

Topic 1 – The Algebra I Regents Exam and Strategies to Pass It

If you are reading this, it means that either you or someone you know has a very limited time to prepare to pass the Algebra I exam. You may be feeling defeated, stressed, overwhelmed. The purpose of this section is to hopefully relieve some of these anxious feelings. Take a look at New York State’s best kept secret, the scoring chat:

Raw Score	Scale Score	Performance Level
86	100	5
85	99	5
84	98	5
83	97	5
82	96	5
81	95	5
80	94	5
79	93	5
78	92	5
77	91	5
76	90	5
75	89	5
74	88	5
73	88	5
72	87	5
71	86	5
70	86	5
69	86	5
68	85	5
67	84	4
66	84	4
65	84	4
64	83	4
63	83	4
62	83	4
61	82	4
60	82	4
59	82	4
58	81	4

Raw Score	Scale Score	Performance Level
57	81	4
56	81	4
55	81	4
54	80	4
53	80	4
52	80	4
51	79	3
50	79	3
49	79	3
48	79	3
47	78	3
46	78	3
45	78	3
44	77	3
43	77	3
42	76	3
41	76	3
40	75	3
39	75	3
38	74	3
37	74	3
36	73	3
35	72	3
34	72	3
33	71	3
32	70	3
31	69	3
30	68	3
29	67	3

Raw Score	Scale Score	Performance Level
28	66	3
27	65	3
26	64	2
25	63	2
24	62	2
23	60	2
22	59	2
21	57	2
20	56	2
19	55	2
18	52	1
17	51	1
16	49	1
15	47	1
14	45	1
13	42	1
12	40	1
11	38	1
10	35	1
9	32	1
8	30	1
7	27	1
6	23	1
5	20	1
4	17	1
3	13	1
2	9	1
1	5	1
0	0	1

Notice that there are only 86 total raw points on the test. Earning all 86 points will scale to a 100. Take a moment to locate how many raw points it takes to earn a passing scaled score of 65. Did you find it? It’s 27. **You only need to earn 27 points to pass the Algebra I Regents exam.**

So how do we earn at least 27 points? The test is broken into sections.

Section 1 has 24 multiple choice questions worth 2 points each

Section 2 has 8 questions worth 2 points each

Section 3 has 4 questions worth 4 points each

Section 4 has 1 question worth 6 points

Look at this a little bit more closely:

Section	# of questions	Total earnable points
1 Multiple Choice	24	48
2 Free Response	8	16
3 Free Response	4	16
4 Free Response	1	6

Do you notice anything?

- **Over half (56%) of the total points on the test are multiple choice questions**
- **You can pass the test by answering 14 multiple choice questions correctly**

This is great news for someone with limited time. The reality of the Algebra I exam is this: **anyone can pass this exam with the right strategies.**

My experience working with students struggling to pass this exam is that students simply don't know how to take a multiple-choice test. This is our approach for this particular exam and is useful in some situations in life. However, "cramming for the test" is not a strategy for every exam, or for getting through life.

When you get the exam, **you need to highlight ten multiple choice problems that you know you can get right.** These are the questions you will do first. Then, with the time left, you will answer the rest of the questions.

Why ten questions? Think of it this way: ten multiple-choice questions are worth 20 points. Of the 14 multiple-choice questions remaining, you are likely to guess correctly on 25% of them, or about 3 or 4 questions. Even if you only get three more questions correct, that's six more points, which brings your total to 26 points. Remember, you only need 27 points to pass the exam. So if you get ten questions right, you will not need very many points on long response in order to pass.

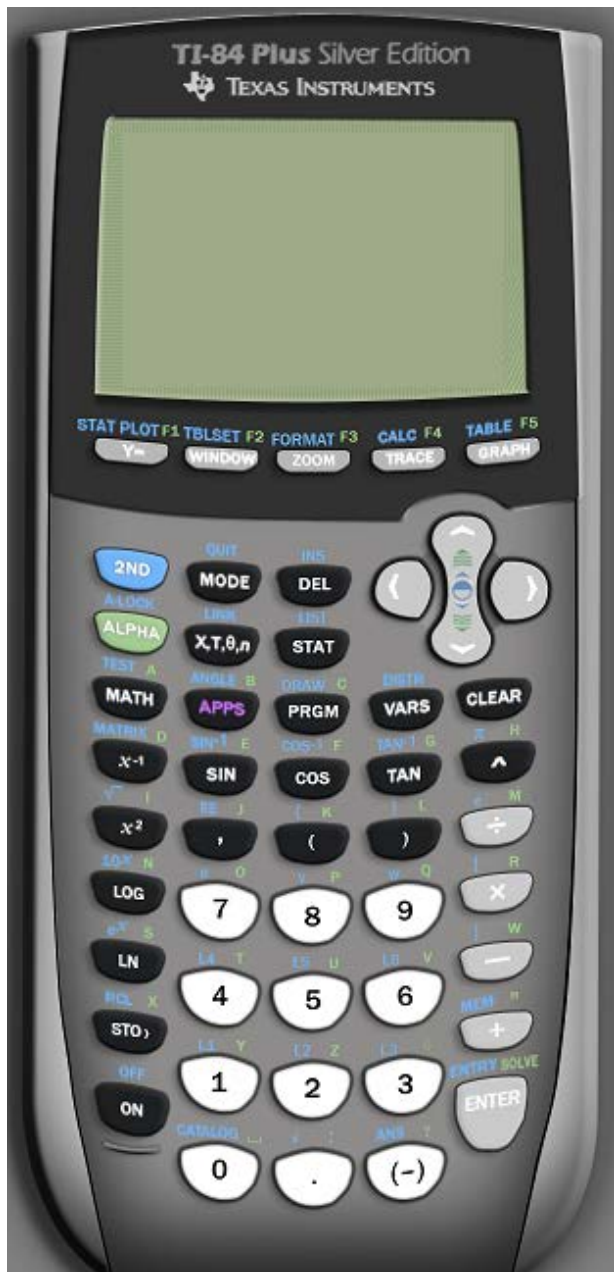
This brings up a few things.

- 1) **The graphing calculator is your best resource!** If you really understand how the calculator works, you won't just *think* you got a question right, you will *know* you got a question right.
- 2) **You need to know which questions you can answer!** We will be working on strategies to identify questions that can be supported or answered by the graphing calculator. This will take practice, but you can do this!!
- 3) **You can't leave anything blank!** Usually, students who fail the Algebra I exam are leaving most of the long response questions blank. Even though we are focusing on multiple choice questions, you need to fight for every last point on this test, even if you don't think you know the answer. Passing or failing really might come down to a single point.

I hope that you feel at least a little better. You really can do this!

So, what are the strategies and how do we know which ten questions to choose?

This lesson, we are going to focus on two main calculator strategies: using tables and graphs to solve equations and to identify equivalent expressions. But first, let's make sure you understand a few things about the calculator.



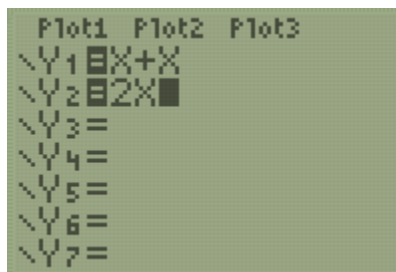
- If you want to do something with tables and graphs, you are going to use one of the buttons in the top row. You almost always have to press **Y=** first.
 - There are hidden things if you press ALPHA and one of the top buttons. For instance, if you want to make a fraction, press ALPHA, Y=, ENTER
- Every button does TWO or THREE things. If you want to do the thing that's in blue, you press the blue 2nd button first, then the button you want. If you want to do something that's in green, you have to press ALPHA first.
- If you ever get lost in the calculator, quit to the home screen by pressing 2nd MODE

Equivalence

What does it mean to be equivalent? **Two expressions are equivalent if they are true for all values of the variable.**

Consider Jose and Eunice once again. Both students are trying to remember what $x + x$ is equivalent to.

- Jose can't remember if $x + x = 2x$ or if $x + x = x^2$. He takes a guess and writes $x + x = x^2$
- Eunice can't remember if $x + x = 2x$ or if $x + x = x^2$. She decides to use her calculator.



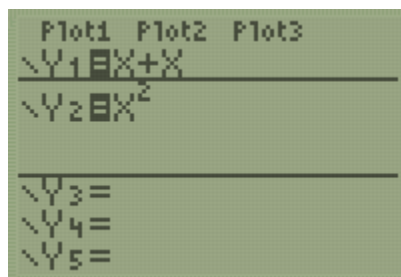
Eunice presses Y=
 She types the left side, $x + x$, as Y1. She types the right side, $2x$ as Y2.

X	Y1	Y2
-5	-10	-10
-4	-8	-8
-3	-6	-6
-2	-4	-4
-1	-2	-2
0	0	0
1	2	2

Press + for Δ tbl

She checks the TABLE by pressing 2nd, GRAPH.
 She notices that $Y1 = Y2$ for every value of x .

Eunice is confident that $x + x = 2x$, but to be sure, she follows the same process to check the other possibility.



She types one side of the = sign as Y1 and the other as Y2.

X	Y1	Y2
-5	-10	25
-4	-8	16
-3	-6	9
-2	-4	4
-1	-2	1
0	0	0
1	2	1

Press + for Δ tbl

This time, Eunice sees that Y1 and Y2 are not the same.
 She concludes that $x + x$ is not equivalent to x^2 .

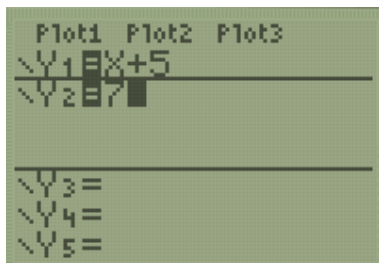
Try to answer the following Regents questions using this strategy. As you read each question, write down all the language used by the exam to indicate that they are testing equivalence.

3 The expression $3(x^2 + 2x - 3) - 4(4x^2 - 7x + 5)$ is equivalent to

- (1) $-13x - 22x + 11$ (3) $19x^2 - 22x + 11$
 (2) $-13x^2 + 34x - 29$ (4) $19x^2 + 34x - 29$

We can solve equations using almost the same approach as equivalence. Consider how to solve the equation $x + 5 = 7$.

Method 1 – Using a table



Press Y=, enter one side of the = sign as Y1 and the other side as Y2

X	Y1	Y2
-2	3	7
-1	4	7
0	5	7
1	6	7
2	7	7
3	8	7
4	9	7

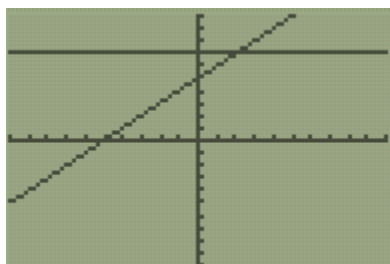
X=4

Find the x where Y1 equals Y2

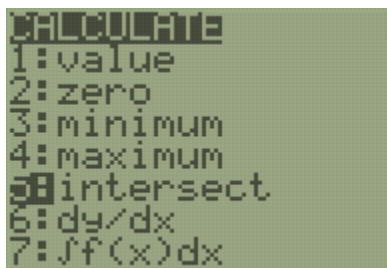
Notice that $Y1 = Y2$ when $x = 2$. **The x-value is the solution to the equation.**

Method 2 – Using a graph

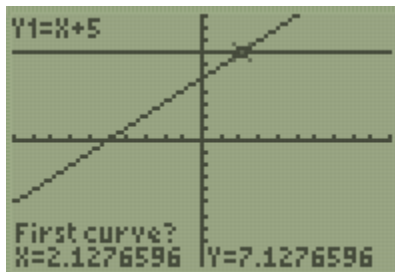
Do the same first step as before: Press Y=, enter one side of the = sign as Y1 and the other side as Y2.



Press GRAPH. The **solution is the point where the graphs intersect.**

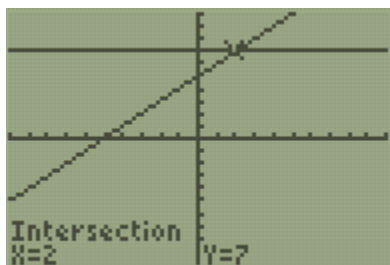


Press 2nd, TRACE, use the down arrow to choose choice 5: intersect, ENTER



Use the left or right arrow to move the blinker close to the desired point.

Then press ENTER, ENTER, ENTER.



The graph intersects at the point (2,7). The solution to the equation is the x-value 2.

We need both the table method and the graph method to solve equations. The table method is fast and simple, but if the solution to the equation is a decimal or a fraction, then the solution will not show up in the standard table. The graphing method will always work, but it is a little more complicated.

Try to answer the following Regents questions by using either tables or graphs.

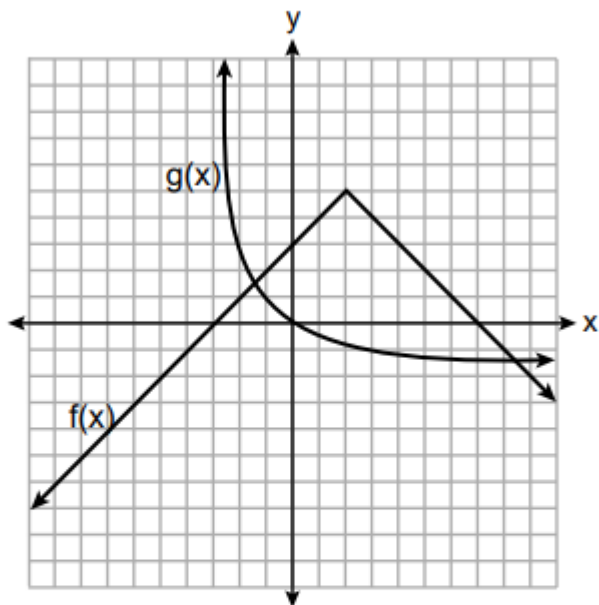
22 How many real-number solutions does $4x^2 + 2x + 5 = 0$ have?

- (1) one
- (2) two
- (3) zero
- (4) infinitely many

4 The solution to $-2(1 - 4x) = 3x + 8$ is

- (1) $\frac{6}{11}$
- (2) 2
- (3) $-\frac{10}{7}$
- (4) -2

19 The functions $f(x)$ and $g(x)$ are graphed below.



Based on the graph, the solutions to the equation $f(x) = g(x)$ are

- (1) the x -intercepts
- (2) the y -intercepts
- (3) the x -values of the points of intersection
- (4) the y -values of the points of intersection

Topic 2 – Analyzing Functions

Using Tables and Graphs

Function Definition and Representations

A **function** is one of the most important concepts in Algebra. Consider the following example. At East High School, students are assigned to a guidance counselor based on the first letter of their last name.

First letter of student's last name	Guidance Counselor
A through K	Ms. Burnell
L through O	Mr. Matos
P through Z	Mrs. Gilbert

Something is a **function** if every x -value (or input) in the domain is assigned to only one y -value (or output) in the range.

In this example, the letters of students' names, A through Z, are the domain. These students are assigned to a guidance counselor in the range. Notice how each student only has one guidance counselor, or in other words, no student has two guidance counselors.

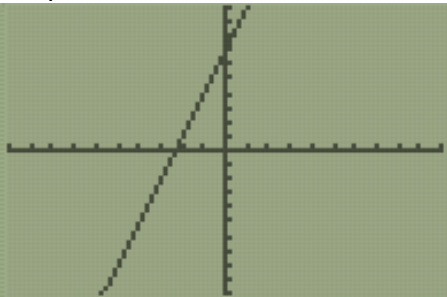
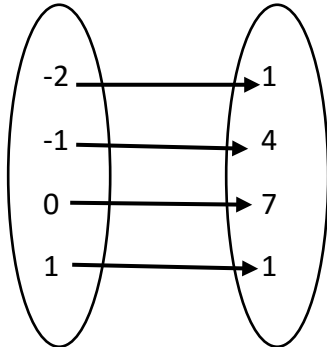
A function is usually some mathematical rule that tells you what to do with x in order to get y .

Consider the linear function

$$y = 3x + 7$$

This rule tells you to multiply each x -value by 3 then add 7 to get the y -value.

Here are other ways to represent $y = 3x + 7$.

Graph	Table	Arrow Diagram (mapping)																
	<table border="1" style="border-collapse: collapse;"> <thead> <tr> <th>X</th> <th>Y₁</th> </tr> </thead> <tbody> <tr><td>-3</td><td>-2</td></tr> <tr><td>-2</td><td>1</td></tr> <tr><td>-1</td><td>4</td></tr> <tr><td>0</td><td>7</td></tr> <tr><td>1</td><td>10</td></tr> <tr><td>2</td><td>13</td></tr> <tr><td>3</td><td>16</td></tr> </tbody> </table> <p>$x=0$</p>	X	Y ₁	-3	-2	-2	1	-1	4	0	7	1	10	2	13	3	16	
X	Y ₁																	
-3	-2																	
-2	1																	
-1	4																	
0	7																	
1	10																	
2	13																	
3	16																	

Every x -value has exactly one y -value.

Solve this next Regents question.

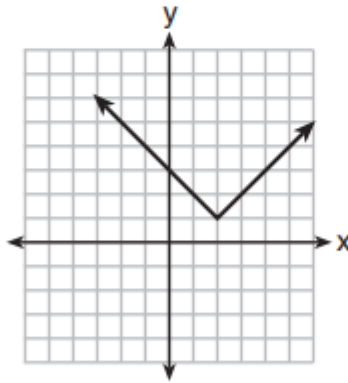
7 Which relation does *not* represent a function?

x	1	2	3	4	5	6
y	3.2	4	5.1	6	7.4	8.8

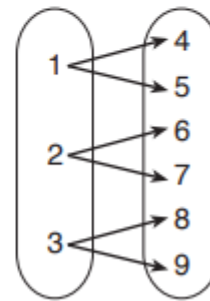
(1)

$$y = 3\sqrt{x+1} - 2$$

(3)



(2)



(4)

The answer is choice (4). Every value of x goes to two values of y . Choice (4) is not a function.

An equivalent way to write this linear function is with function notation: $f(x) = 3x + 7$.

You say “f of x” when you see $f(x)$ and it is another way of writing the variable y .

Function notation is used to input numbers for x .

For instance, consider $f(8)$. This means, “Find the y -value when x equals 8.” For a simple linear function such as $f(x) = 3x + 7$, you may be able to find y in your head, or with the home screen of your calculator.

$$f(8) = 3 \cdot 8 + 7$$

$$f(8) = 24 + 7$$

$$f(8) = 31$$

Determine the value of $f(3)$, $f(25)$, and of $f(-6)$.

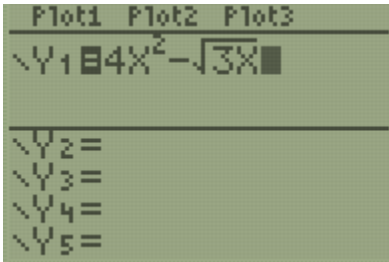
Did you get it right? Did you make a mistake? How long did that take you?

Remember, linear functions like $f(x) = 3x + 7$ are the easiest ones to work with.

Using the TABLE

Consider $g(x) = 4x^2 - \sqrt{3x}$. Notice that this function has been named $g(x)$ “ g of x ”. We can use any letter to name a function, which is helpful when a problem involves more than one function. Let’s explore $g(x)$ with the calculator.

Every function can be represented with a table or a graph.



All function rules can be put into **Y=**.

The 2 on top of the x is an exponent. To get an exponent, press the \wedge key then the value of the exponent. Since 2 is a very common exponent, you can also press the x^2 key. Notice that anything you type will stay in the exponent until you press the right arrow button \rightarrow .

Notice that this function involves the square root of $3x$. This symbol $\sqrt{\quad}$ is called a radical. Find the radical symbol on your calculator by pressing **2nd**, x^2 .

From the Y= menu, we can find specific values of $g(x)$ by using tables or graphs.

Alejandro wants to find $g(4)$ using a table.

X	Y1
1	2.2679
2	13.551
3	33
4	60.536
5	96.127
6	139.76
7	191.42

X=4

Press **2nd**, **GRAPH**

Use the up and down arrows until you get to $x = 4$.

$$g(4) = 60.536$$

Next, Alejandro wants to find $g(528)$ using a table. He definitely doesn’t want to press the down arrow that many times.

TABLE SETUP
TblStart=528
ΔTbl=1
Indent: Auto Ask
Depend: Auto Ask

He presses **2nd**, **WINDOW** and changes TblStart to 528

$\Delta Tbl = 1$ means that the x -values will count by 1.

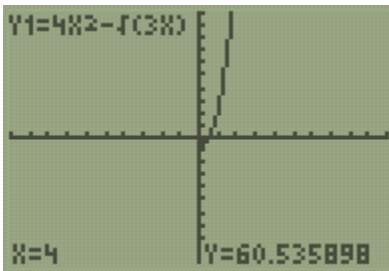
Don’t worry about the rest.

X	Y ₁
528	1.12E6
529	1.12E6
530	1.12E6
531	1.13E6
532	1.13E6
533	1.14E6
534	1.14E6

Y₁=1115096.2005

When Alejandro presses **2nd GRAPH**, his table starts with $x = 528$. The y -value is too large to display properly unless he highlights it using the arrow keys. This sometimes happens when x is very large.

Using the GRAPH



Tatiana is working with the same function, $g(x) = 4x^2 - \sqrt{3x}$

Tatiana wants to find $g(4)$ using a graph.

Press **GRAPH**. If your viewing window is different, press **ZOOM, 6** to get a standard window.

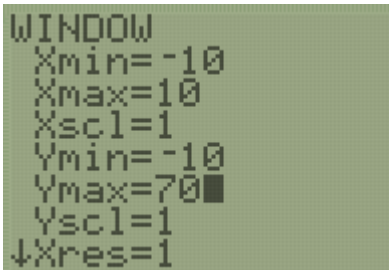
Press **TRACE, 4, ENTER**

Tatiana notices that when $x = 4$, $y = 60.535898$.

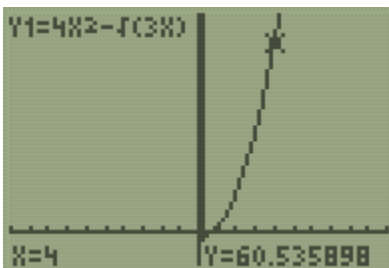
Alejandro's and Tatiana's answers for $g(4)$ are slightly different, but that's okay.

Tatiana notices that her calculator tells her that $g(4) = 60.535898$ at the bottom of her screen, but that she can't see the y -value 60.535898 on the graph of the function. She counts along the y -axis and notices that it only goes up to 10.

She decides to make her graph show more y -values.



She presses **WINDOW** and sets her maximum y -value to 70.



When she presses **GRAPH**, the y -axis goes up to 70.

When she presses **TRACE, 4, ENTER**, she can see the location of $g(4)$ on the graph.

In general, make the Xmax and Ymax higher than the number you want and make the Xmin and Ymin lower than the number you want. You never need to change the Xscl or Yscl, but if you do, they tell the calculator how far to space the marks on the x-axis and y-axis.

Try the following Regents questions. Try to use tables and graphs, and what you know about functions to answer them.

1.

The function $g(x)$ is defined as $g(x) = -2x^2 + 3x$. The value of $g(-3)$ is

- | | |
|---------|--------|
| (1) -27 | (3) 27 |
| (2) -9 | (4) 45 |

2.

If $k(x) = 2x^2 - 3\sqrt{x}$, then $k(9)$ is

- | | |
|---------|---------|
| (1) 315 | (3) 159 |
| (2) 307 | (4) 153 |

3.

Marc bought a new laptop for \$1250. He kept track of the value of the laptop over the next three years, as shown in the table below.

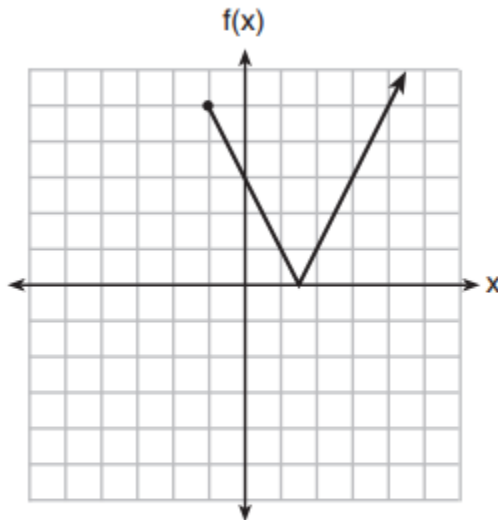
Years After Purchase	Value in Dollars
1	1000
2	800
3	640

Which function can be used to determine the value of the laptop for x years after the purchase?

- (1) $f(x) = 1000(1.2)^x$ (3) $f(x) = 1250(1.2)^x$
 (2) $f(x) = 1000(0.8)^x$ (4) $f(x) = 1250(0.8)^x$

4.

The function $f(x)$ is graphed below.

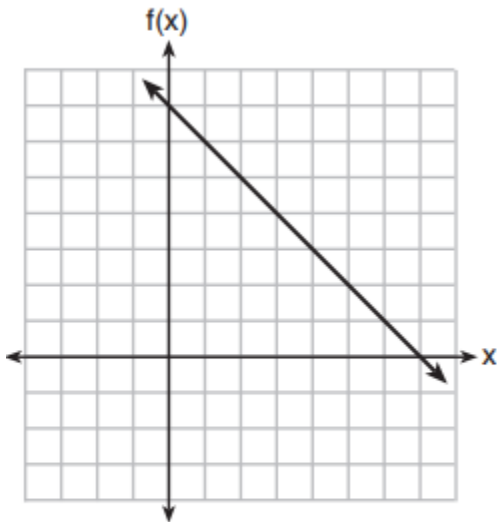


The domain of this function is

- (1) all positive real numbers (3) $x \geq 0$
 (2) all positive integers (4) $x \geq -1$

5.

The functions $f(x)$, $q(x)$, and $p(x)$ are shown below.



$$q(x) = (x - 1)^2 - 6$$

x	$p(x)$
2	5
3	4
4	3
5	4
6	5

When the input is 4, which functions have the same output value?

- (1) $f(x)$ and $q(x)$, only (3) $q(x)$ and $p(x)$, only
 (2) $f(x)$ and $p(x)$, only (4) $f(x)$, $q(x)$, and $p(x)$

6.

Materials A and B decay over time. The function for the amount of material A is $A(t) = 1000(0.5)^{2t}$ and for the amount of material B is $B(t) = 1000(0.25)^t$, where t represents time in days. On which day will the amounts of material be equal?

- (1) initial day, only (3) day 5, only
 (2) day 2, only (4) every day

In this lesson you learned some very important ideas:

- A function assigns every x to exactly one y -value.
- Functions can be represented by algebraic rules, tables, and graphs.
- How to view function tables in the calculator, how to adjust both the start value, and how much x changes by.
- How to view graphs in the calculator, change the window to get a better picture, and trace an input x to determine an output y .

Topic 3 – Function Families

Congratulations! You are nearing the end of this four topic series. This lesson will aim to deepen your understanding of functions by grouping them into three main “families.”

Think about your own family. Everyone in your family has things about them that make them unique. But now think about what makes your family similar. Maybe it’s the way you speak, or dress, or behave that makes your family similar.

Just like you, certain functions belong to a family. In Algebra I, we focus on three main families: linear, exponential, and quadratic.

Linear Functions

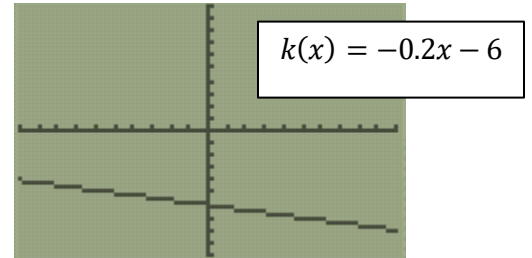
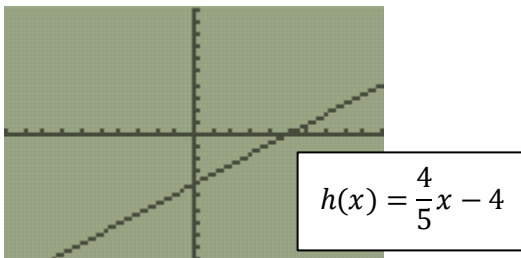
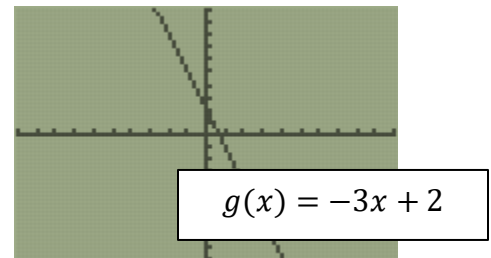
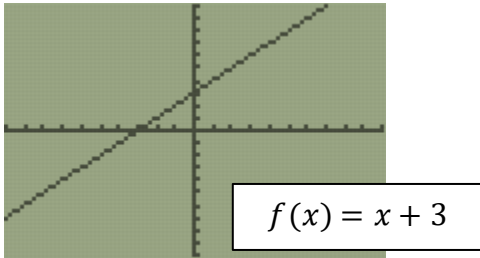
The following four situations are examples of linear functions.

- Pedro earns \$12 per hour at his job.
- A plant’s height increases 3 inches per week.
- Water is emptying out of a bathtub such that its volume is decreasing at a rate of 2 gallons per minute.
- A car’s distance traveled is changing at a constant speed of 55 miles per hour.

Write all the ways these situations are similar and how they are different.

Similar	Different
<hr/>	<hr/>
<hr/>	<hr/>
<hr/>	<hr/>
<hr/>	<hr/>
<hr/>	<hr/>
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The following graphs and rules are linear functions.



What is similar about the graphs of linear functions? What are some differences between the graphs?

Similar

Different

How are the linear function rules similar to one another? How are the linear function rules different?

Similar

Different

Use your calculator the fill in tables for f , g , h , and k from $x = -5$ to $x = 5$.

x	$f(x)$ $= x + 3$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

x	$g(x)$ $= -3x + 2$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

x	$h(x)$ $= \frac{4}{5}x - 4$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

x	$f(x)$ $= -0.2x - 6$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

What similarities and differences do you notice about the tables of linear function rules?

Similar

Different

Every linear function rule can be written in the form

$$y = mx + b$$

The m and b represent numbers. Here are important ideas about them that you may have noticed from the similarities and differences work you just did.

- | | |
|--|---|
| m | b |
| <ul style="list-style-type: none"> • Also called the <u>rate of change</u>. • It is the <u>slope</u> of the graph. • How much the y-value increases or decreases by when the x-value increases by 1 • When m is positive, the graph <u>increases</u> as x increases from left to right. • When m is negative, the graph <u>decreases</u> as x increases from left to right. | <ul style="list-style-type: none"> • The value of y when $x = 0$. • It is where the graph intersects the y-axis. • The starting amount for linear situations. |

Exponential Functions

Exponential functions are a type of nonlinear function, because their patterns do not result in straight lines. The following situations are examples of exponential functions.

- A.** 500 bacteria double every hour.
- B.** A car bought for \$25,000 is worth half of its value each year.
- C.** A savings account has \$1000 initially and gains 5% interest annually.
- D.** A school with 900 students is decreasing its enrollment by 2% each year.

What do each of these situations have in common? How are they different?

Same	Different

Here are rules for each of the situations A through D.

$$A(x) = 500(2)^x$$

$$B(x) = 25000\left(\frac{1}{2}\right)^x$$

$$C(x) = 1000(1.05)^x$$

$$D(x) = 900(0.98)^x$$

How are these exponential rules similar? How are they different?

Similar	Different

What connections do you see between the situation described and its function rule?

A. _____

B. _____

C. _____

D. _____

Here are tables for each of the rules.

$$A(x) = 500(2)^x$$

X	Y ₁
0	500
1	1000
2	2000
3	4000
4	8000
5	16000
6	32000

$$B(x) = 25000\left(\frac{1}{2}\right)^x$$

X	Y ₁
0	25000
1	12500
2	6250
3	3125
4	1562.5
5	781.25
6	390.63

$$C(x) = 1000(1.05)^x$$

X	Y ₁
0	1000
1	1050
2	1102.5
3	1157.6
4	1215.5
5	1276.3
6	1340.1

$$D(x) = 900(0.98)^x$$

X	Y ₁
0	900
1	882
2	864.36
3	847.07
4	830.13
5	813.53
6	797.26

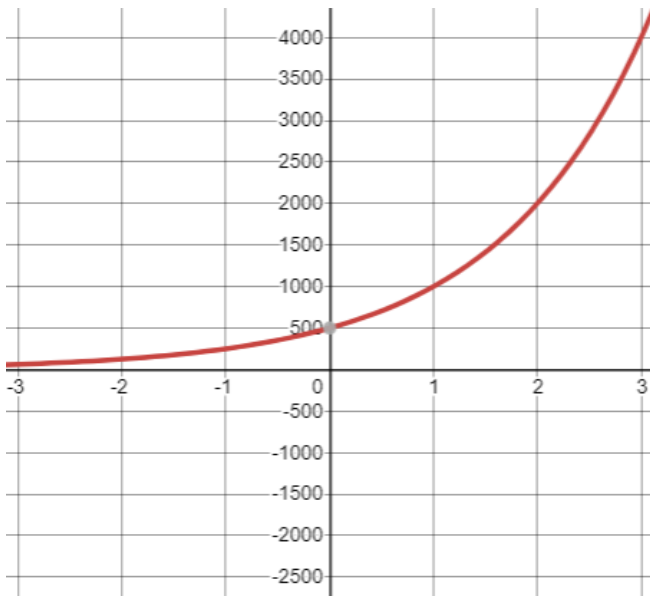
What similarities and differences do you notice between the exponential patterns in the tables?

Similar

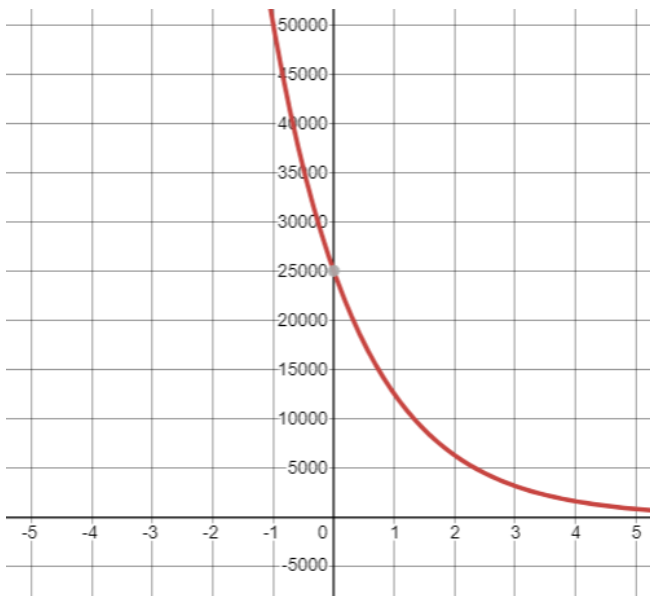
Different

What connections do you notice about the function rule and the table of values?

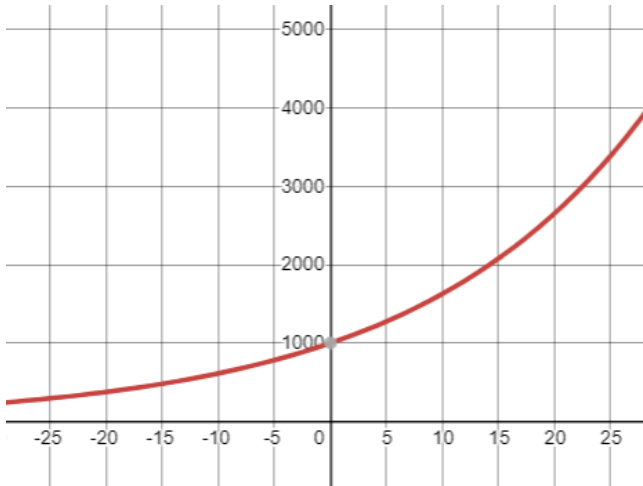
Here are graphs of each exponential function.



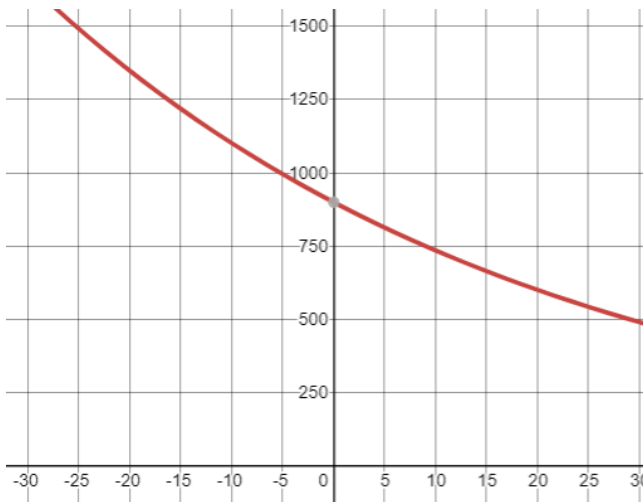
$$A(x) = 500(2)^x$$



$$B(x) = 25\,000\left(\frac{1}{2}\right)^x$$



$$C(x) = 1000(1.05)^x$$



$$D(x) = 900(0.98)^x$$

What are some similarities and differences you notice about how each graph is shaped?

Similar

Different

How do the exponential graphs compare to the linear graphs?

What are some observations you can make about the how the exponential function rules and their graphs are related?

Every exponential function rule can be written in the form

$$y = a(b)^x$$

Where a and b are numbers. Here are some important ideas about how these numbers affect the behavior of tables and graphs of exponential function rules.

- | a | b |
|---|---|
| <ul style="list-style-type: none"> • This is the initial (starting) value of the function. • On a table, this is the y-value when $x = 0$. • On a graph, this is where the graph will intersect the y-axis (vertical axis). | <ul style="list-style-type: none"> • This is the <u>multiplier</u> of the function. If the function is growing or shrinking by a percent, b will be equal to $1 + \frac{\%}{100}$ or $1 - \frac{\%}{100}$ • To get the next y-value in the table, multiply the current y-value by b. • If b is greater than 1, the graph will be increasing from left to right. • If b is between 0 and 1, the graph will be decreasing from left to right. |

Quadratic Functions

The third major function family, quadratic functions, are also nonlinear. Situations that are modeled with quadratic function rules typically involve gravity or motion. For example:

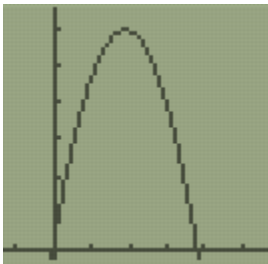
- Jonathan kicks a football into the air. The height in feet at any given time (t) can be modeled by the function rule $h(t) = -16t^2 + 60t + 3$.
- A skydiver jumps from a height of 3,500 meters. The total distance fallen at any given time (t) can be modeled by the function rule $d(t) = 3500 - 9.8t^2$.
- A concert venue's revenue depends on the ticket price, x . The concert revenue can be modeled by the quadratic function rule $R(x) = x(200 - 20x)$.
- The path of a comet as it slingshots around planet Earth can be modeled by the quadratic function $p(x) = (x - 3)^2 + 4$.

What similarities and differences do you notice about these function rules?

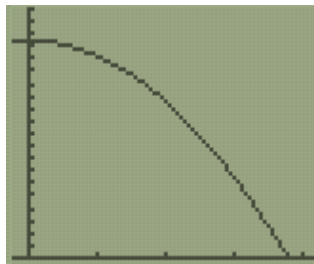
Similar

Different

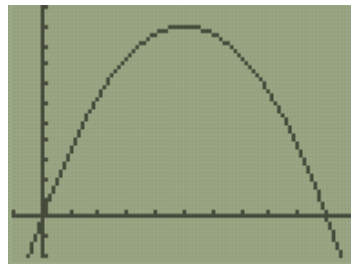
Quadratic function rules appear in three major forms. We are not going to focus on how to convert between these forms, but it makes it more difficult to recognize what exactly makes function rules quadratic. Did you notice how three rules involved an exponent of 2? Quadratic function rules do have x^2 in them. It is less obvious that $R(x) = x(200 - 20x)$ is quadratic, but it is. Let's take a look at each of their graphs.



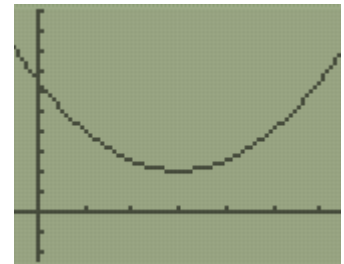
$h(t)$



$d(t)$



$R(x)$



$p(x)$

Which graphs are similar? How are they similar? Which graph is different? How is it different?

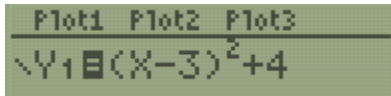
Similar

Different

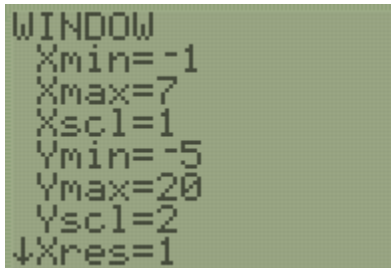
Looking at a graph will always reveal whether a rule is quadratic. These U-shaped graphs are called **parabolas**. Notice that it is not obvious that $d(t)$ is a parabola. Next lesson, we will discuss how to adjust the calculator WINDOW in greater detail.

Notice that linear and exponential functions either always increase or always decrease from left to right. Parabolas change directions. This turning point is called the **vertex**. If the quadratic graph opens up (shaped like a cup), the vertex is the minimum of the graph. If the parabola opens down (shaped like a frown), the vertex is a maximum point.

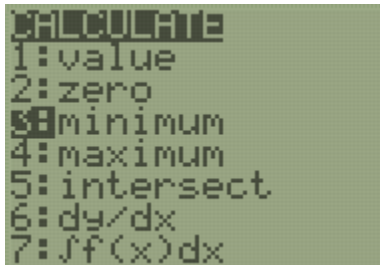
Suppose you want to find the minimum point of $p(x)$.



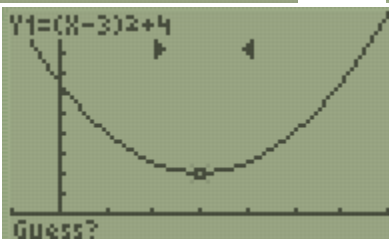
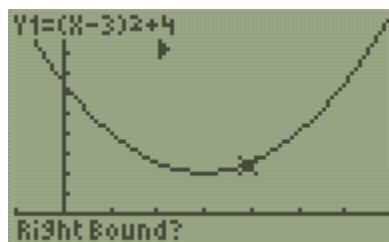
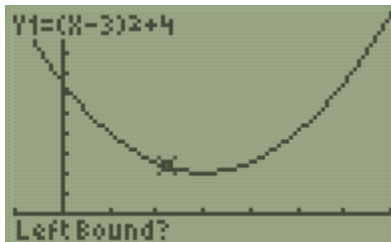
Press **Y=** and enter the function rule for Y1.



To follow along with this example, press **WINDOW** and enter the following settings.



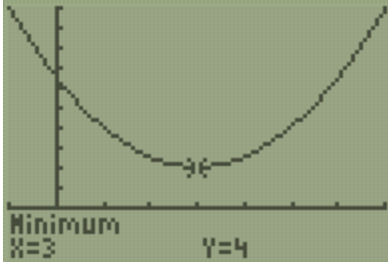
Press **2nd**, **TRACE**, and choose choice **3** for the minimum finder.



Move the cursor using the arrow keys until it is on the left side of the minimum, then press **ENTER**.

Move the cursor to the right side of the minimum and press **ENTER**.

Finally, move the cursor to somewhere close to the minimum point and press **ENTER**.



The coordinates of the minimum point of $p(x)$ are $(3,4)$.

This process works nearly the same for finding a maximum.

Try to find the coordinates of the maximum point on $h(t) = -16t^2 + 60t + 3$. Note that when you put the rule into **Y=** you always use x as your variable.

```
WINDOW
Xmin=-1
Xmax=7
Xscl=1
Ymin=-5
Ymax=65
Yscl=10
↓Xres=1
```

These settings produce a nice viewing window for h . Again, we will discuss strategies for how to do this next lesson.

Use these Window settings and write the coordinates of the maximum point of $h(t) = -16t^2 + 60t + 3$.

The Regents exam also loves to ask about the **zeros** of quadratic functions. **Zeros** are x -values for which $y = 0$. Sometimes we can find zeros on a table.

Locate the zeros of $R(x) = x(200 - 20x)$ using your table.

Hopefully you noticed that the zeros are $x = 0$ and $x = 10$.

From this example, we see that quadratics can have two zeros.

The zeros of a function are the **x -intercepts** of its graph. Notice how the graph of $R(x)$ intersects the x -axis at two locations, $x = 0$ and $x = 10$

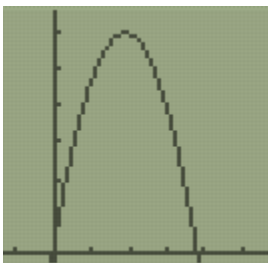


Sketch the graph of a quadratic function that has one zero.

Sketch the graph of a quadratic function that has no zeros.

Is it possible for a quadratic function to have more than two zeros? Explain your reasoning.

Consider the function $h(t) = -16t^2 + 60t + 3$. By looking at its graph, we know that $h(t)$ has two zeros.



However, when we examine a table of $h(t)$ values, the location of the zeros are not obvious.

Fill out the table of values for $h(t)$ for integers $x = -2$ through $x = 5$

x	-2	-1	0	1	2	3	4	5
$h(x)$								

Between what values of x must the zeros be “hiding”? Explain how you can tell.

This example shows that we can't rely on tables to find the zeros of every quadratic function.

To find the zeros on a graph, press **2nd**, **TRACE**, and choose option **2: zero**.

Then, follow the same process to locate a max or min.

Use the following WINDOW for a nice picture of the graph.

```

WINDOW
Xmin=-2
Xmax=6
Xscl=1
Ymin=-5
Ymax=70
Yscl=10
↓Xres=1
  
```

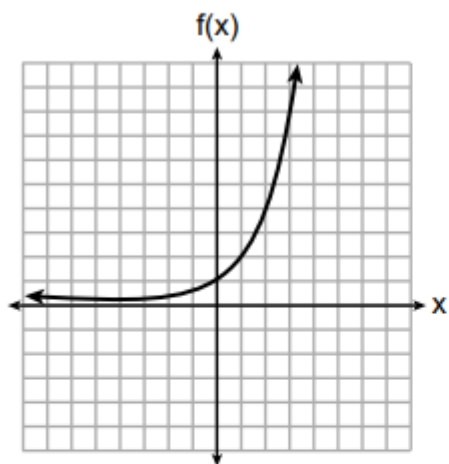
Try this process now to locate the zeros of $h(t) = -16t^2 + 60t + 3$. Note that they will be decimal values.

Great job so far! You have made it through the third topic. Next, particular questions have been chosen from the August 2018 Regents exam that you should be able to correctly answer by applying strategies and ideas from each lesson so far. Some questions may look a little different than what you have experienced so far. That's okay; use your strategies and what you know to pick answers that make the most sense. You will notice that not all questions from the test are here. Next lesson, you will be expected to decide which questions to leave out. Good luck, and remind yourself that you only need at least ten correct answers! **You can do this!!**

8 Compared to the graph of $f(x) = x^2$, the graph of $g(x) = (x - 2)^2 + 3$ is the result of translating $f(x)$

- (1) 2 units up and 3 units right
- (2) 2 units down and 3 units up
- (3) 2 units right and 3 units up
- (4) 2 units left and 3 units right

11 Three functions are shown below.



$$g(x) = 3^x + 2$$

x	h(x)
-5	30
-4	14
-3	6
-2	2
-1	0
0	-1
1	-1.5
2	-1.75

Which statement is true?

- (1) The y -intercept for $h(x)$ is greater than the y -intercept for $f(x)$.
- (2) The y -intercept for $f(x)$ is greater than the y -intercept for $g(x)$.
- (3) The y -intercept for $h(x)$ is greater than the y -intercept for both $g(x)$ and $f(x)$.
- (4) The y -intercept for $g(x)$ is greater than the y -intercept for both $f(x)$ and $h(x)$.

13 If $y = 3x^3 + x^2 - 5$ and $z = x^2 - 12$, which polynomial is equivalent to $2(y + z)$?

- (1) $6x^3 + 4x^2 - 34$
- (2) $6x^3 + 3x^2 - 17$
- (3) $6x^3 + 3x^2 - 22$
- (4) $6x^3 + 2x^2 - 17$

16 If $f(x) = 2x^2 + x - 3$, which equation can be used to determine the zeros of the function?

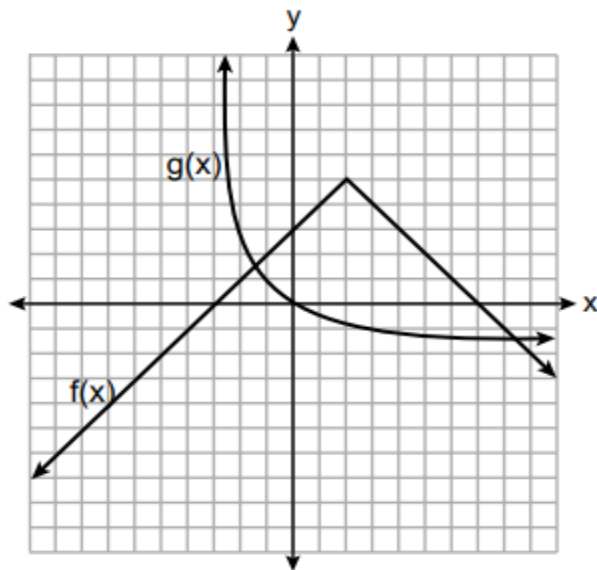
- (1) $0 = (2x - 3)(x + 1)$ (3) $0 = 2x(x + 1) - 3$
 (2) $0 = (2x + 3)(x - 1)$ (4) $0 = 2x(x - 1) - 3(x + 1)$

17 Each day, a local dog shelter spends an average of \$2.40 on food per dog. The manager estimates the shelter’s daily expenses, assuming there is at least one dog in the shelter, using the function $E(x) = 30 + 2.40x$.

Which statements regarding the function $E(x)$ are correct?

- I. x represents the number of dogs at the shelter per day.
 II. x represents the number of volunteers at the shelter per day.
 III. 30 represents the shelter’s total expenses per day.
 IV. 30 represents the shelter’s nonfood expenses per day.
- (1) I and III (3) II and III
 (2) I and IV (4) II and IV

19 The functions $f(x)$ and $g(x)$ are graphed below.



Based on the graph, the solutions to the equation $f(x) = g(x)$ are

- (1) the x -intercepts
 (2) the y -intercepts
 (3) the x -values of the points of intersection
 (4) the y -values of the points of intersection

20 For the sequence $-27, -12, 3, 18, \dots$, the expression that defines the n th term where $a_1 = -27$ is

(1) $15 - 27n$

(3) $-27 + 15n$

(2) $15 - 27(n - 1)$

(4) $-27 + 15(n - 1)$

23 Which of the three situations given below is best modeled by an exponential function?

I. A bacteria culture doubles in size every day.

II. A plant grows by 1 inch every 4 days.

III. The population of a town declines by 5% every 3 years.

(1) I, only

(3) I and II

(2) II, only

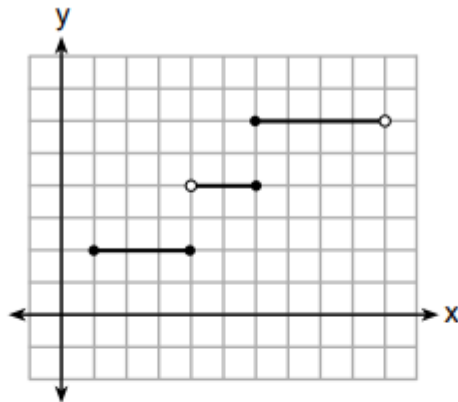
(4) I and III

Next are some “gettable” long response questions. We have not spent much time discussing the long response questions. This is because the majority of your time and effort should be spent making sure you are answering multiple choice questions correctly. But long response questions can only help you earn points. There is also partial credit available. If a question says “explain” it means that you must write words. If a question says “justify” it means to use words and/or show **evidence**. The words you write can describe calculator steps and the evidence you show can be tables and sketches of graphs that your calculator produces. You should try your best on the long response, and don’t worry if some of them seem challenging to you. These are considered your “reach” questions to get as many points as possible. Try the ones you know and don’t get discouraged! Show your thinking on paper.

25 Explain how to determine the zeros of $f(x) = (x + 3)(x - 1)(x - 8)$.

State the zeros of the function.

26 Four relations are shown below.



I

$$\{(1,2), (2,5), (3,8), (2,-5), (1,-2)\}$$

II

x	y
-4	1
0	3
4	5
6	6

III

$$y = x^2$$

IV

State which relation(s) are functions.

Explain why the other relation(s) are *not* functions.

30 Solve the following equation by completing the square:

$$x^2 + 4x = 2$$

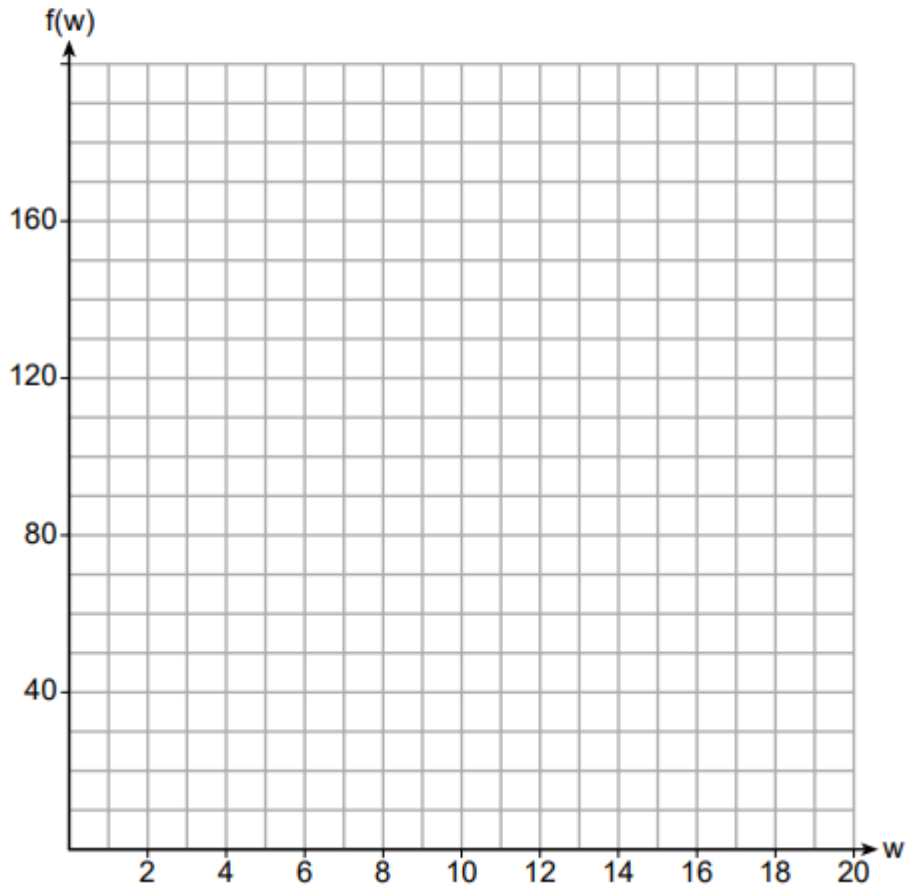
- 34** A car was purchased for \$25,000. Research shows that the car has an average yearly depreciation rate of 18.5%.

Create a function that will determine the value, $V(t)$, of the car t years after purchase.

Determine, to the *nearest cent*, how much the car will depreciate from year 3 to year 4.

36 Paul plans to have a rectangular garden adjacent to his garage. He will use 36 feet of fence to enclose three sides of the garden. The area of the garden, in square feet, can be modeled by $f(w) = w(36 - 2w)$, where w is the width in feet.

On the set of axes below, sketch the graph of $f(w)$.



Explain the meaning of the vertex in the context of the problem.

Topic 4 – Some Final Thoughts

For the final topic, we are going to talk more about how to adjust the viewing window in your calculator, and discuss some last minute ideas that are really nice to know for the exam. You have seen all the major ideas that will be tested on the Regents. At the end of this lesson, your instructor will give you an entire blank exam and you will decide which questions you must complete.

The Calculator WINDOW

When you press **ZOOM, 6**, your calculator graph is set up so that the x- and y-axes go from -10 to 10.

```
WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
↓Xres=1
```

Xmin and Xmax tell the calculator where to start and stop the x-axis, or how far left and right to show.

Ymin and Ymax tell the calculator where to start and stop the y-axis, or how far up and down to show.

The Xscl and Yscl tell the graph how far to space the marks on each axis. It is helpful to change the scales, but not necessary.

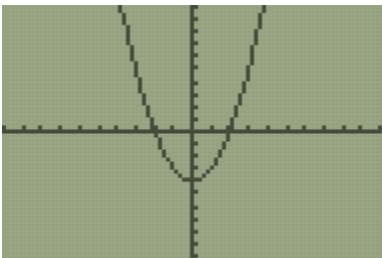
You will never adjust Xres or any other window settings.

For most functions, the ZOOM 6 viewing window gives a good picture of the graph.

```
Plot1 Plot2 Plot3
Y1=(X+2)(X-2)
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
```

For the function rule

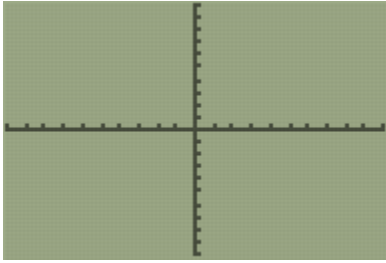
$$y = (x + 2)(x - 2)$$



When we use ZOOM 6, we can see the full graph. It is easy to tell that this is a quadratic function; its vertex is a minimum at the point (0, -4) and its zeros are $x = -2$ and $x = 2$.

Sometimes the standard viewing window doesn't provide a great picture.

Consider the situation from Topic 3: a school with 900 students is decreasing its enrollment by 2% each year. The function rule to model the population of the school over t years is $p(t) = 900(0.98)^t$.



When we look at the standard ZOOM 6 window, the graph of $p(t)$ is not showing.

We need to adjust our window settings.

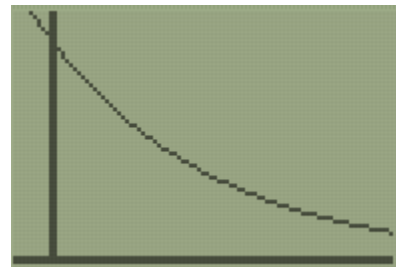
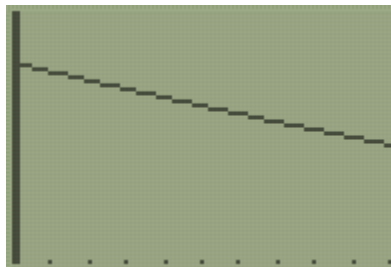
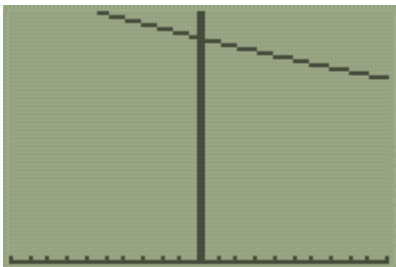
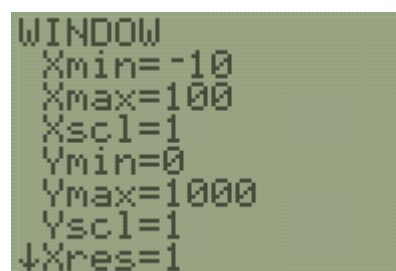
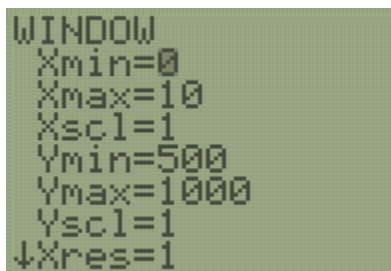
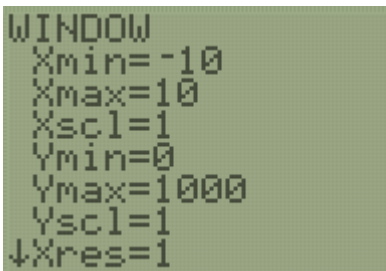
The best way to get an idea of the numbers to set for the WINDOW is by looking at the table.

X	Y ₁
-2	937.11
-1	918.37
0	900
1	882
2	864.36
3	847.07
4	830.13

Notice that the y-values are in the 900s.

The x-values are generally in the single digits.

There is no one way to set up your window. Here are a few different settings and the graphs they produce.



Each window setting gives a different looking picture of $p(t) = 900(0.98)^t$, but in each case we can see the graph.

Adjust the calculator window until you get a good picture of the graph and sketch what it looks like below each function rule. Write the window settings that you use.

$$f(x) = -16x^2 + 70x + 3$$

$$g(x) = 25x - 200$$

$$h(x) = 25(1.02)^x$$

Some tips for WINDOW setting:

- Mins should be lower than your lowest value.
- Maxs should be higher than your highest value.
- When in doubt, set your minimums to zero.
- If your graph doesn't look right, change one setting at a time and look back and forth from the WINDOW and the GRAPH until you get a good picture.

Another Function Family

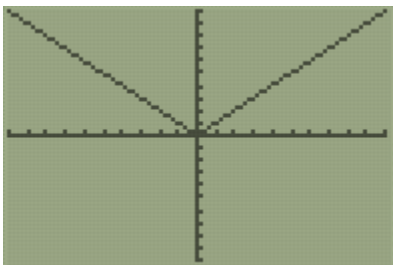
There is another family (in addition to linear, exponential, and quadratic) that is worth knowing – the absolute value function.

The **absolute value** of a number a , written as $|a|$, is that number's distance from 0. Since distance is always positive, the absolute value of a number is always positive.

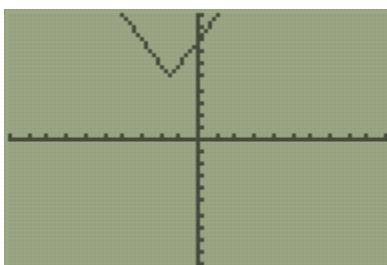
For example, the absolute value of negative four is four $|-4| = 4$ because negative four is four units away from zero on a number line.

Absolute value functions have similarities in the shape of their graphs.

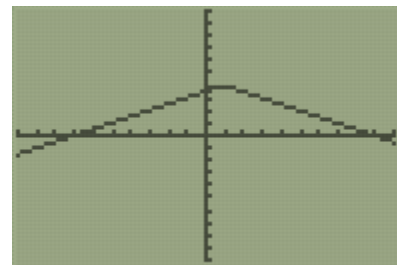
$$y = |x|$$



$$y = |2x + 3| + 5$$



$$y = 0.5|x - 1| + 4$$



Get the absolute value bars in the calculator by pressing **ALPHA, WINDOW, ENTER**.

What are some similarities between each function rule?

What similarities do you notice in each graph?

How Will I Remember Where Everything is in the Calculator???

By now you are noticing that the calculator can do many things. It is not obvious where everything is inside the calculator. You will probably forget where something is. Usually, you can find what you are looking for by doing one of the following things:

1. ALPHA, and one of the top buttons

These reveal hidden menus in the calculator. Play around with them to see what they can do. Here are a few highlights.



ALPHA, Y=, ENTER will create a fraction



ALPHA, WINDOW, ENTER will make absolute value bars

2. It's probably the top row of buttons

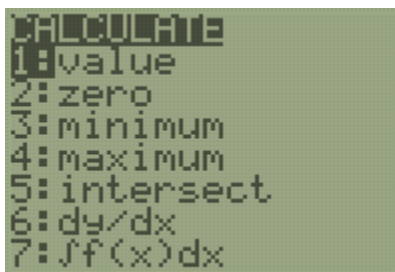
When you're working with graphs or tables, you always use the top row of buttons. You know that **Y=** is where you go to enter function rules, and **GRAPH** is what to press to look at a graph. Here are a few other features that you might find helpful.



2nd WINDOW will allow you to change the starting x-value of the table.

ΔTbl adjusts how much the x-value changes by.

If you highlight "Indpnt: Ask" and press ENTER, when you visit your table (2nd GRAPH), it will be blank. You can type whatever x-values you want and the calculator will tell you the y-value. This sometimes confuses students, so only use this feature if it makes sense to you.



2nd TRACE is where all the tools for analyzing graphs are located.

Linear and Exponential Regression

Many students consider this an easy topic to earn points. The process of regression will generate a function rule that fits a set of data points.

Consider the question on the next page from the January 2019 exam:

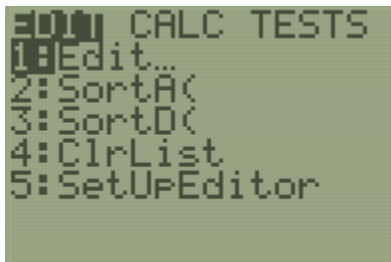
34 The data given in the table below show some of the results of a study comparing the height of a certain breed of dog, based upon its mass.

Mass (kg)	4.5	5	4	3.5	5.5	5	5	4	4	6	3.5	5.5
Height (cm)	41	40	35	38	43	44	37	39	42	44	31	30

Write the linear regression equation for these data, where x is the mass and y is the height. Round all values to the *nearest tenth*.

State the value of the correlation coefficient to the *nearest tenth*, and explain what it indicates.

Solution:



Press **STAT**, **ENTER**

L1	L2	L3	3
4.5	41		
5	40		
4	35		
3.5	38		
5.5	43		
5	44		
5	37		

Type the x -values in L1 and the y -values in Y2.

```

EDIT [MENU] TESTS
1:1-Var Stats
2:2-Var Stats
3:Med-Med
4:LinReg(ax+b)
5:QuadReg
6:CubicReg
7↓QuartReg
    
```

Press **STAT**, →

Since this problem asks for a linear regression, choose **4**.

(Exponential regression is choice **0**.)

```

LinReg(ax+b)
Xlist:L1
Ylist:L2
FreqList:
Store RegEQ:
Calculate
    
```

Highlight “Calculate” and press **ENTER**

```

LinReg
y=ax+b
a=1.917355372
b=29.79889807
r²=.1136307339
r=.3370915809
    
```

The first line, $y = ax + b$, is the function rule.

The next lines tell you the numbers to write in place of a and b . x and y will remain letters.

Ignore r^2

The r is called the **correlation coefficient**, and it is always between 0 and 1. The closer r is to 1, the better fit this data is to the type of function rule you chose.

Notice that this question asked us to round the values of the function rule to the nearest tenth.

The linear regression equation is $y = 1.9x + 29.8$.

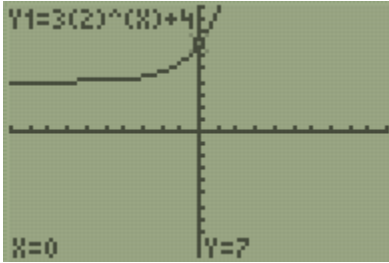
The correlation coefficient, to the nearest tenth, is .3.

Since .3 is not close to 1, this function rule is not a very good fit for the data.

The y-intercept

This is a really easy one.

Consider $y = 3(2)^x + 4$



The y-intercept is the location where the graph intersects the y-axis.

Press **TRACE**, **0**, **ENTER**

X	Y1
-1.5	5.0607
-1	5.5
-.5	6.1213
0	7
.5	8.2426
1	10
1.5	12.485

It is the **y-value when $x = 0$** in the TABLE.

The y-intercept of $y = 3(2)^x + 4$ is $y = 7$.

You're almost there. I hope that by now you feel some confidence toward passing this test. Next, your teacher is going to give you a copy of the June 2017 exam.

(<http://www.jmap.org/JMAPRegentsExamArchives/ALGEBRAIEXAMS/0617ExamAI.pdf>)

It is now up to you to highlight ten problems that you can answer correctly by using the calculator strategies and ideas that we have discussed over these four lessons. Take your time to answer these questions first, and then focus on answering the other questions. Try the long response questions too.

If you have the time, try to spend at least two hours taking this practice exam. Pretend it's the real thing. Your teacher can help you read the questions, but they cannot tell you whether your answers or work are right or wrong.

Taking a Regents

You may have been told that “it never hurts to ask.” But never ask if something is right while you’re taking a test. Students usually interpret a proctor saying “I can’t tell you” to mean “what you’re doing is wrong.” Test proctors are simply not allowed to tell you whether what you are doing is right or wrong. For some reason, students tend to think this means that they are on the wrong track. Don’t put your teacher or proctor in that position! Trust yourself.

If there is something you’re having trouble with, but you are confident that you might be able to get it, put a star by that question and move on. The star will remind you to come back to that question later.

If you start to get sleepy, frustrated, or feel like your brain is fried, ask to get a drink of water or go to the bathroom and splash your face with water. While you’re in the hallway, stretch your muscles and get your blood flowing. Focus on breathing slowly and deeply, then get back to work.

The night before the test, get to bed on time and eat a good breakfast. Bring a bottle of water with you to the test.

Try your hardest, and use as much time as you need. Don’t feel like you are behind even if other kids are leaving the exam. Everyone processes at different speeds.

Don’t rush. You have three hours. This is more than enough time to work slowly and carefully.

There are going to be questions that you cannot do. That is completely fine and expected. Remember, we are highlighting ten questions only, doing those, then finishing the rest. This is the key to your success.

On the long response, if you think you might have some idea, or you can do only part of the question, try it and write down your process.

Read every word of each question carefully. Sometimes only one word, such as “*not*”, changes the meaning of a question completely.

Trust that the work you put into preparing for this test will pay off.

Good luck!