Topic 3 Parallel and Perpendicular Lines

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$$\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope

positive   negative   zero   undefined
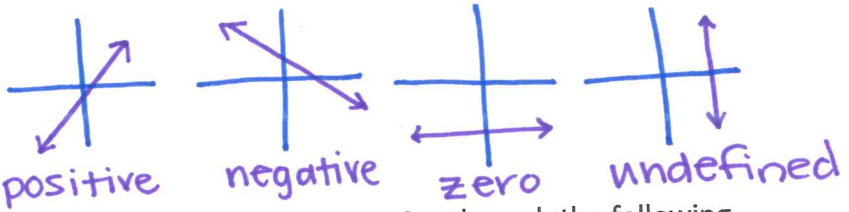
Practice: Use the slope formula to find the slope of the line passing through the following points.

$x_1 \, y_1 \qquad x_2 \, y_2$

1. (0, 0) and (4,8)

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 0}{4 - 0} = \frac{8}{4} = \boxed{\frac{2}{1}}$$

$x_1 \, y_1 \qquad x_2 \, y_2$

2. (1,5) and (3, 9)

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 5}{3 - 1} = \frac{4}{2} = \boxed{\frac{2}{1}}$$

$x_1 \, y_1 \qquad x_2 \, y_2$

3. (-1, 3) and (2,3)

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 3}{2 - {}^- 1} = \frac{0}{3} = \boxed{0}$$

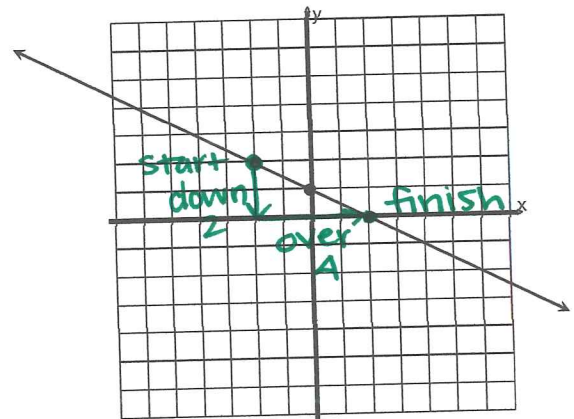$x_1 \, y_1 \qquad x_2 \, y_2$

4. (4,2) and (4,1)

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 2}{4 - 4} = \frac{-1}{0} = \boxed{\text{undefined}}$$

For each of the following, count the slope of the line on the graph.

*remember:
$$\frac{\text{rise}}{\text{run}}$$

over 3
finish
up
6
start

start down
2
over 4
finish
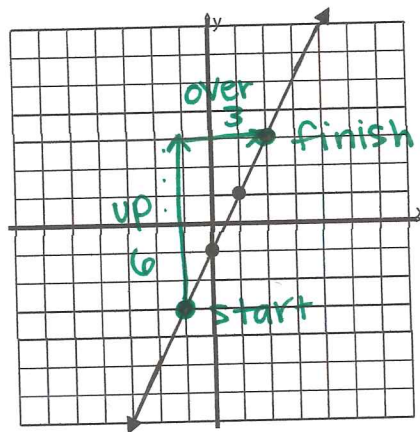
$$m = \frac{6}{3} = \boxed{\frac{2}{1}}$$
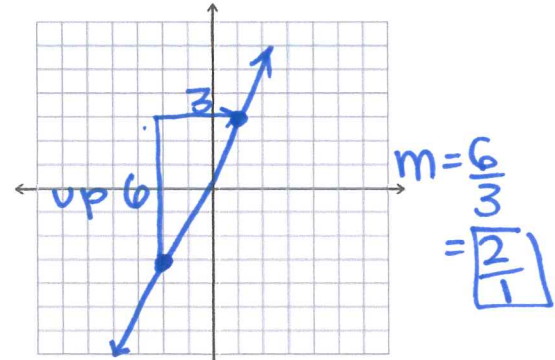
$$m = \frac{-2}{4} = \boxed{\frac{-1}{2}}$$

Which method do you like better?

Practice

Directions: Use the slope formula to find the slope. Then check on the graph by counting.

$$x_1 \; y_1 \qquad x_2 \; y_2$$
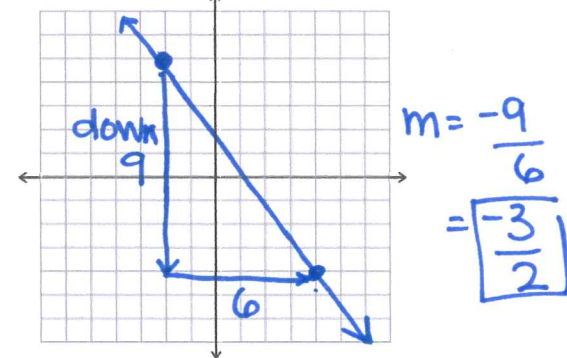
1. Find the slope of the line between (-2, -3) and (1, 3).

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - -3}{1 - -2} = \frac{6}{3} = \boxed{\frac{2}{1}}$$

up 6   3

$$m = \frac{6}{3} = \boxed{\frac{2}{1}}$$

$$x_1 \; y_1 \qquad x_2 \; y_2$$

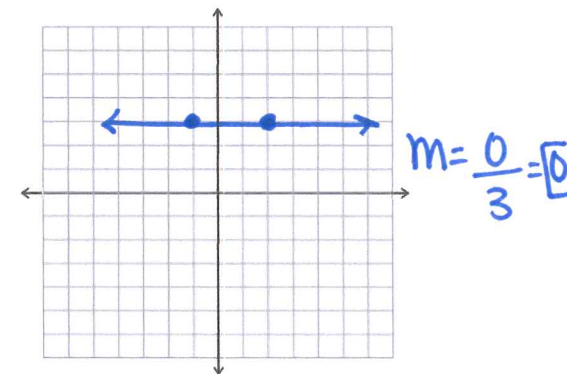2. Find the slope of the line between (-2, 5) and (4, -4).

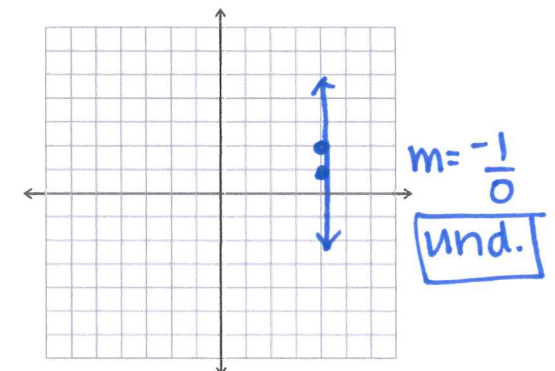$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-4 - 5}{4 - -2} = \frac{-9}{6} = \boxed{\frac{-3}{2}}$$

down 9   6

$$m = \frac{-9}{6} = \boxed{\frac{-3}{2}}$$

$$x_1 \; y_1 \qquad x_2 \; y_2$$

3. Find the slope of the line between (-1, 3) and (2,3)

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 3}{2 - -1} = \frac{0}{3} = \boxed{0}$$

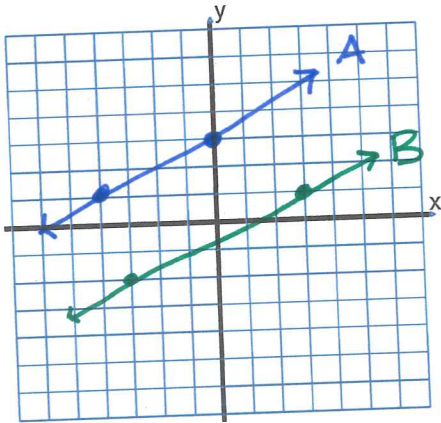$$m = \frac{0}{3} = \boxed{0}$$

$$x_1 \; y_1 \qquad x_2 \; y_2$$

4. Find the slope of the line between (4,2) and (4,1)

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 2}{4 - 4} = \frac{-1}{0} = \boxed{\text{undefined}}$$

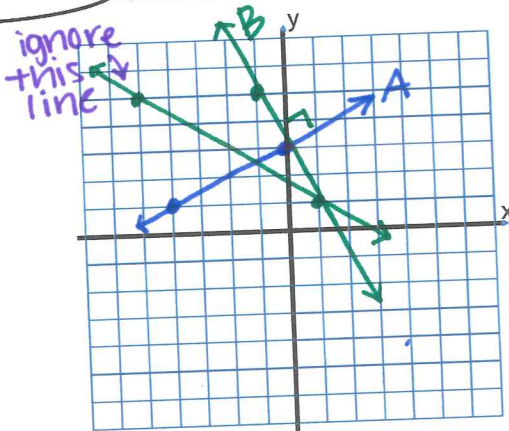$$m = \frac{-1}{0}$$

$$\boxed{\text{und.}}$$

Parallel vs. Perpendicular

ignore
this
line

Graph Line A to pass through the points
(-4, 1) and (0, 3)

Graph Line B to pass through the points
(-3, -2) and (3, 1)

Lines A and B are __Parallel__

Graph Line A to pass through the points
(-4, 1) and (0, 3)

Graph Line B to pass through the points

(-1,5) (-5,5) and (1, 1)

Lines A and B are __PERPENDICULAR__

Parallel lines have **SAME** slopes.

Perpendicular lines have **NEGATIVE RECIPROCAL** slopes.
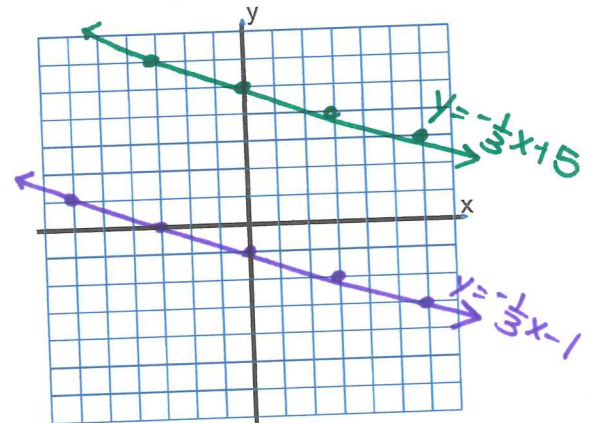
opposite
signs

fraction
flip

Examples

1. Are the lines $y = -\frac{1}{3}x + 5$ and $y = -\frac{1}{3}x - 1$ parallel, perpendicular or neither? Graph the lines to prove your answer.

mx + b
$$y = -\frac{1}{3}x + 5$$
$$m = -\frac{1}{3}$$

mx + b
$$y = -\frac{1}{3}x - 1$$
$$m = -\frac{1}{3}$$

$y = -\frac{1}{3}x + 5$

$y = -\frac{1}{3}x - 1$

Parallel! Slopes are the SAME!

2. Are the lines $y = \dfrac{3}{2}x$ and $y = -\dfrac{2}{3}x + 3$ parallel, perpendicular or neither? Graph the lines to prove your answer.
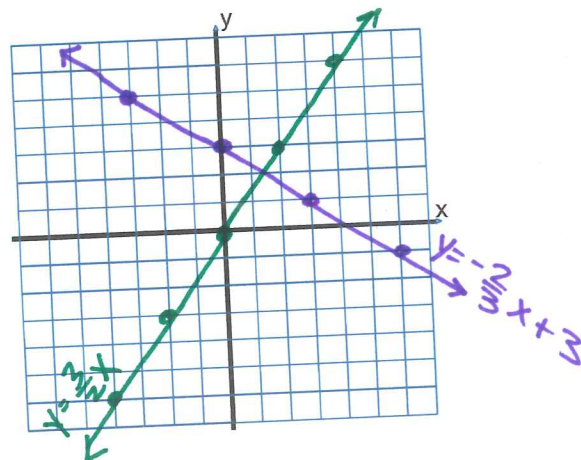
$y = \dfrac{3}{2}x$ $y = -\dfrac{2}{3}x + 3$

$m = \dfrac{3}{2}$ $m = -\dfrac{2}{3}$

PERPENDICULAR...the slopes are negative reciprocals!

3. Are the lines $y = \dfrac{1}{2}x - 4$ and $y = 2x - 3$ parallel, perpendicular or neither? Graph the lines to prove your answer.
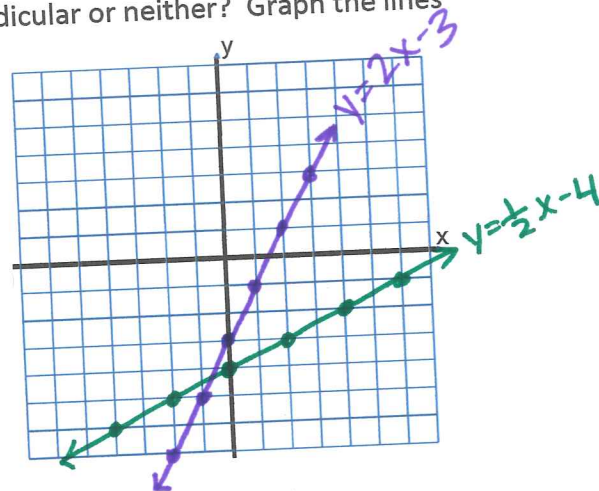
$y = \dfrac{1}{2}x - 4$ $y = 2x - 3$

$m = \dfrac{1}{2}$ $m = 2$

NEITHER... the slopes are reciprocals, but not opposite signs (so not perpendicular). The slopes aren't the same, so not parallel!

4. Which line is perpendicular to the line whose equation is $5y + 6 = -3x$?

1) $y = -\dfrac{5}{3}x + 7$ $m = \dfrac{-5}{3}$

2) $y = \dfrac{5}{3}x + 7$ $m = \dfrac{5}{3}$

3) $y = -\dfrac{3}{5}x + 7$ $m = \dfrac{-3}{5}$

4) $y = \dfrac{3}{5}x + 7$ $m = \dfrac{3}{5}$

$\dfrac{5y}{5} = \dfrac{-3x}{5} - \dfrac{6}{5}$

$y = \dfrac{-3}{5}x - \dfrac{6}{5}$

$m = \dfrac{-3}{5}$

Perpendicular means OPPOSITE/NEGATIVE RECIPROCALS

Practice

For each of the following, decide whether the set of lines are parallel, perpendicular or neither. Place the letter in the appropriate box.

Parallel	Perpendicular	Neither
B, E	A, F	C, D, G
(same slope)	(opp/neg reciprocals)	

A.
$$y = 3x - 1 \quad (m = \tfrac{3}{1})$$
$$y = \tfrac{-1}{3} + 4 \quad (m = \tfrac{-1}{3})$$

B.
$$y = 2x \quad (m = \tfrac{2}{1})$$
$$y = 2x + 4 \quad (m = \tfrac{2}{1})$$

C.
$$y = 2x \quad (m = \tfrac{2}{1})$$
$$y = -2x + 4 \quad (m = \tfrac{-2}{1})$$

D.
$$y = \tfrac{1}{2}x \quad (m = \tfrac{1}{2})$$
$$y = 2x + 4 \quad (m = \tfrac{2}{1})$$

E.
$$y = \tfrac{1}{2}x \quad (m = \tfrac{1}{2})$$
$$\frac{2y}{2} = \frac{x}{2} + \frac{4}{2}$$
$$y = \tfrac{1}{2}x + 2 \quad (m = \tfrac{1}{2})$$

F.
$$\frac{3y}{3} = \frac{-2x}{3} + \frac{6}{3}$$
$$2x + 3y = 6 \quad (m = \tfrac{-2}{3})$$
$$2y - 3x = 9$$
$$\frac{2y}{2} = \frac{3x}{2} + \frac{9}{2}$$
$$y = \tfrac{3}{2}x + \tfrac{9}{2} \quad (m = \tfrac{3}{2})$$

G.
$$\frac{3y}{3} = \frac{x}{3} + \frac{6}{3}$$
$$-x + 3y = 6$$
$$y - 3x = 9$$
$$y = \tfrac{1}{3}x + 2 \quad (m = \tfrac{1}{3})$$
$$y = 3x + 9 \quad (m = 3)$$

What is the slope of a line that is parallel to the line that passes through the points (-3,-1) and (1,2)?
$x_1 y_1$ ... $x_2 y_2$

Find slope:

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 2}{-3 - 1} = \frac{-3}{-4} = \boxed{\tfrac{3}{4}}$$

Parallel lines = same slope!

$$\boxed{m = \tfrac{3}{4}}$$

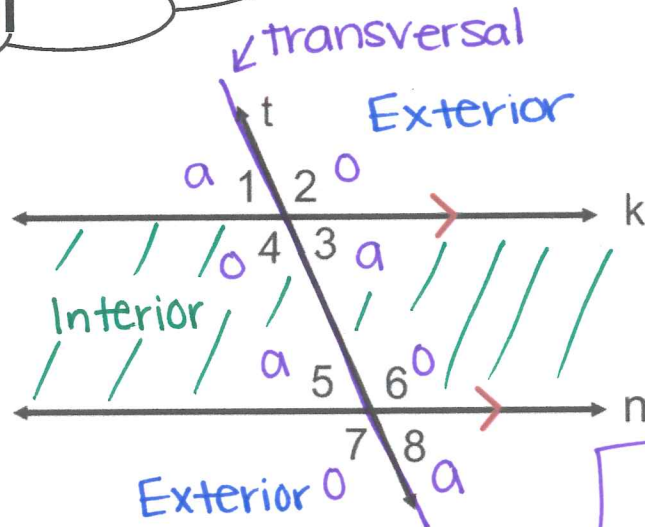What is the slope of a line that is perpendicular to the line that passes through the points (-3,-1) $x_1 y_1$ and (1,2)? $x_2 y_2$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - -1}{1 - -3} = \frac{3}{4}$$

Perpendicular lines have negative reciprocal slopes!

$$\boxed{m = \tfrac{-4}{3}}$$

Parallel Lines cut by a Transversal

↙ transversal

Parallel
↓
$k \parallel n$

t   Exterior

a 1 2 o

                    k

o 4 3 a

Interior

a 5 6 o

                    n

7 8

Exterior o   a

Key
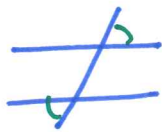a = a
o = o
a + o = 180°

Types of Angles that are formed:

✱ Alternate Interior Angles — 4,6
3,5   alt. int. ∢'s are ≅
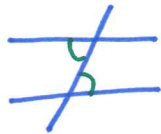
Alternate Exterior Angles — 1,8
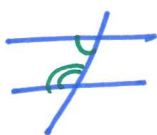2,7   alt. ext. angles are ≅

Corresponding Angles — 1,5   2,6   corresponding
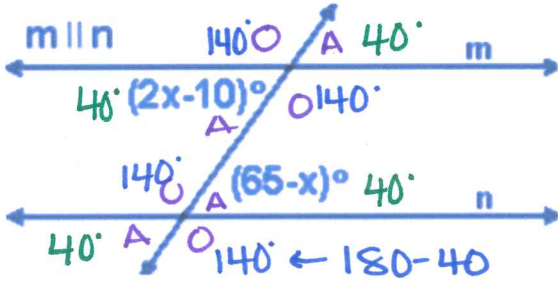4,7   3,8   angles are ≅
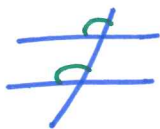
Same Side Interior Angles — 3,6   Same side int.
4,5   angles are
SUPPLEMENTARY

Examples

Directions: Solve for *x* and each angle.
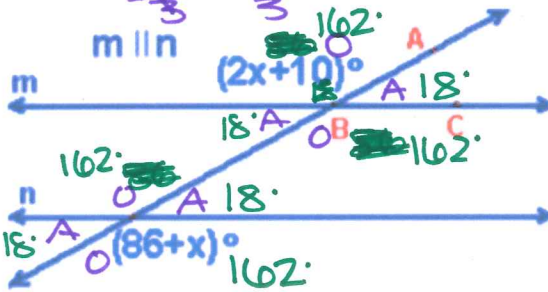
Ex. 1 Alt. Int. Angles

m ‖ n   140° ∠A 40°   m
40° (2x-10)° ∠A 140°
140° ∠A (65-x)° 40°   n
40° ∠A 140° ← 180-40

$$2x-10 = 65 - x$$
$$+x \qquad\qquad +x$$
$$3x-10 = 65$$
$$+10 \quad +10$$
$$3x = 75$$
$$\frac{3x}{3} = \frac{75}{3}$$
$$\boxed{x = 25}$$

Ex. 2 CORRESPONDING

m ‖ n   80° ∠A 100°   m
100° (x+80)° ∠A 80°
80° ∠A 100°   n
100° (5x)° ∠A 80°

$$x+80 = 5x$$
$$-x \qquad -x$$
$$80 = 4x$$
$$\frac{80}{4} = \frac{4x}{4}$$
$$\boxed{20 = x}$$

Ex. 3 ALT. EXT. ANGLES

m ‖ n   162° ∠A   A
(2x+10)° B ∠A 18°
18° ∠A ∠B 162° C
162° ∠A 18°   n
18° ∠A (86+x)° 162°

$$2x+10 = 86 + x$$
$$-x \qquad -x$$
$$x+10 = 86$$
$$-10 \quad -10$$
$$\boxed{x = 76}$$

Ex. 4 SAME SIDE INTERIOR

130° ∠A 50   m
50° (3x-10)° ∠A 130°
130° (5x+30)° ∠A 50°   m ‖ n
50° ∠A 130°   n

$$3x-10+5x+30 = 180$$
$$8x+20 = 180$$
$$-20 \quad -20$$
$$8x = 160$$
$$\frac{8x}{8} = \frac{160}{8}$$
$$\boxed{x = 20}$$

Ex. 5

m ‖ n   114° ∠A
(3x+15)° ∠A 66°
66° ∠A 114°
114° ∠A (2x)° 66°   n
66° ∠A 114°
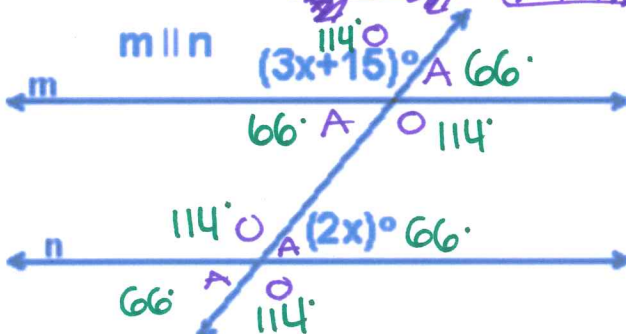
$$3x+15+2x = 180$$
$$5x+15 = 180$$
$$-15 \quad -15$$
$$\frac{5x}{5} = \frac{165}{5}$$
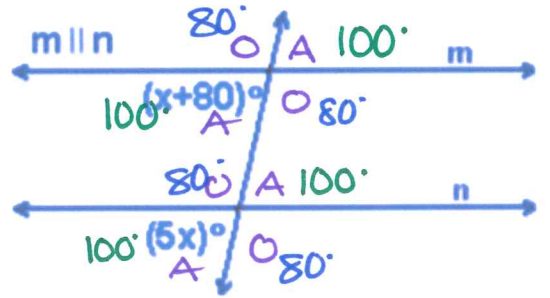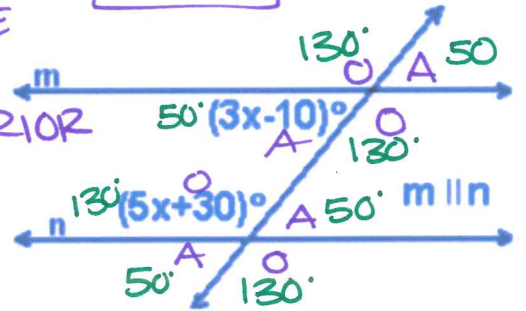
Practice

For each of the following, identify the type of angles and then solve each of the following for x.
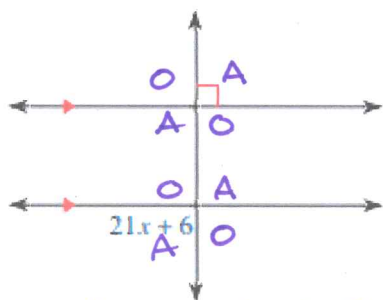
opposite exterior

$90 = 21x + 6$

$84 = 21x$

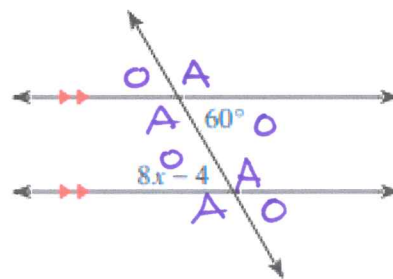$\dfrac{84}{21} = \dfrac{21x}{21}$

$\boxed{4 = X}$
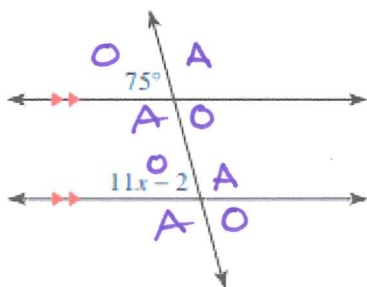
Corresponding

$75 = 11x - 2$

$+2 \qquad +2$

$77 = 11x$

$\dfrac{77}{11} = \dfrac{11x}{11}$

$\boxed{7 = X}$

Alt. Int. Angles

$60 = 8x - 4$

$+4 \qquad +4$

$64 = 8x$

$\dfrac{64}{8} = \dfrac{8x}{8}$

$\boxed{8 = X}$
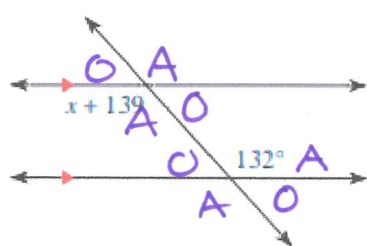
Alt. Int. Angles

$x + 139 = 132$

$-139 \quad -139$

$\boxed{X = -7}$
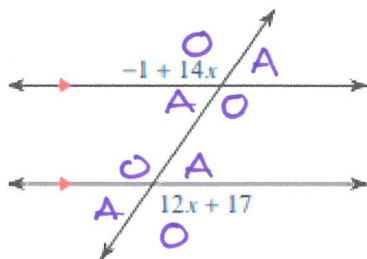
Alt. Ext. Angles

$-1 + 14x = 12x + 17$

$-12x \quad -12x$

$-1 + 2x = 17$

$+1 \qquad +1$

$\dfrac{2x}{2} = \dfrac{18}{2}$

$\boxed{X = 9}$
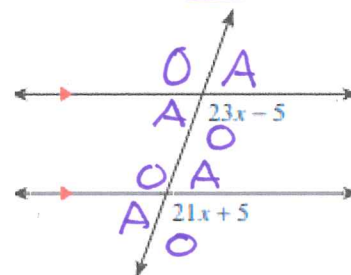
Corresponding

$23x - 5 = 21x + 5$

$-21x \qquad -21x$

$2x - 5 = 5$

$+5 \quad +5$

$\dfrac{2x}{2} = \dfrac{10}{2}$

$\boxed{X = 5}$