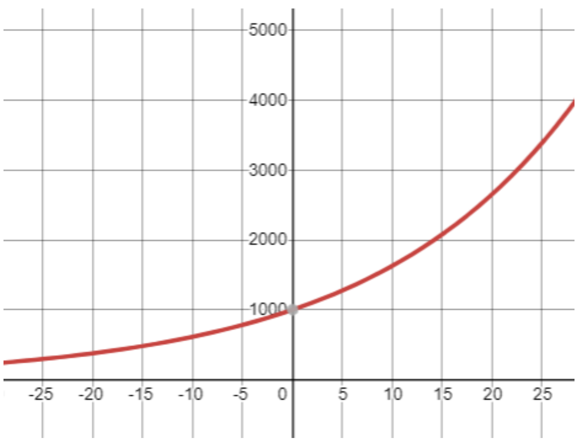
Here are graphs of each exponential function.



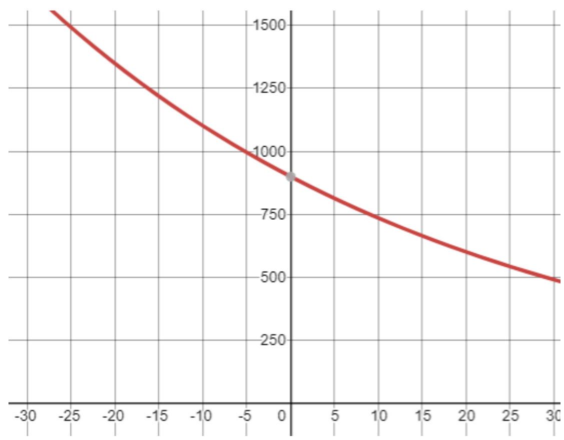
$A(x) = 500(2)^x$



$B(x) = 25\,000\left(\frac{1}{2}\right)^x$



$C(x) = 1\,000(1.05)^x$



$D(x) = 900(0.98)^x$

What are some similarities and differences you notice about how each graph is shaped?

How do the exponential graphs compare to the linear graphs?

What are some observations you can make about the how the exponential function rules and their graphs are related?

Every exponential function rule can be written in the form

$$\boldsymbol{y = a(b)^x}$$

Where **a** and **b** are numbers. Here are some important ideas about how these numbers affect the behavior of tables and graphs of exponential function rules.

a

- This is the initial (starting) value of the function
- On a table, this is the y-value when x = 0
- On a graph, this is where the graph will intersect the y-axis (vertical axis)

b

- This is the multiplier of the function
  If the function is growing or shrinking by a percent, b will be equal to $1 + \frac{\%}{100}$ or $1 - \frac{\%}{100}$
- To get the next y-value in the table, multiply the current y-value by b
- If b is greater than 1, the graph will be increasing from left to right
- If b is between 0 and 1, the graph will be decreasing from left to right

Decide whether the rule belongs to the linear function family, the exponential function family, or neither. Use evidence from the calculator or your knowledge of mathematics to justify your answer.

A. $y = 4 + 2x$

B. $y = 2(x+1)^2 - 5$

C. $f(x) = 3^x$

D. $g(x) = |x+1| - 3$

E. $k(x) = 0.5x - 6 + x$

F. $p(t) = 5000(0.87)^x$

Decide which situations below are best modeled by linear functions, exponential functions, or neither. Explain your reasoning.

i. A bamboo plant grows at a constant rate of 3 inches per day.

ii. An amusement park allows 50 people to enter every 30 minutes

iii. The value of a cell phone decreciates by 3.5% each year.

iv. A baseball tournament eliminates half of the teams after each round.

v. A football is kicked into the air from an initial height of 4 feet. It reaches a maximum height of 82 feet before it returns to the ground.

Linear Regression

This technique will transform a set of data into a line of best fit.

For example, in January 2019, the Regents exam gave data on the height of a certain breed of dog based upon its mass (weight).

First, make a prediction. What do you think is the relationship between the height of a dog and its weight? Would a linear function make sense to model this relationship?

Here is the question:

34 The data given in the table below show some of the results of a study comparing the height of a certain breed of dog, based upon its mass.

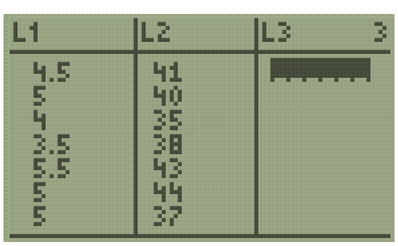
Mass (kg)	4.5	5	4	3.5	5.5	5	5	4	4	6	3.5	5.5
Height (cm)	41	40	35	38	43	44	37	39	42	44	31	30

Write the linear regression equation for these data, where x is the mass and y is the height. Round all values to the *nearest tenth*.

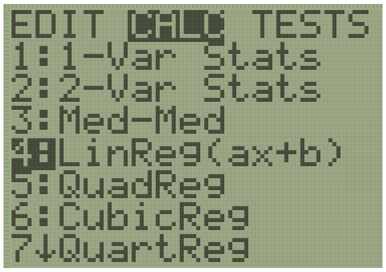
State the value of the correlation coefficient to the *nearest tenth*, and explain what it indicates.

The solution starts here. Follow along on your calculator as well.

Press **STAT**, **ENTER**



Type the x-values in **L1** and the y-values in **L2**

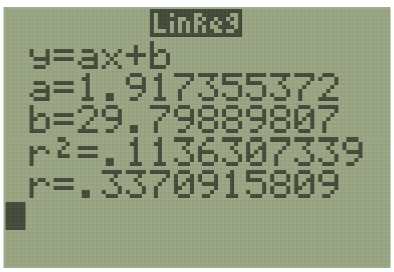


Press **STAT**, →

Since this problem asks for a <u>linear</u> regression, choose **4**

(<u>Exponential</u> regression is choice **0**)

Highlight "Calculate" and press **ENTER**



The first line, **y = ax + b**, is the function rule.

The next lines tell you the numbers to write in place of **a** and **b**. **x** and **y** will remain letters.

Ignore r^2

"r" is called the **correlation coefficient**, and it is always between 0 and 1. The closer **r** is to 1, the better fit this data is to the type of function rule you chose.

Notice that this question asked us to round the values of the function rule to the nearest tenth.

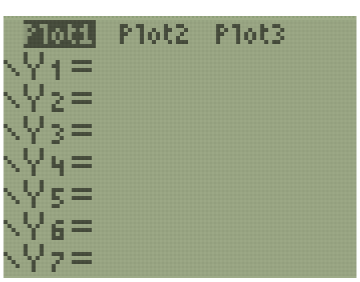
The linear regression equation is $\boldsymbol{y = 1.9x + 29.8}$

The correlation coefficient, to the nearest tenth, is **0.3**

Since 0.3 is not close to 1, this is a <u>weak</u> correlation.

Notice how the *r*-value 0.3 was closer to zero than one. This tells us that there is a weak relationship between the breed's height and weight.

To better see this, do the following:

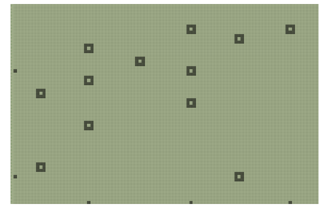


Step 1: When you press **Y=** be sure that **"Plot1"** is highlighted. If it isn't, use the **arrow keys** to highlight it and press **ENTER**

All **Y=** rules should be blank.

Step 2: Press **ZOOM**, and choose **“9: ZoomStat”** and then press **ENTER**

Describe the overall pattern of the data points. How do they compare to a linear function?



Next, to see how well our function fits the data, type $\boldsymbol{y = 1.9x + 29.8}$ into **Y1** by pressing **Y=**. This is the line that best fits our data. You can see that the line traces the pattern in the data, but that the points are relatively spread out. This makes for a weak correlation.

When working with real world measurements, or data, it is rare that the measurements make perfectly linear patterns. However, data is often close to linear, which is why the correlation coefficient is important. It tells us how well our data fits a line.

The correlation coefficient, **r**, is either strong or weak depending on whether it is closer to zero or one.

Complete the following problems on your own.

36 The percentage of students scoring 85 or better on a mathematics final exam and an English final exam during a recent school year for seven schools is shown in the table below.

Percentage of Students Scoring 85 or Better	
Mathematics, x	**English, y**
27	46
12	28
13	45
10	34
30	56
45	67
20	42

Write the linear regression equation for these data, rounding all values to the *nearest hundredth*.

State the correlation coefficient of the linear regression equation, to the *nearest hundredth*. Explain the meaning of this value in the context of these data.

35 The table below shows the number of hours ten students spent studying for a test and their scores.

Hours Spent Studying (x)	0	1	2	4	4	4	6	6	7	8
Test Scores (y)	35	40	46	65	67	70	82	88	82	95

Write the linear regression equation for this data set. Round all values to the *nearest hundredth*.

State the correlation coefficient of this line, to the *nearest hundredth*.

Lesson 3 – Piecing it Together

You are becoming more familiar with linear and exponential functions. This lesson, we will introduce two more important families: **quadratic** and **absolute value** functions. Finally, we will learn about **piecewise** functions.

Quadratic Functions

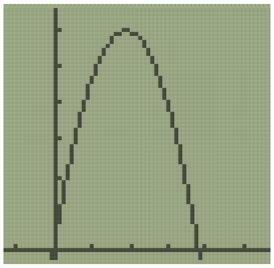
The third major function family, quadratic functions, is also nonlinear. Situations that are modeled with quadratic function rules many times involve gravity or motion. Here are a few examples:

- Jonathan kicks a football into the air. The height in feet at any given time *t* can be modeled by the function rule $\boldsymbol{h(t) = -16t^2 + 60t + 3}$
- A skydiver jumps from a height of 3,500 meters. The total distance fallen at any given time *t* can be modeled by the function rule $\boldsymbol{d(t) = 3500 - 9.8t^2}$
- A concert venue's revenue depends on the ticket price, *x*. The concert revenue can be modeled by the quadratic function rule $\boldsymbol{R(x) = x(200 - 20x)}$
- The path of a comet as it slingshots around planet Earth can be modeled by the quadratic function $\boldsymbol{p(x) = (x - 3)^2 + 4}$

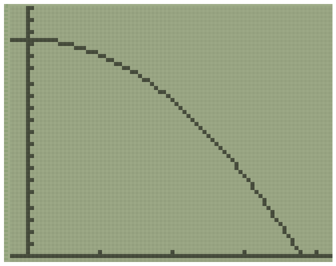
What similarities and differences do you notice about these function rules?

Based on the situations what shape do you predict quadratic graphs to have? Make a sketch.

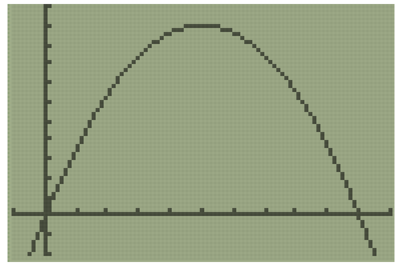
Quadratic function rules appear in three major forms. We are going to learn how to convert between these forms later, but it makes it more difficult to recognize what exactly makes function rules quadratic. Did you notice how three rules involved an exponent of 2? Quadratic function rules do have x^2 in them. It is less obvious that $\boldsymbol{R(x) = x(200 - 20x)}$ is quadratic, but it is. Let's take a look at each of their graphs.



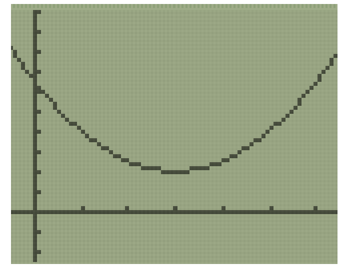
$h(t)$



$d(t)$



$R(x)$



$p(x)$

Which graphs are similar? How are they similar? Which graph is different? How is it different?

Karla answered the previous question by saying that $d(t)$ is the graph that is different. She said,

"I think $d(t)$ is different because it decreases the whole time. Every other graph has one part that increases and another part that decreases."

Robert and Karla exchange papers and read each other's responses. Robert disagrees with Karla and tells her,

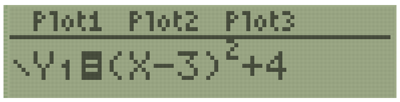
"If you graph it on the left, $d(t)$ is similar to h and R. I think p is the different one because it goes up on both sides."

Explain Robert's thinking and expand upon it in your own words. What evidence could Robert have shown Karla to help support his response?

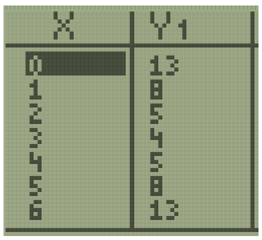
Looking at a graph will always reveal whether a rule is quadratic. These U-shaped graphs are called **parabolas**. Notice that it is not obvious that $\boldsymbol{d(t)}$ is a parabola. We will discuss how to adjust the calculator **WINDOW** in greater detail in the future.

Notice that linear and exponential functions either always increase or always decrease from left to right. Parabolas change directions. This turning point is called the **vertex**. If the quadratic graph opens up (shaped like a cup), the vertex is the minimum of the graph. If the parabola opens down (shaped like a frown), the vertex is a maximum point.

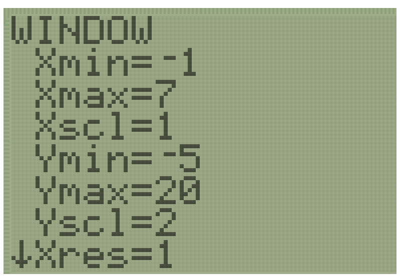
Suppose you want to find the minimum point of $\boldsymbol{p(x)}$.



Press **Y=** and enter the function rule for **Y1**

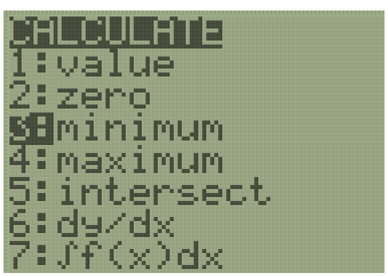


To get an idea of the correct window settings, press **2nd, GRAPH**. Decide on the lowest and highest values for *x*- and *y*-values.

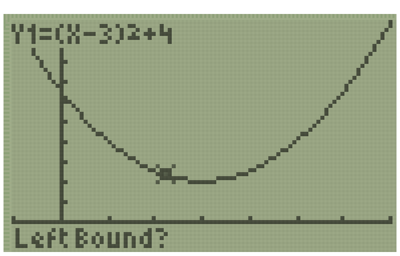


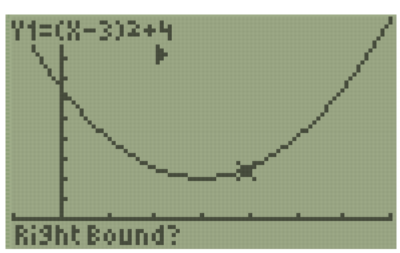
To follow along with this example, press **WINDOW** and enter the following settings.

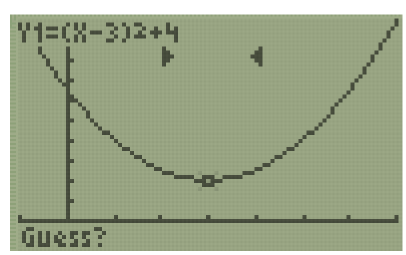
Are these choices close to the ones you made based on looking at the table?



Press **2nd**, **TRACE**, and choose choice **3** for the minimum finder.



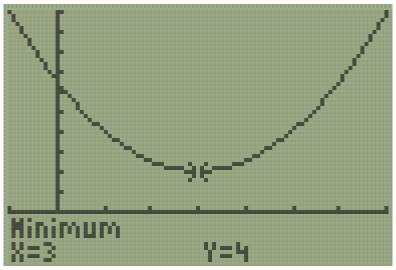




Move the cursor using the arrow keys until it is on the left side of the minimum, then press **ENTER**.

Move the cursor to the right side of the minimum and press **ENTER**

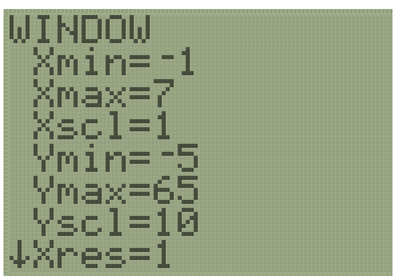
Finally, move the cursor to somewhere close to the minimum point an press **ENTER.**



The coordinates of the minimum point of $\boldsymbol{p(x)}$ are $\boldsymbol{(3, 4)}$.

This process works nearly the same for finding a maximum.

Try to find the coordinates of the maximum point on $\boldsymbol{h(t) = -16t^2 + 60t + 3}$. Note that when you put the rule into **Y=** you always use *x* as your variable.



These settings produce a nice viewing window for ***h***. Again, we will discuss strategies for how to do this in the future.

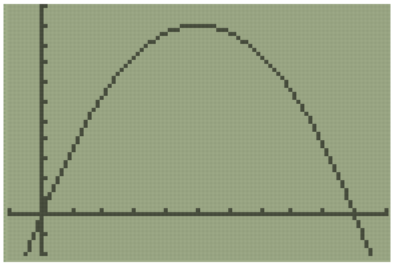
Use these Window settings and write the coordinates of the maximum point of $\boldsymbol{h(t) = -16t^2 + 60t + 3}$.

The Regents Exam also loves to ask about the **zeros** of quadratic functions. **Zeros** are x-values for which y = 0. Sometimes we can find zeros on a table.

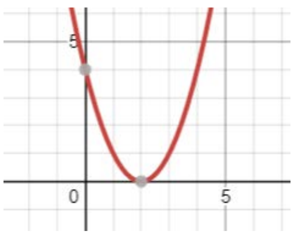
Locate the zeros of $\boldsymbol{R(x) = x(200 - 20x)}$ using your table.

From this example, we see that quadratics can have two zeros.

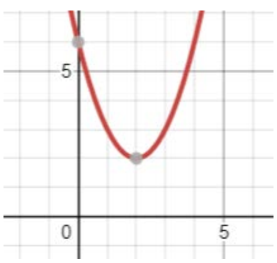
The zeros of a function are the **x-intercepts** of its graph. Notice how the graph of $\boldsymbol{R(x)}$ intersects the x-axis at two locations, **x = 0** and **x = 10**



Sketch the graph of a quadratic function that has one zero.

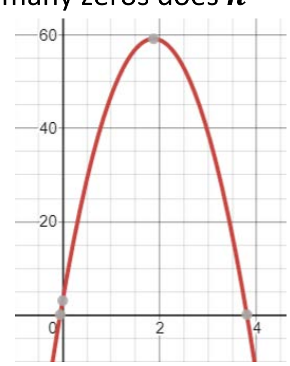


Sketch the graph of a quadratic function that has no zeros.



Is it possible for a quadratic function to have more than two zeros? Explain your reasoning.

Consider the function $\boldsymbol{h(t) = -16t^2 + 60t + 3}$. View a graph of this function. How many zeros does $\boldsymbol{h}$ have?



Fill out the table of values for $\boldsymbol{h(t)}$ for integers **x = -2** through **x = 5**

x	-2	-1	0	1	2	3	4	5
$h(x)$								

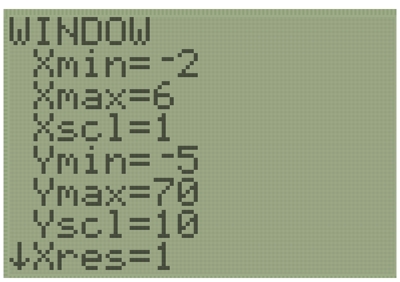
It's not immediately obvious what exact *x*-values the zeros are. Between what values of ***x*** must the zeros be "hiding"? Explain how you can tell.

This example shows that we can't rely on tables to find the zeros of every quadratic function.

To find the zeros on a graph, press **2^{nd}**, **TRACE**, and choose option **2: zero**.

Then, follow the same process to locate a max or min.

Use the following **WINDOW** for a nice picture of the graph.



Try this process now to locate the zeros of $\boldsymbol{h(t) = -16t^2 + 60t + 3}$. Note that they will be decimal values.

Absolute Value Functions

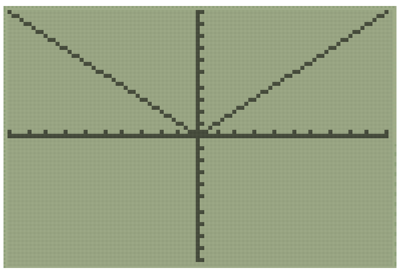
There is another family, in addition to linear, exponential, and quadratic, that is worth knowing – the absolute value function.

The **absolute value** of a number a, written as $|a|$, is that number's distance from 0. Since distance is always positive, the absolute value of a number is always positive.

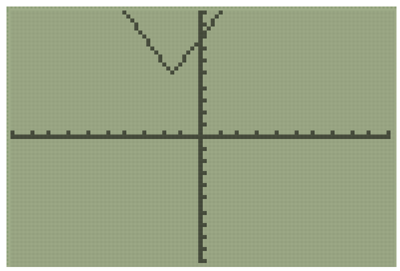
For example, the absolute value of negative four is four because negative four is four units away from zero on a number line. This is written as $|-4| = 4$

Absolute value functions have similarities in the shape of their graphs.

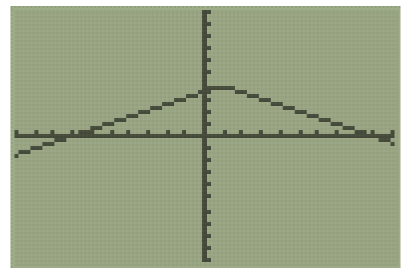
$y = |x|$



$y = |2x + 3| + 5$



$y = -0.5|x - 1| + 4$



Get the absolute value bars in the calculator by pressing **ALPHA**, **WINDOW**, **ENTER**.

What are some similarities between each absolute value function rule?

What similarities do you notice in each graph?

What similarities and differences do you notice between absolute value functions and quadratic function graphs? Rules?

Domain and Graphing

In your own words, explain the idea of a function's domain and range.

For many situations, there are only certain input values that make sense. For example, consider the example from the beginning of the lesson involving revenue:

- A concert venue's revenue depends on the ticket price, $\boldsymbol{x}$. The concert revenue can be modeled by the quadratic function rule $\boldsymbol{R(x) = x(200 - 20x)}$

What does ***x*** represent in this situation?

What is one value of ***x*** that does not make sense for this situation?

Select the inequality that best represents the domain for this situation. Explain your reasoning for your choice.

$\{x|x = 5\}$ $\{x|x > 0\}$ $\{x|x \geq 0\}$ $\{x|x < 0\}$

Tyrece is exploring $\boldsymbol{R(x) = x(200 - 20x)}$ using a table of values on his calculator. He notices a pattern and writes it down.

As the ticket price increases, the money or revenue increases for a little while, then it decreases and becomes negative.

Do you agree with the pattern Tyrece noticed?

Explain why it makes sense in the context of the problem that as ticket price increases, the revenue will eventually decrease.

Explain why it makes sense in the context of the problem that revenue will eventually be negative.

Tyrece wants to consider the domain of ticket prices for which revenue is positive. Using words, which ticket prices give positive revenues?

Which inequality below represents the domain you described? Explain your reasoning.

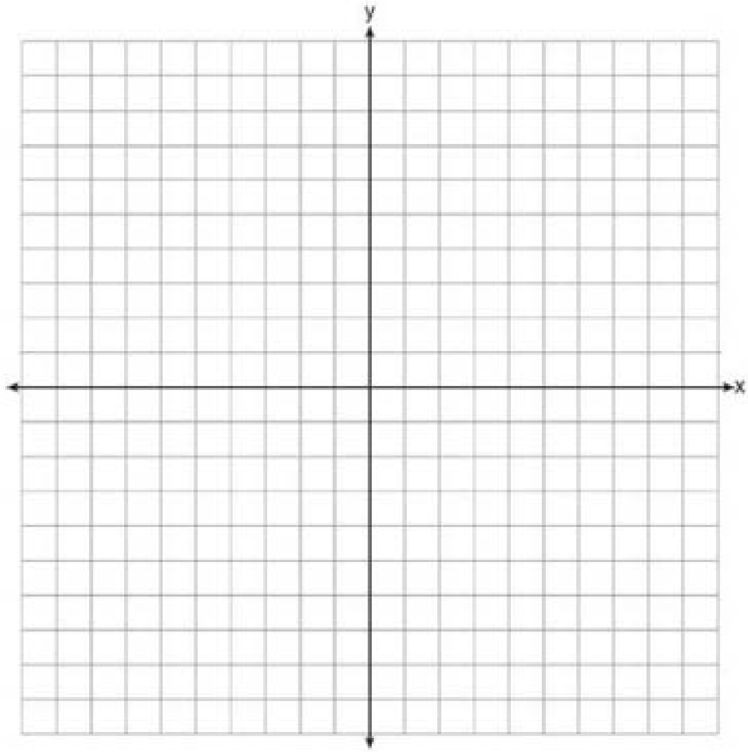
$\{0 < x < 10\}$ $\{x < 10\}$ $\{0 > x > 10\}$

Many students struggle to interpret these types of inequalities. To help you, fill in the chart below:

Inequality	**Literally**	**In my own words**
$0 < \boldsymbol{x} < \mathbf{10}$	Zero is less than x which is less than ten	All the *x*-values between zero and ten, not including zero or ten
$\mathbf{2} \leq \boldsymbol{x} < \mathbf{5}$		
$-\mathbf{1} < \boldsymbol{x} \leq \mathbf{2}$		
$\boldsymbol{x} > \mathbf{10}$		
		All *x*-values between 4 and 7, not including 4 but including 7
		All x-values smaller than 8

Complete the following Regents question.

On the set of axes below, graph the function $y = |x + 1|$.



State the range of the function. State the domain over which the function is increasing.

Piecewise Functions

Brendalee receives a model rocket kit for her birthday. The rocket has a GPS signal that allows her to track its distance from the ground using a phone app.

- For the first five seconds, the engine causes the rocket's distance from the ground in meters to increase according to the exponential function $= \mathbf{5(2)^{t} - 4.5}$.
- When the fuel runs out, the rocket free falls toward the ground for two seconds. Its distance from the ground decreases according to the quadratic function $\boldsymbol{y = -9.8(t-5)^2 + 155.5}$
- Finally, the parachute opens, and the rocket's distance from the ground decreases at a constant rate of $\mathbf{-10}$ meters per second according to the linear function $\boldsymbol{y = 186.3 - 10t}$ until the rocket hits the ground.

Let $\boldsymbol{D(t)}$ represent the distance from the ground of the rocket for the entire time between launch to when it hits the ground. Sketch what you think the graph of $\boldsymbol{D(t)}$ looks like.

During which times will Brendalee's rocket follow the exponential function rule? The quadratic function rule? The linear function rule? Write inequalities to represent each of these intervals.

Brendalee uses a **piecewise function** to define $D(t)$.

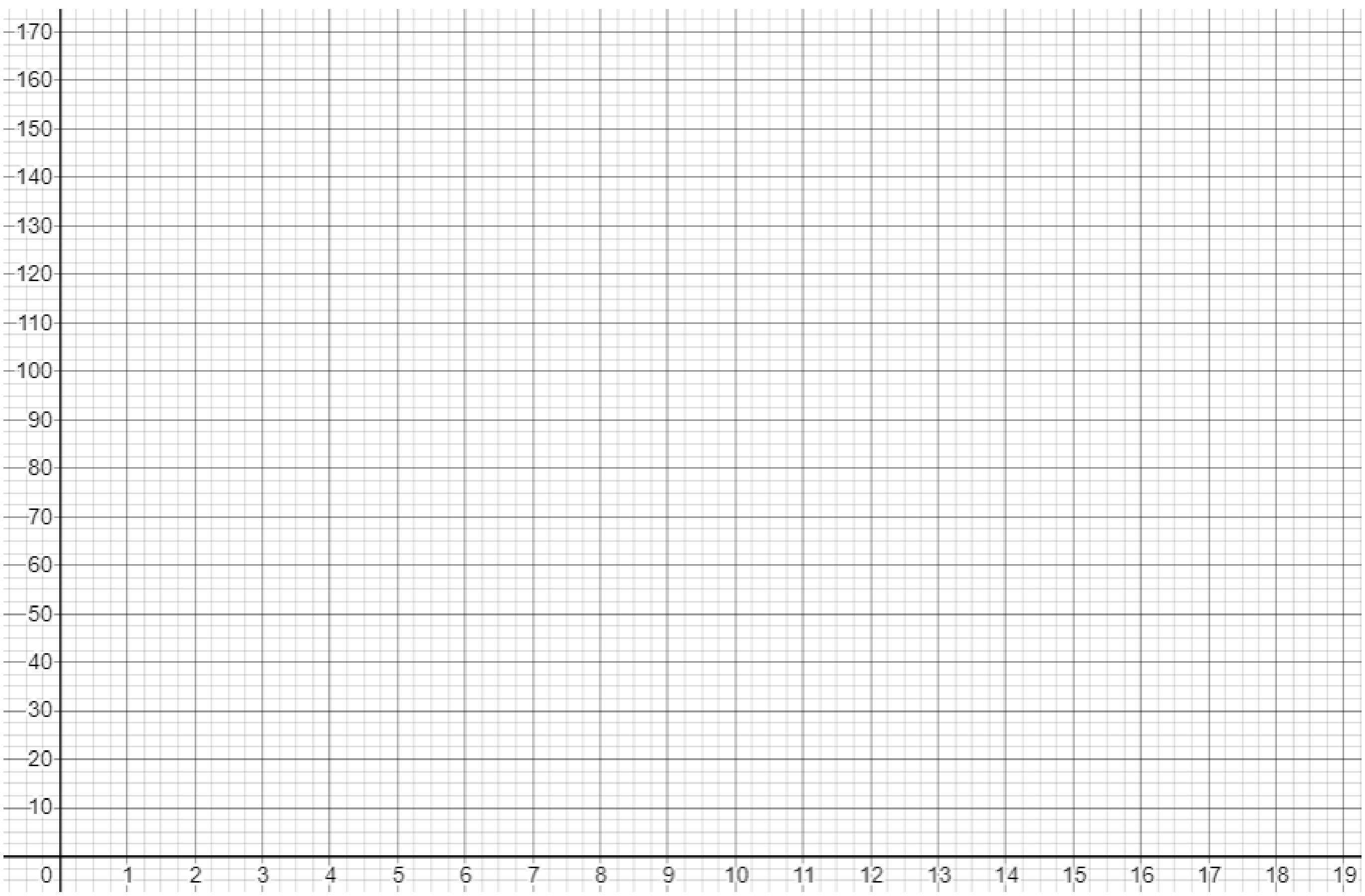
$$D(t) = \begin{cases} 5(2)^t - 4.5 & 0 \leq \mathrm{t} \leq 5 \\ -9.8(t-5)^2 + 155.5 & 5 < t \leq 7 \\ 186.3 - 10t & 7 < t < 18.6 \end{cases}$$

Explain what this notation means in your own words.

According to the piecewise function Brendalee wrote, which of the three function rules should be applied when $\boldsymbol{t = 2}$? What about when $\boldsymbol{t = 6}$? How do you know?

What is the domain of $\boldsymbol{D(t)}$**?**

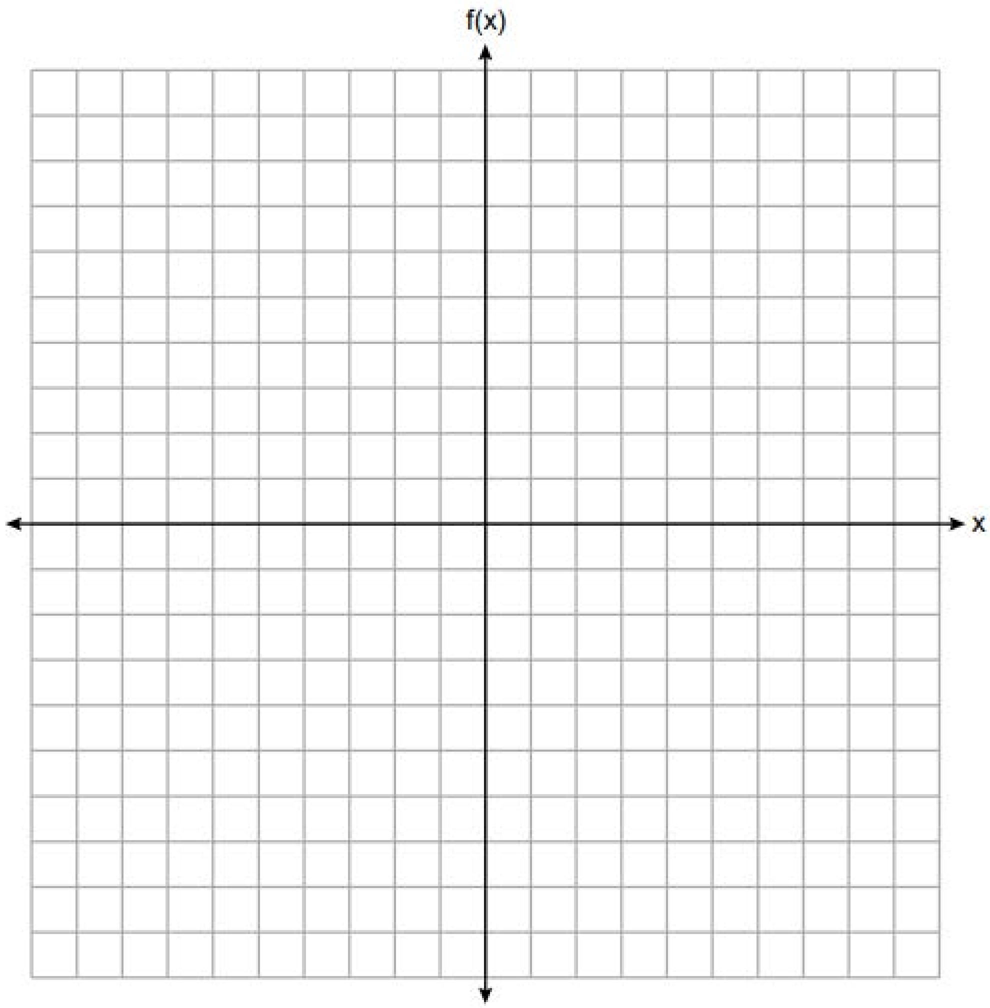
Make a graph of $\boldsymbol{D(t)}$ below.



Did you notice that each rule matched up to form one continuous graph? Not all piecewise functions do this.

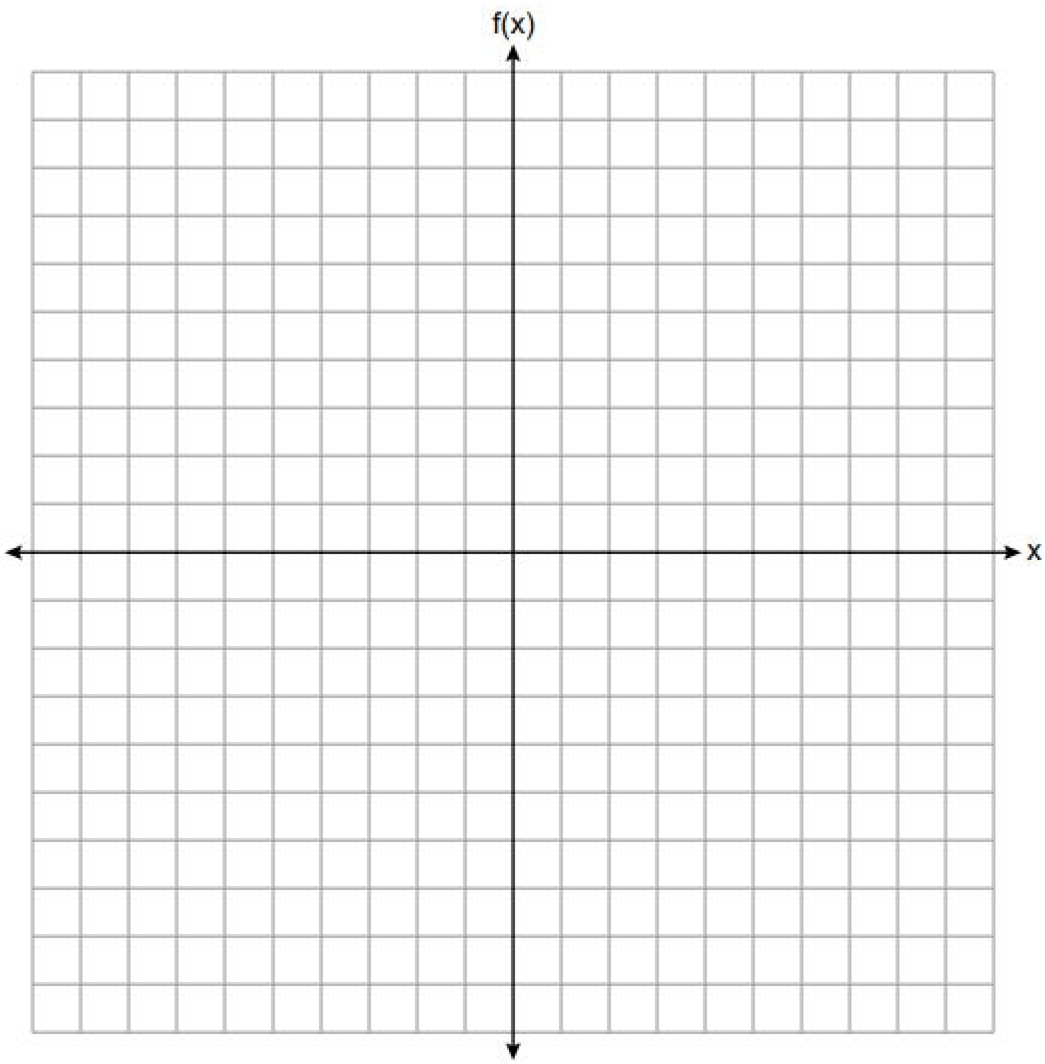
27 Graph the following piecewise function on the set of axes below.

$$f(x) = \begin{cases} |x|, & -5 \le x < 2 \\ -2x + 10, & 2 \le x \le 6 \end{cases}$$



32 On the set of axes below, graph the piecewise function:

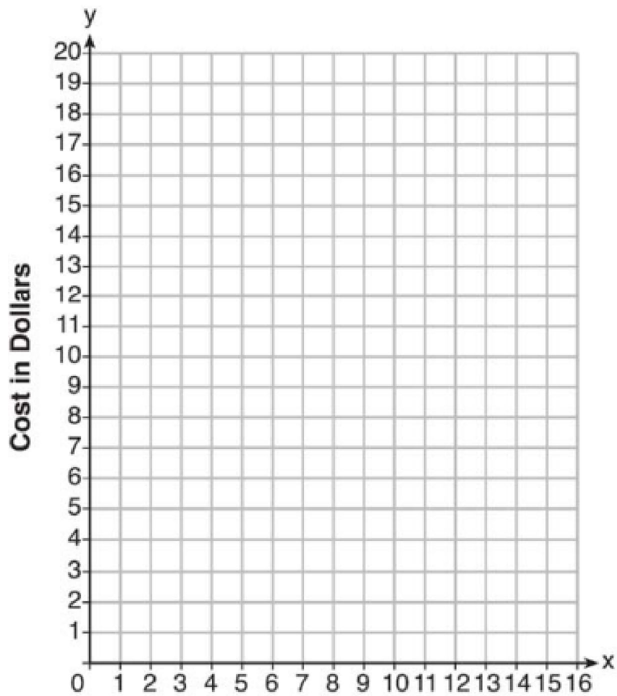
$$f(x) = \begin{cases} -\frac{1}{2}x, & x < 2 \\ x, & x \geq 2 \end{cases}$$



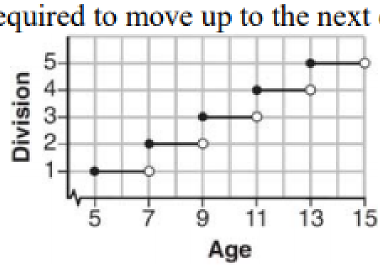
At an office supply store, if a customer purchases fewer than 10 pencils, the cost of each pencil is \$1.75. If a customer purchases 10 or more pencils, the cost of each pencil is \$1.25. Let c be a function for which $c(x)$ is the cost of purchasing x pencils, where x is a whole number.

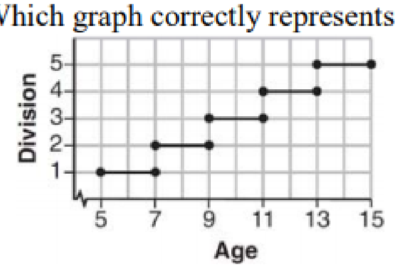
$$c(x) = \begin{cases} 1.75x, \text{ if } 0 \leq x \leq 9 \\ 1.25x, \text{ if } x \geq 10 \end{cases}$$

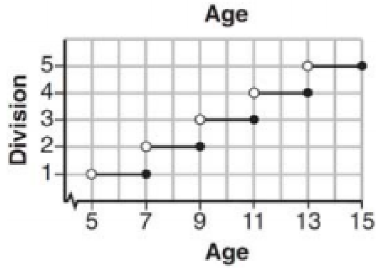
Create a graph of c on the axes below.

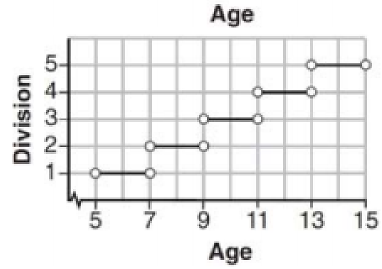


Morgan can start wrestling at age 5 in Division 1. He remains in that division until his next odd birthday when he is required to move up to the next division level. Which graph correctly represents this information?

a. 

c. 

b. 

d. 

Lesson 4 – Solving Linear Equations

For the past three lessons, we learned about functions. A function is a rule that tells you what to do with an *x* in order to get a *y*-value. But how are functions different from **equations**?

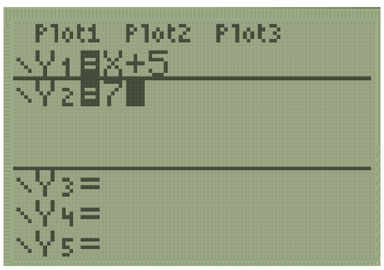
When told to solve an equation for a variable, such as solving $3x + 2 = x - 1$, we are looking for a single value (sometimes more) that will makes both sides of the equation have the same *y*-value.

Most students solve *algebraically*. They do operations on both sides of the equation until the variable is by itself and a number is on the other side of the equal sign. However, all equations can be solved using graphs and tables as well. This lesson, we will practice solving linear equations.

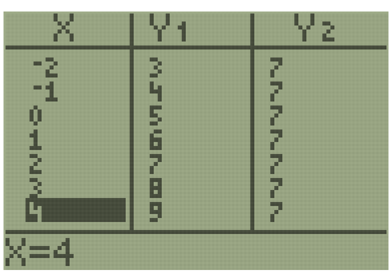
Solving (mostly) linear equations

Consider how to solve the equation $x + 5 = 7$.

Method 1 – Using a table



Press **Y=**, enter one side of the **=** sign as **Y1** and the other side as **Y2**

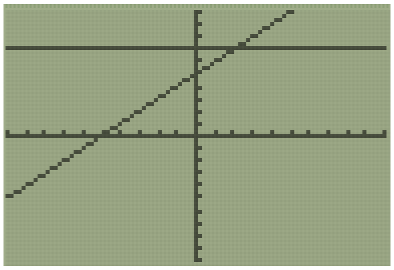


Find the x-value where **Y1** equals **Y2**

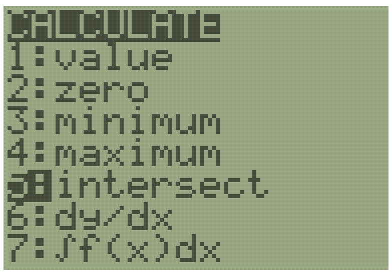
Notice that Y1 = Y2 when x = 2. **The x-value is the solution to the equation**.

Method 2 – Using a graph

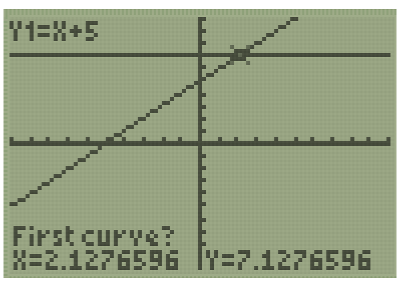
Do the same first step as before: Press **Y=**, enter one side of the **=** sign as **Y1** and the other side as **Y2**.



Press **GRAPH**. **The solution is the point where the graphs intersect**.

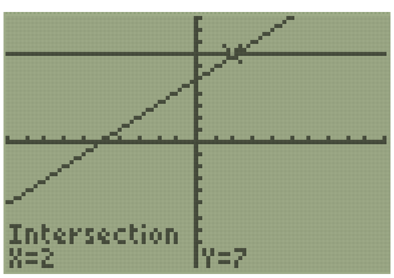


Press **2nd**, **TRACE**, use the **down arrow** to choose choice **"5: intersect,"** and press **ENTER**



Use the left or right arrow to move the blinker close to the desired point.

Then press **ENTER**, **ENTER**, **ENTER**.



The graph intersects at the point **(2,7).** The solution to the equation is the **x-value 2.**

We need both the table method and the graph method to solve equations. The table method is fast and simple, but if the solution to the equation is a decimal or a fraction, then the solution will not show up in the standard table. The graphing method will always work, but it is a little more complicated.

1. Solve the equation $\mathbf{3x + 2 = x - 1}$ using a table or graph. Show evidence of your method to support your answer.

You also have the tools to solve equations involving the other nonlinear function families using graphs or tables. **Solve** the following equations using graphs or tables in your calculator. Some equations have more than one solution. ***One equation has no solution!***

2. $2|x - 1| - 8 = 2$

4. $2(1.5)^x + 1 = -3$

3. $-x^2 + 4x = 4 - x$

5. $3x^2 + 2x = 0$

6. Summarize how to determine a solution to an equation using a graph or table. Explain how you can tell when an equation has no solution.

Method 3 – Solving linear equations algebraically

Marcos solved the equation $\mathbf{7x - 3(x + 1) = 2(x + 4)}$ algebraically below.

7. Explain each step of his process.

$7x - 3(x + 1) = 2(x + 4)$

$7x - 3x - 3 = 2x + 8$

$4x - 3 = 2x + 8$

$2x - 3 = 11$

$2x = 14$

$x = 7$

8. Show how Marcos could check that his answer is correct using a graphing calculator.

9. Solve each equation algebraically. Check that your answer is correct using a graphing calculator.
   a. $4x + 10 = 2x + 7$
   c. $12 - 2(x + 5) = 4$
   b. $2(5x - 3) = 3(4x + 1)$
   d. $17 - 3(3x - 5) = 2 + x$

10. Write a general summary explaining how to solve linear equations algebraically.

You are also expected to solve linear equations algebraically that contain fractions.

11. Explain each step of the process to solve $\mathbf{6-\frac{2}{3}(x+5)=4x}$.

$$6-\frac{2}{3}(x+5)=4x$$

$$6-\frac{2}{3}x-\frac{10}{3}=4x$$

$$\frac{8}{3}-\frac{2}{3}x=4x$$

$$\frac{8}{3}=\frac{14}{3}x$$

$$x=\frac{4}{7}\ or\ .5714285714$$

12. Show how to check that the answer is correct using the graphing calculator.

13. Solve each linear equation for the variable algebraically. Check that your answers are correct using a graphing calculator.
    a. $-\frac{2}{3}(x+12)+\frac{2}{3}x=-\frac{5}{4}x+2$

    b. $\frac{3}{4}x-6=\frac{1}{2}x-9$

    c. $\frac{1}{6}y+8=10-\frac{2}{3}y$

Lesson 5 – Systems of Equations & Inequalities

In the last lesson, you practiced solving a single linear equation. In this lesson, we will focus on **systems of two equations**. A **system of equations** are two or more equations. The *solution* to a system of equations are the numbers for each variable that make all equations true at once.

This lesson, we will go over three methods to solve a system of equations, then practice solving systems of inequalities graphically.

A group of students were given the following situation and asked to solve:

A baseball team is planning a special promotion at its first game. Fans who arrive early will get a team athletic bag or cap, as long as supplies last. The promotion manager from the team can buy athletic bags for \$9 each and caps for \$5 each. The total budget for buying bags and caps is \$25,500. The team plans to give a bag or cap, but not both, to the first 3,500 fans. The promotion manager wants to know: *How many caps and bags should be given away?*

1. When given a problem in context, the first step is to always translate it into math equations.
   a. All students first wrote two equations: one for the total cost of bags and caps, and one for the total number of bags and caps. Fill in the coefficients below.

$$\_\, b + \_\, c = \$25{,}500$$
$$\_\, b + \_\, c = 3{,}500$$

   b. What is the meaning of the variables ***b*** and ***c***?

2. Tyrece chose to solve for *b* and *c* using the **substitution method**.
   a. Follow Tyrece's work and explain his process in your own words.

$b + c = 3500$

$b = 3500 - c$

$9b + 5c = 25500$

$9(3500 - c) + 5c = 25500$

$31500 - 9c + 5c = 25500$

$31500 - 4c = 25500$

$6000 = 4c$

$c = 1500$

$b + c = 3500$

$b + 1500 = 3500$

$b = 2000$

b. Write a sentence to explain the meaning of Tyrece's answer.

c. Another student, Clair, also used the substitution method to solve this problem. However, her first step was to write $\boldsymbol{c = 3500 - b}$. Is she also correct? Use the substitution method to show that Clair will reach the same answer as Tyrece.

4. Eddie and Monica used a different method, the **elimination method**, to solve the same system of equations. Eddie's work is shown below. Follow Eddie's work and discuss his process.

$$9b + 5c = 25500$$
$$b + c = 3500$$

Multiply the second equation by -9 →

$$9b + 5c = 25500$$
$$-9b - 9c = -31500$$

Add these equations together

$$-4c = -6000$$

$$c = 1500$$

$$b + 1500 = 3500$$

$$b = 2000$$

a. Explain why you think Eddie chose to multiply the second equation by **-9**.

b. Monica looks at Eddie's work and says, "I got the same answer, but I eliminated the c's first." Show the work that Monica must have used to get the same answer as Eddie.

c. Eddie and Monica are trying to solve the following systems of equations using **elimination,** but they are stuck on what to multiply by. Their teacher gives them a hint, "Sometimes you have to multiply each equation by a different number."

$$-2a + 6b = 6$$
$$-7a + 8b = -5$$

i. Eddie looks at the ***a*** coefficients and thinks aloud to his partner, "What if we multiplied the first equation by negative 7 and the second equation by positive 2?"
What new system of equations will result from following Eddie's idea? Will this first step be productive in finding the correct answer? Explain.

ii. Monica's idea is slightly different. She wants to eliminate the ***b*'s**. What should she multiply by in order to help eliminate the ***b*'s**?

iii. Use either Eddie or Monica's first step and solve for ***a*** and ***b*** using the elimination method.

5. Rita decides to solve the original system of equation **graphically**.

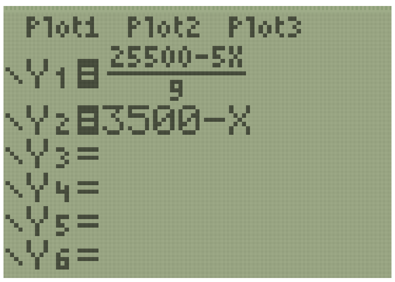
$\mathbf{9b + 5c = 25500} \quad \mathbf{b + c = 3500}$

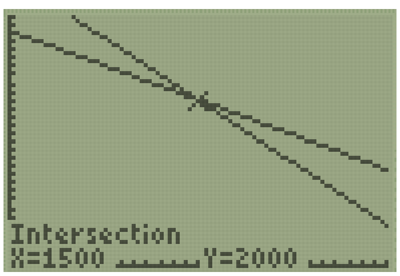
a. Follow Rita's work and explain her process.

$9b + 5c = 25500$ $\qquad$ $b + c = 3500$

$9b = 25500 - 5c$ $\qquad$ $b = 3500 - c$

$b = \frac{25500-5c}{9}$





$\mathbf{c = 1500}$ **and** $\mathbf{b = 2000}$

b. Based on her calculator work, explain how Rita knew that **c** equaled 1500 and not the other way around.

6. After reviewing the **substitution**, **elimination**, and **graphing** method, identify one advantage and one disadvantage of each method. When might it make the most sense to use each method?

7. Solve each problem involving a system of two linear equations algebraically by using either substitution or elimination.

a. $2x - 6y = 28$
$-x - y = 14$

c. $-5x - 14y = -9$
$-10x - 7y = 3$

b. $-2x + 2y = -16$
$-3x - 2y = 11$

d. $y = 4x - 5$
$-x - 8y = 7$

8. To participate in a school trip, Kim had to earn $85 in one week. Kim could earn $8 per hour babysitting and $15 dollars per hour for yard work. Kim's parents limit work time to 8 hours per week. How many hours should Kim work at each job in order to meet her income goal and work exactly eight hours?

9. Solve the following Regents question from the June 2019 exam.

When visiting friends in a state that has no sales tax, two families went to a fast-food restaurant for lunch. The Browns bought 4 cheeseburgers and 3 medium fries for $16.53. The Greens bought 5 cheeseburgers and 4 medium fries for $21.11.

Using c for the cost of a cheeseburger and f for the cost of medium fries, write a system of equations that models this situation.

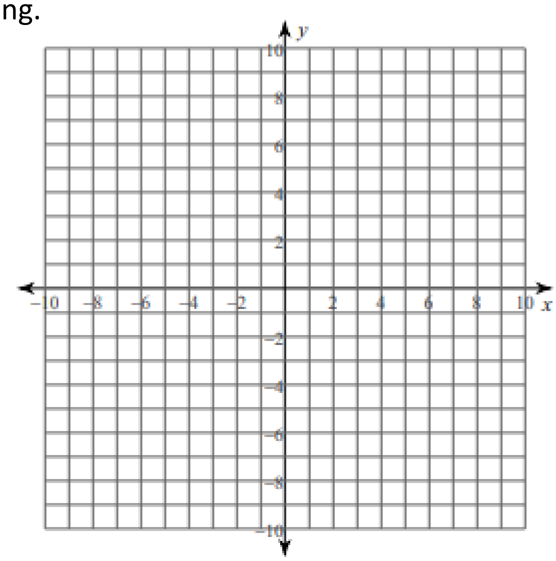
The Greens said that since their bill was $21.11, each cheeseburger must cost $2.49 and each order of medium fries must cost $2.87 each. Are they correct? Justify your answer.

Using your equations, algebraically determine both the cost of one cheeseburger and the cost of one order of medium fries.

10. Solve the following systems using graphing.

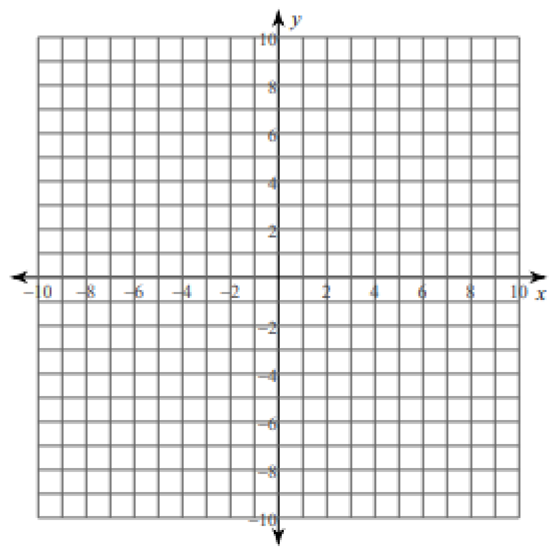
a.

$$y = -\frac{5}{6}x + 2$$
$$y = \frac{1}{6}x + 8$$



b.

$$y = \frac{3}{2}x + 1$$
$$y = -\frac{1}{4}x - 6$$



Jess was reading the following part from problem 8:

To participate in a school trip, Kim had to earn $85 in one week. Kim could earn $8 per hour babysitting and $15 dollars per hour for yard work. Kim's parents limit work time to 8 hours per week.

11. Jess thought aloud to her partner, "Kim wants to earn at least $85, but she could make more than that. Also, her parents limit her work to 8 hours, but she could work less than that if she wanted."

a. Assuming Kim makes $8 babysitting and $15 per hour for yard work, determine three different ways she could work to earn at least $85 and also work 8 hours or less.

b. The system of equations from problem 8 is written below, but the equal signs have been removed. Replace them with the proper inequality symbol <, >, ≤, ≥.

$$b + y \qquad 8$$

$$8b + 15y \qquad 85$$

c. In Algebra 1, we solve **systems of inequalities** graphically. The graph to this system of inequalities is below. On the graph, plot your three answers to part a as points on the graph. What do you notice about the location of your three points?

d. One of the inequality graphs uses a dotted line and the other uses a solid line. Why do you think this is?

e. The dotted-line-inequality graph is shaded below the dotted line and the solid-line-inequality graph is shaded above the solid line. What features about the algebraic inequalities from part b explain why this is?

f. The student who produced the graph in part c, Ernest, explained his process. "First, I solved each inequality for *y* just like I would solve a normal equation. Then I typed those into Y= and looked at the table to plot the points of each line graph. Then I used the inequality symbols to decide which line had to be dotted or solid, and which way to shade."

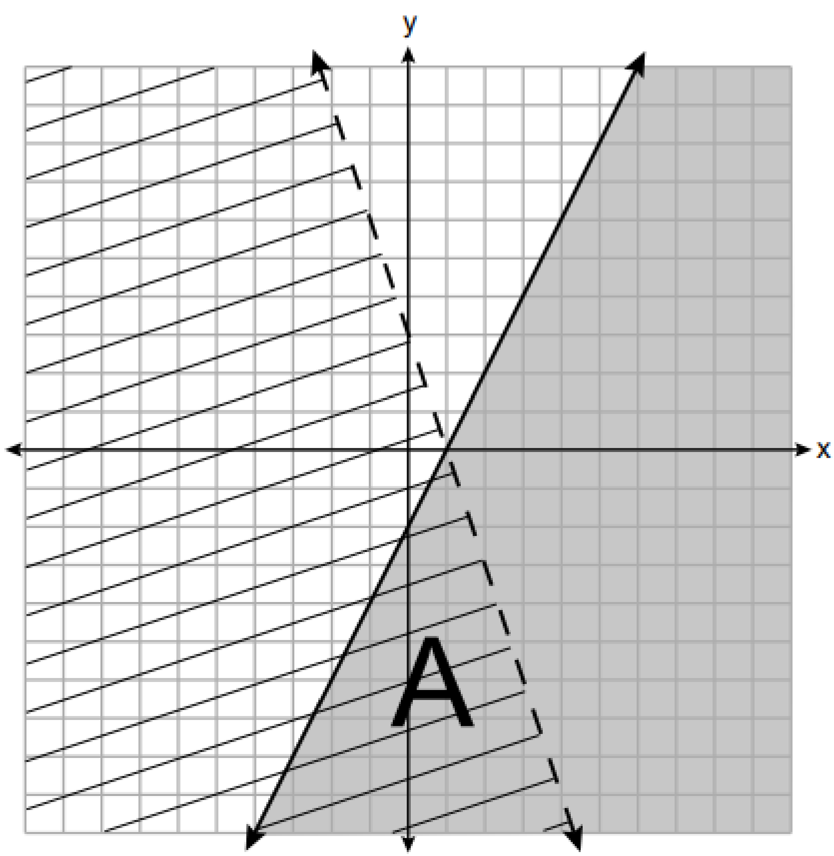
i. Show the algebraic work Ernest might have used to solve each inequality for *y*.

ii. What did Ernest mean when he wrote, "I used the inequality symbols to decide which line had to be dotted or solid, and which way to shade"?

g. The double-shaded region on the graph is called the **solution set**. Explain what the solution set of this graph represents in the context of Kim's school trip situation.

12. Solve the following question from the June 2019 Regents exam.

A system of inequalities is graphed on the set of axes below.



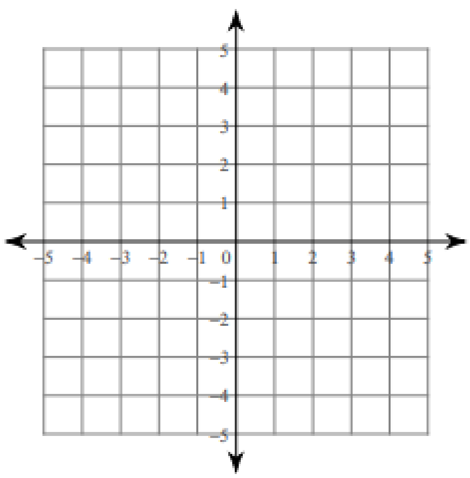
State the system of inequalities represented by the graph.

State what region A represents.

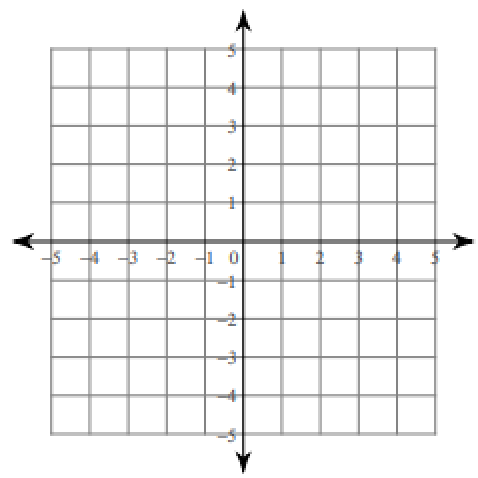
State what the entire gray region represents.

13. Sketch the solution to each system of inequalities. **Label each solution set with the letter S**.

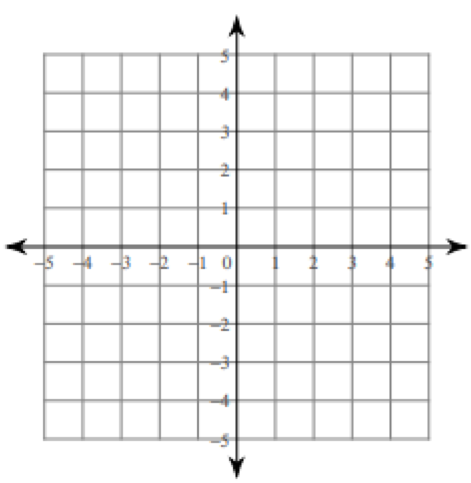
a. $y > 4x - 3$
$y \geq -2x + 3$



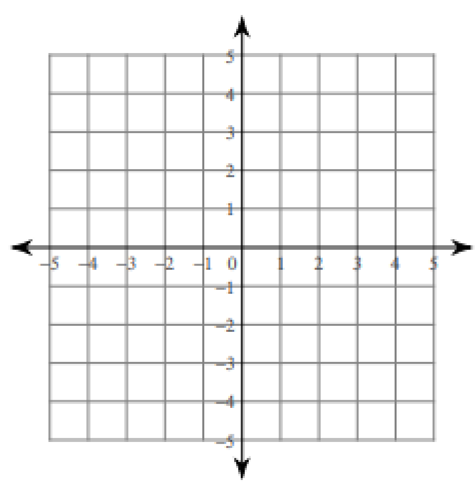
c. $x + y \geq 2$
$4x + y \geq -1$



b. $4x - 3y < 9$
$x + 3y > 6$



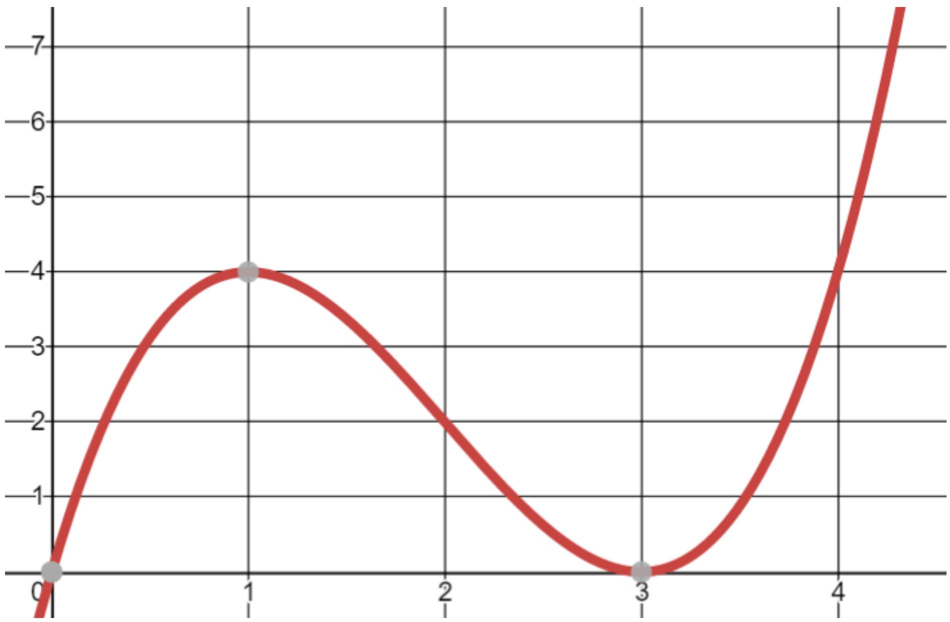
d. $3x + y \geq -3$
$x + 2y \leq 4$



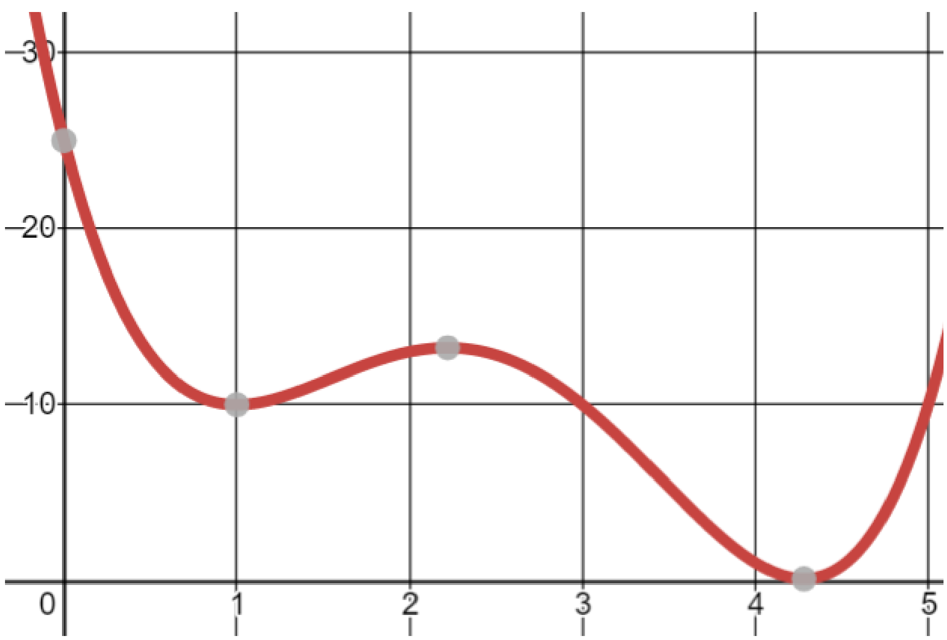
Lesson 6 – An Overview of Polynomials

Francisca is an engineer who designs roller coasters. She is designing two sections of a new roller coaster for Six Flags theme park. Using ***regression***, she generates two **polynomials** to model her track designs.

Section 1 $f(x) = x^3 - 6x^2 + 9x$



Section 2 $g(x) = x^4 - 10x^3 + 32x^2 - 38x + 25$



Polynomials are a very important family of functions that include two types of functions you have studied so far. All linear and quadratic functions are part of the polynomial function family. Polynomial rules can be written in **standard** or sometimes in **factored** form. Both $\boldsymbol{f(x)}$ and $\boldsymbol{g(x)}$ are standard form. Consider all the following examples written in standard form.

$f(x) = x^3 - 6x^2 + 9x$

$g(x) = 4x^4 - 10x^3 + 32x^2 - 38x + 25$

$h(x) = -2x^2 - 7x + 15$

$k(x) = 5x + 1$

$m(x) = x^6 + 4x^3 - 7x$

$n(t) = 16t^8 - 4$

1) What do you believe are the important characteristics of a standard form polynomial?

2) You may have noticed that polynomials all have exponents on the variable. Even $k(\boldsymbol{x})$ has an exponent of 1 on the **x** that is usually not written. A polynomial's **degree** is the value of its highest exponent. Next to each polynomial function above, write its degree next to its function rule. For example, you can write ***"degree 3"*** or "**deg 3**" next to $\boldsymbol{f(x)}$.

The **highest degree term** is the term that has the biggest exponent on the variable. Standard form polynomials are always written from the highest degree term to the lowest degree term. A term with no variable is called a **constant**.

3) A coefficient is the constant that multiplies the variable. The coefficient of $\boldsymbol{-6x^5}$ **is** $\boldsymbol{-6}$. The coefficient of the highest degree term is called the **leading coefficient**. Write leading coefficient for each standard form polynomial above. For example, on page 1, in Section 2, the leading coefficient of $\boldsymbol{g(x)}$ is **4** so you could write "**l.c. is 4**" next to the rule for $\boldsymbol{g(x)}$**.**

4) Rewrite each polynomial below in standard form. Then state its degree and the value of its leading coefficient.

a. $4x^3 + 5x^5 - 4 - 2x^2$

b. $12 - 3x^2 + 7x - 3x^4$

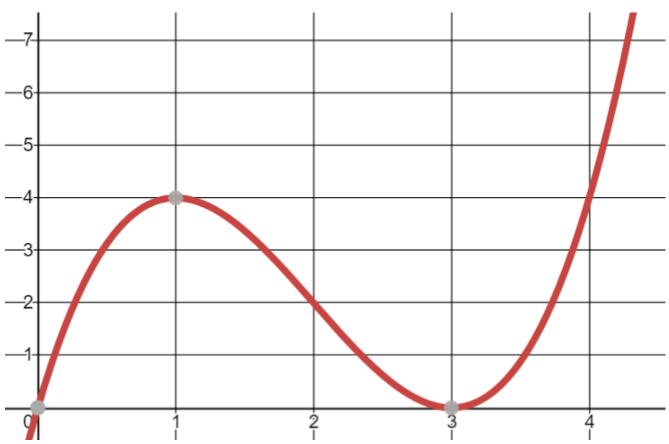
Here are a selection of polynomials written in **factored form.**

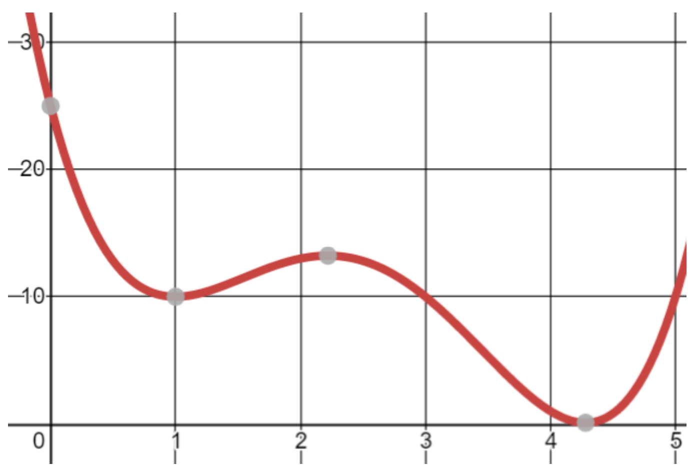
$$a(x) = (x+2)(x-7)$$
$$b(x) = x(x+3)(x+5)$$
$$c(x) = (x-4)(x+2)(x-1)$$

What do you think are the important characteristics of a factored form polynomial?

Polynomials have some important features. Recall the vertex of a quadratic function was its turning point. Polynomial graphs can have many turning points. When a turning point makes a little "hill" it is called a **local maximum**. When a turning point makes a "valley" we call it a **local minimum**. Similarly, polynomials also have a **y-intercept**, and their **x-intercepts** are called **zeros** or **roots**.

5) Label Francisca's roller coaster track designs' local minimum(s), local maximum(s), roots, and zeros below and write their coordinates.





y-intecept:

zeros:

local maximum(s):

local minimum(s):

y-intercept:

zero:

local minimum(s):

local maximum(s):

6) Try to notice some patterns.

a. What do the coordinates of the y-intecepts share in common?

b. What do the coordinates of the zeros share in common?

The important features of polynomials can be located using the graphing calculator and analyzing either a table or graph. The next problem seeks to highlight some of the advantages to representing polynomials in either factored or standard form.

7) In each part below, you will be given the standard form and equivalent factored form of a polynomial function. Then use a calculator table or graph to locate the coordinates of the function's y-intercept, root(s), and local maximum and/or minimum points. State the degree and the value of its leading coefficient. Sketch a graph of each polynomial function. Provide evidence for how you found your coordinates.

a. $\boldsymbol{f(x) = (x+5)(x-7)}$ = $\boldsymbol{x^2 - 2x - 35}$

Degree: **Leading Coefficient:**

Sketch:

y-intercept: **zero(s):**

Local maximum(s): **Local minimum(s):**

b. $g(x) = x(x+2)(x-3)$ = $x^3 - x^2 - 6x$

Degree: **Leading Coefficient:**

Sketch:

y-intercept: **zero(s):**

Local maximum(s): **Local minimum(s):**

c. $k(x) = 2x^4 - 11x^3 + 23x + 10$ = $(x+1)(2x+1)(x-5)(x-2)$

Degree: **Leading Coefficient:**

Sketch:

y-intercept: **zero(s):**

Local maximum(s): **Local minimum(s):**

8) Jared and Karen were working together on problem 7. Jared notices "you can find the zeros if you look at the factored form. Just switch the sign in the middle and it tells you." Karen looked at her work and noticed "that only sometimes work. Look at part c. When I checked with the graph the zeros were -1, 5, 2, but also -0.5."

a. Do you believe that the factored form $k(\boldsymbol{x}) = (\boldsymbol{x}+\mathbf{1})(\mathbf{2}\boldsymbol{x}+\mathbf{1})(\boldsymbol{x}-\mathbf{5})(\boldsymbol{x}-\mathbf{2})$ can be helpful to determine that the zeros are -1, 5, 2, and -0.5? Which factor helps to find each zero?

b. Substitute each number into the factor that you believe it belongs with. What number do you get? Why do you think these numbers are called zeros?

c. Karen had an idea. "We are looking for the *x* that makes each factor zero. What if I make each factor an equation that equals zero and solve for ***x*****.**" Verify that this strategy works for each factor of $\boldsymbol{k}(\boldsymbol{x})$**.**

9) Analyze your results from problem seven. Identify which form is best for finding a polynomial's degree, leading coefficient, x-intercepts, y-intercept, and coordinates of its local maximum and/or minimum point(s). Briefly explain how you can use that form to identify each important characteristic.

10) How can you use the calculator table or graph to find a polynomial's zeros and y-intercept?

11) Write a polynomial function that has zeros at $x = -2$, $x = 4$, and $x = 1$. Use your calculator to verify that the function rule you write has these zeros.

12) Answer the following Regents questions. Use your knowledge of algebra as well as your graphing calculator to reach the correct answer.

a.

Which expression is equivalent to $2(x^2 - 1) + 3x(x - 4)$?

(1) $5x^2 - 5$
(2) $5x^2 - 6$
(3) $5x^2 - 12x - 1$
(4) $5x^2 - 12x - 2$

b.

Josh graphed the function $f(x) = -3(x - 1)^2 + 2$. He then graphed the function $g(x) = -3(x - 1)^2 - 5$ on the same coordinate plane. The vertex of $g(x)$ is

(1) 7 units below the vertex of $f(x)$
(2) 7 units above the vertex of $f(x)$
(3) 7 units to the right of the vertex of $f(x)$
(4) 7 units to the left of the vertex of $f(x)$

c.

The expression $16x^2 - 81$ is equivalent to

(1) $(8x - 9)(8x + 9)$
(2) $(8x - 9)(8x - 9)$
(3) $(4x - 9)(4x + 9)$
(4) $(4x - 9)(4x - 9)$

d.

A ball is thrown into the air from the top of a building. The height, $h(t)$, of the ball above the ground t seconds after it is thrown can be modeled by $h(t) = -16t^2 + 64t + 80$. How many seconds after being thrown will the ball hit the ground?

(1) 5
(2) 2
(3) 80
(4) 144

e.

Which equation is equivalent to $y = x^2 + 24x - 18$?

(1) $y = (x + 12)^2 - 162$
(2) $y = (x + 12)^2 + 126$
(3) $y = (x - 12)^2 - 162$
(4) $y = (x - 12)^2 + 126$

f.

When $(x)(x - 5)(2x + 3)$ is expressed as a polynomial in standard form, which statement about the resulting polynomial is true?

(1) The constant term is 2.

(2) The leading coefficient is 2.

(3) The degree is 2.

(4) The number of terms is 2.

g.

The quadratic functions $r(x)$ and $q(x)$ are given below.

x	r(x)
−4	−12
−3	−15
−2	−16
−1	−15
0	−12
1	7

$$q(x) = x^2 + 2x - 8$$

The function with the *smallest* minimum value is

(1) $q(x)$, and the value is -9

(2) $q(x)$, and the value is -1

(3) $r(x)$, and the value is -16

(4) $r(x)$, and the value is -2