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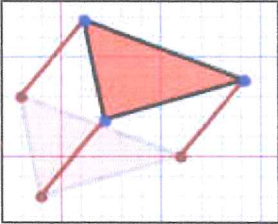
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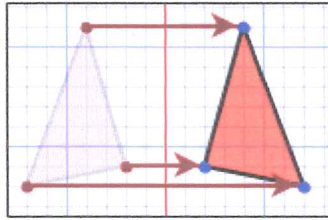
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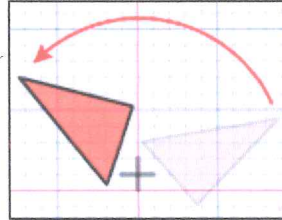
# Transformations



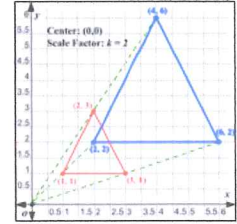
Translation



Reflection



Rotation



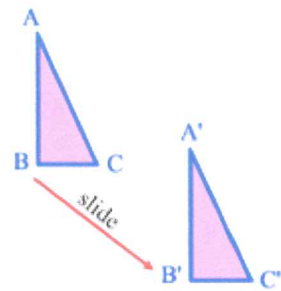
Dilation

## What is a RIGID MOTION?

- transformation that preserves size and shape.
- transformation that preserves distance (length of sides)

### Translation

"SLIDE"

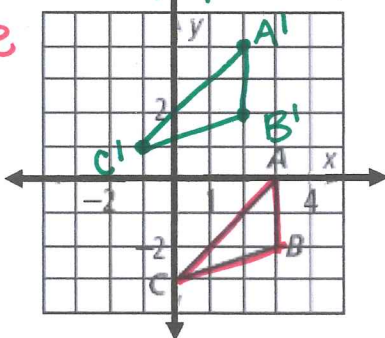


Rule:  $T_{x,y}$  or  $(x,y) \rightarrow (x+a, y+b)$

Practice. Graph the image of each figure under the given translation and state the coordinates of the image figure.

1.  $T_{-1,4}(\triangle ABC)$

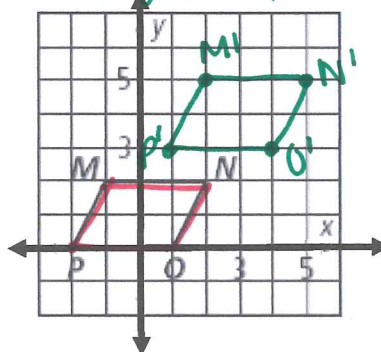
left 1, up 4



- $A'(2,4)$
- $B'(2,2)$
- $C'(-1,1)$

2.  $T_{3,3}(MNOP)$

right 3 up 3

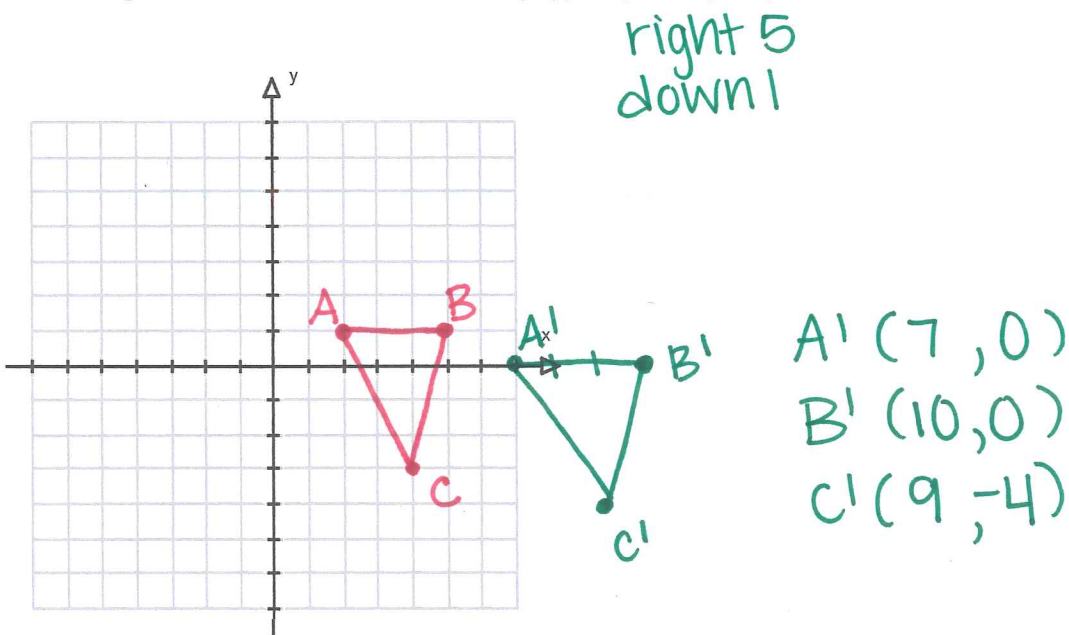


- $M'(2,5)$
- $N'(5,5)$
- $O'(4,3)$
- $P'(1,3)$

Is a translation a RIGID MOTION?

YES!!

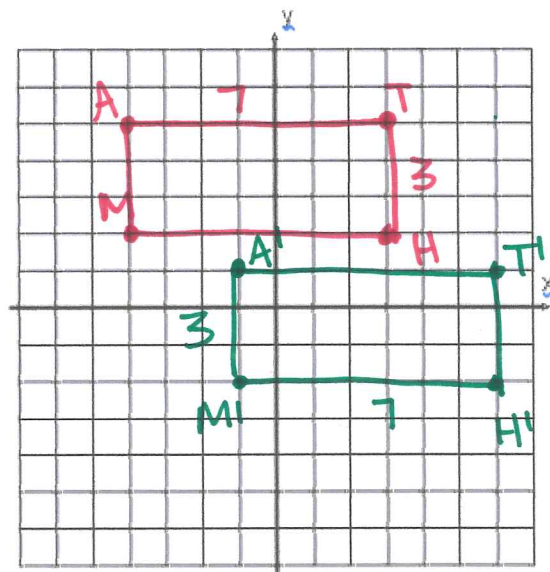
3. Given  $\triangle ABC$  with coordinates  $A(2, 1)$ ,  $B(5, 1)$ , and  $C(4, -3)$ , state the coordinates of  $A'B'C'$ , the image of  $ABC$  after the translation  $(x, y) \rightarrow (x + 5, y - 1)$



4. The vertices of rectangle  $MATH$  are  $M(-4, 2)$ ,  $A(-4, 5)$ ,  $T(3, 5)$  and  $H(3, 2)$ .

a.) Find the area of rectangle  $MATH$  in square units.

$$A = bh = (7)(3) = 21 \text{ units}^2$$

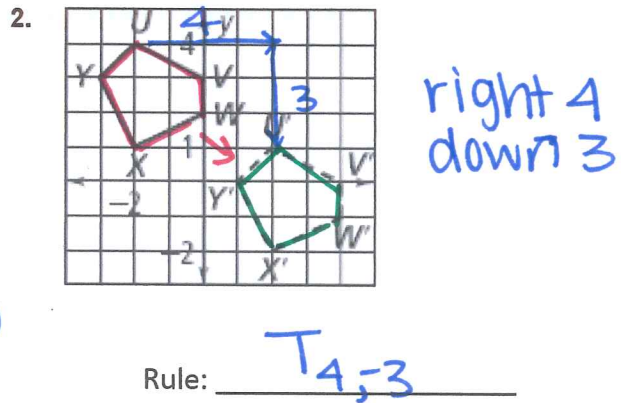
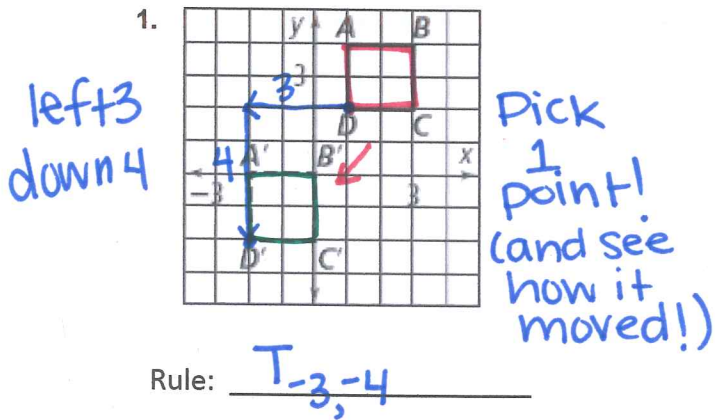


b.) Translate the rectangle 3 units to the right and 4 units down. Find the area of rectangle  $M'A'T'H'$  in square units.

$$A = bh = (7)(3) = 21 \text{ units}^2$$

## Writing a Translation Rule

**Practice:** The dashed-line figure is a translation image of the solid-line figure. Write a rule to describe each translation.

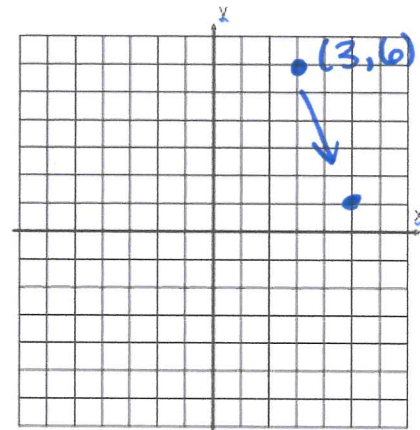


## Practice

1. A translation function is defined by the rule  $(x,y) \rightarrow (x+2, y-5)$ . What are the coordinates of the image of the point  $(3,6)$  under this translation?

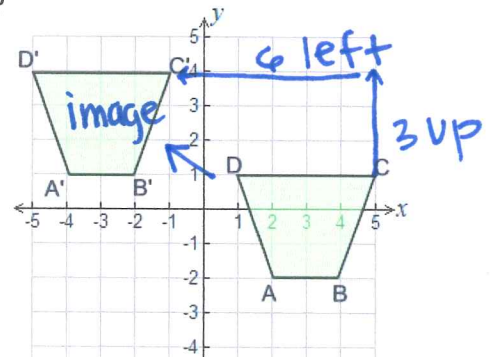
right 2  
down 5

$(5,1)$



2. Which translation mapping is shown in the graph to the right?

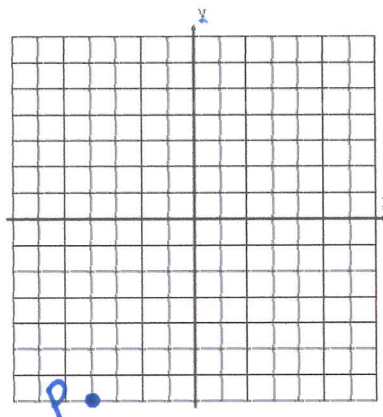
- a.)  $(x, y) \rightarrow (x + 6, y - 3)$   
 b.)  $(x, y) \rightarrow (x - 3, y + 6)$   
 c.)  $(x, y) \rightarrow (x - 6, y + 3)$   
 d.)  $(x, y) \rightarrow (x + 3, y - 6)$



3. Find the coordinates of the image of point  $P(-4, -7)$  after the transformation  $T(-5, 2)$

$(-9, -5)$

$T_{-5, 2}$

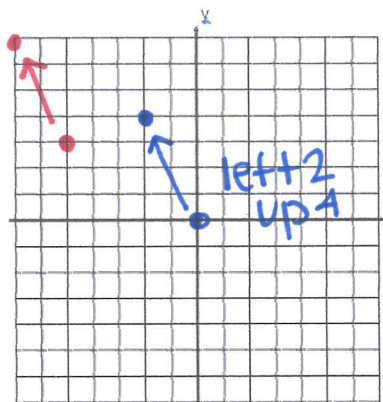


4. A translation maps the origin to the point  $(-2, 4)$ . What is the image of point  $(-5, 3)$  under this same translation?

$(-7, 7)$

$(0, 0)$

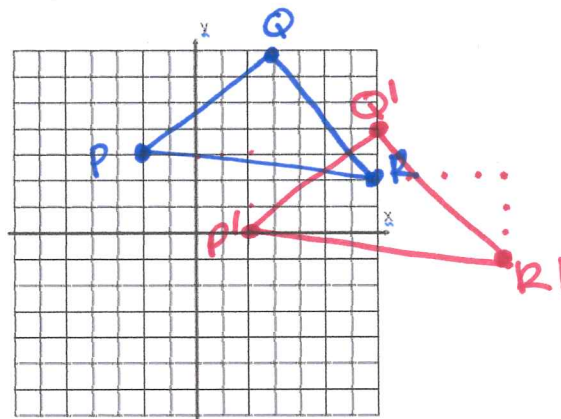
$P'$   
~~point~~



5. Given  $\triangle PQR$  with  $P(-2, 3)$ ,  $Q(3, 7)$ , and  $R(7, 2)$ . Plot  $\triangle P'Q'R'$ , the image of  $\triangle PQR$  after a translation of  $(x, y) \rightarrow (x + 4, y - 3)$ . Is the transformation of  $\triangle PQR$  an example of rigid motion? Explain.

right 4 down 3

yes it is a rigid motion...  
size and shape are preserved!



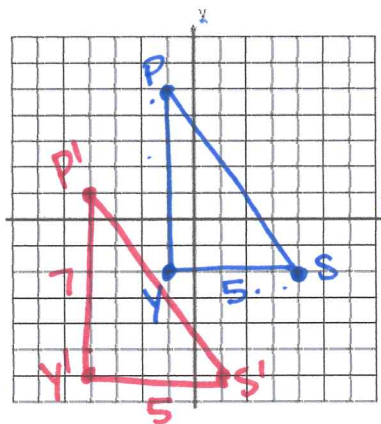
6. The vertices of triangle SPY are  $S(4, -2)$ ,  $P(-1, 5)$ , and  $Y(-1, -2)$ .

- a.) Find the area of triangle SPY in square units.

$A = \frac{1}{2}(5)(7) = 17.5 \text{ units}^2$

- b.) Translate the triangle 3 units to the left and 4 units down. Find the area of triangle  $S'P'Y'$  in square units.

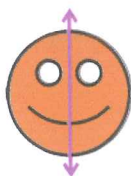
$A = \frac{1}{2}(5)(7) = 17.5 \text{ units}^2$



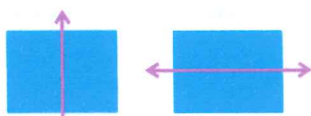
# Symmetry

a line drawn through the middle of a figure such that if the figure was folded on this line, it would map onto itself.

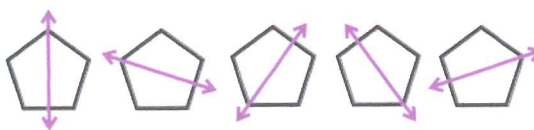
**Line Symmetry / Reflection Symmetry:** If there is a reflection for which the figure is its own image.



one line

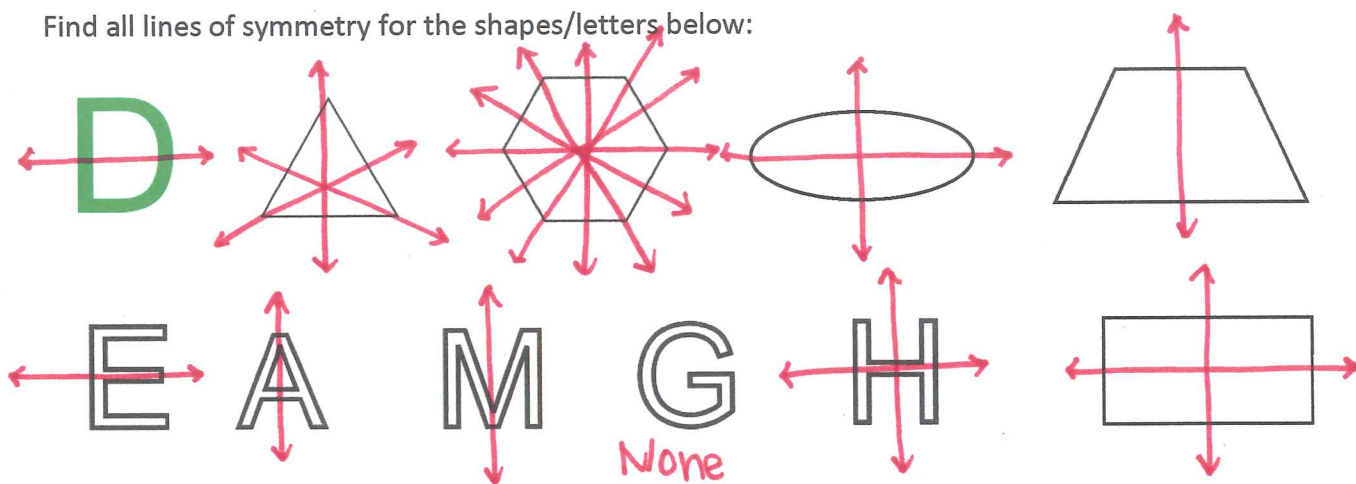


two lines



five lines

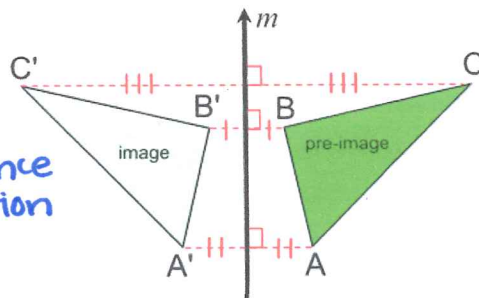
Find all lines of symmetry for the shapes/letters below:



# Reflection "FLIP"

Rule:  
x-axis  
(reflection over x-axis)

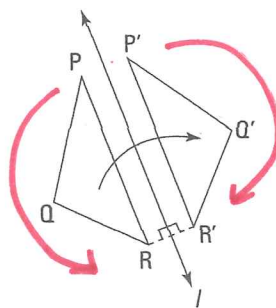
\* image is the same distance from the line of reflection as original figure.



When you reflect a figure across a line, each point of the figure maps to another point the same distance from the line, but on the other side.

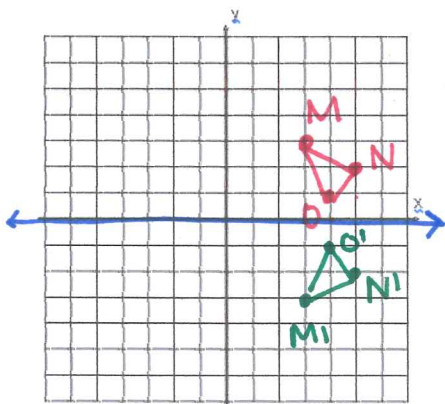
The orientation of the figure reverses.

(order of letters)



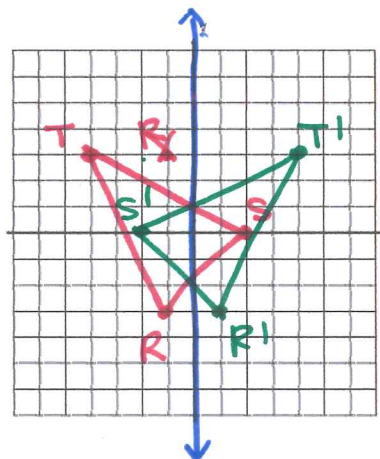
order changed!

1. Given points  $M(3, 3)$ ,  $N(5, 2)$ , and  $O(4, 1)$ , graph  $\triangle MNO$  and its reflection image after  $r_{x\text{-axis}}$ . State the coordinates of the image.



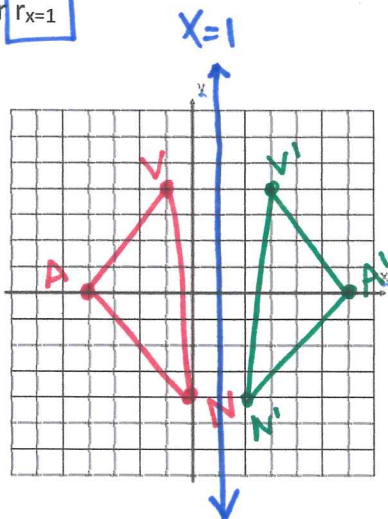
$M'(3, -3)$   
 $N'(5, -2)$   
 $O'(4, -1)$

2. Given points  $R(-1, -3)$ ,  $S(2, 0)$ , and  $T(-4, 3)$ , graph  $\triangle RST$  and its reflection after  $r_{y\text{-axis}}$ . State the coordinates of the image.

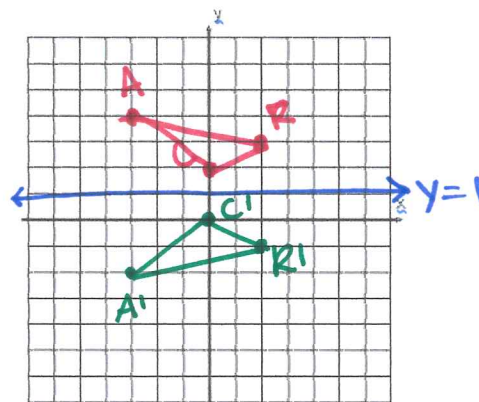


$R'(1, -3)$   
 $S'(-2, 0)$   
 $T'(4, 3)$

3. Given points  $V(-1, 4)$ ,  $A(-4, 0)$ , and  $N(0, -4)$ . Graph  $\triangle V'A'N'$  the image of  $\triangle VAN$  after  $r_{x=1}$



4. Given points  $C(0, 2)$ ,  $A(-3, 4)$ , and  $R(2, 3)$ . Graph  $\triangle C'A'R'$  the image of  $\triangle CAR$  after  $r_{y=1}$

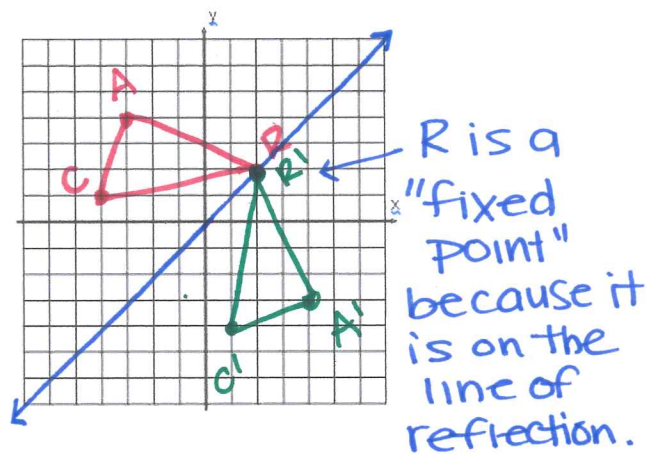
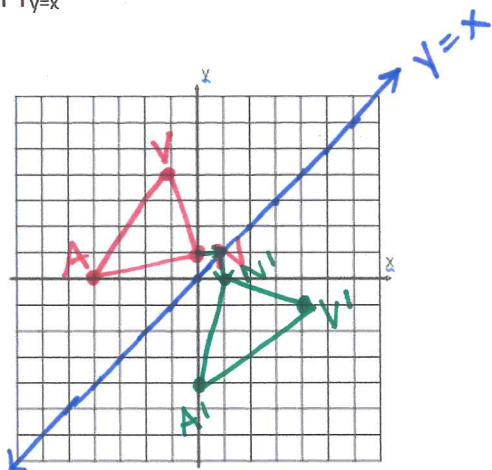


Is a reflection a RIGID MOTION?

YES! size and distance are preserved.

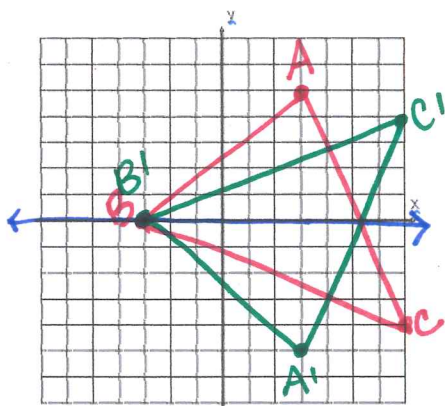
## What about non-horizontal vertical lines?

- Given points  $V(-1,4)$ ,  $A(-4, 0)$ , and  $N(0,1)$ . Graph  $\triangle V'A'N'$  the image of  $\triangle VAN$  after  $r_{y=x}$
- Given points  $C(-4,1)$ ,  $A(-3,4)$ , and  $R(2,2)$ . Graph  $\triangle C'A'R'$  the image of  $\triangle CAR$  after  $r_{y=x}$

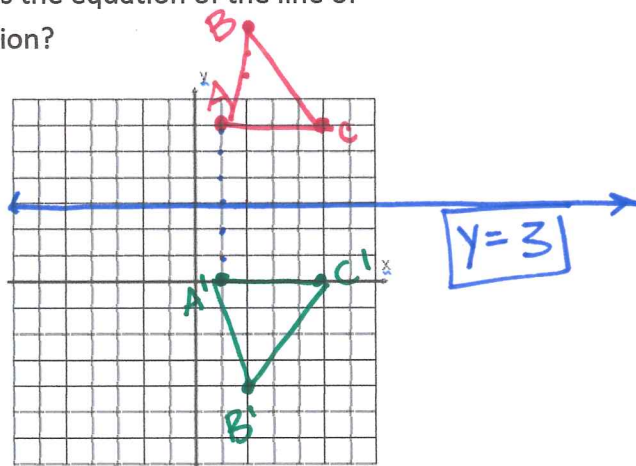


## Practice

- Given  $\triangle ABC$  with vertices  $A(3,5)$ ,  $B(-3,0)$  and  $C(7,-4)$ , graph  $\triangle A'B'C'$ , the image of  $\triangle ABC$  after a reflection in the  $x$ -axis. Which vertex remains fixed?
- $\triangle ABC$  with  $A(1,6)$ ,  $B(2,10)$ , and  $C(5,6)$  is reflected in a line to create image  $\triangle A'B'C'$  with  $A'(1,0)$ ,  $B'(2,-4)$  and  $C'(5,0)$ . What is the equation of the line of reflection?



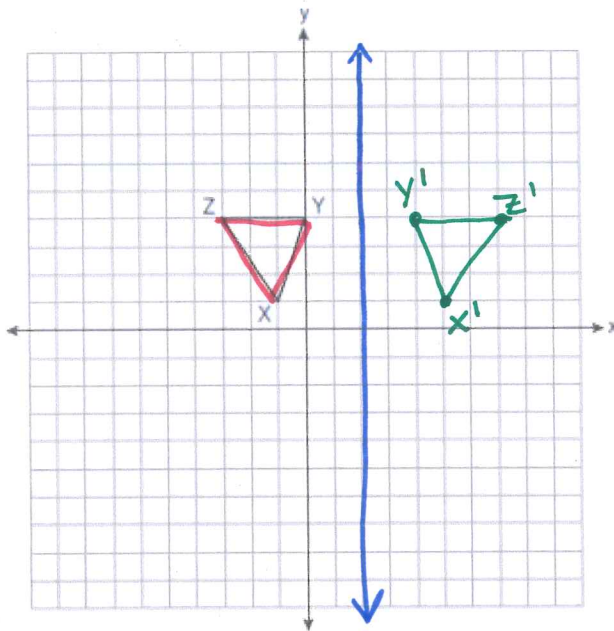
B remains "fixed"



Find the "middle"

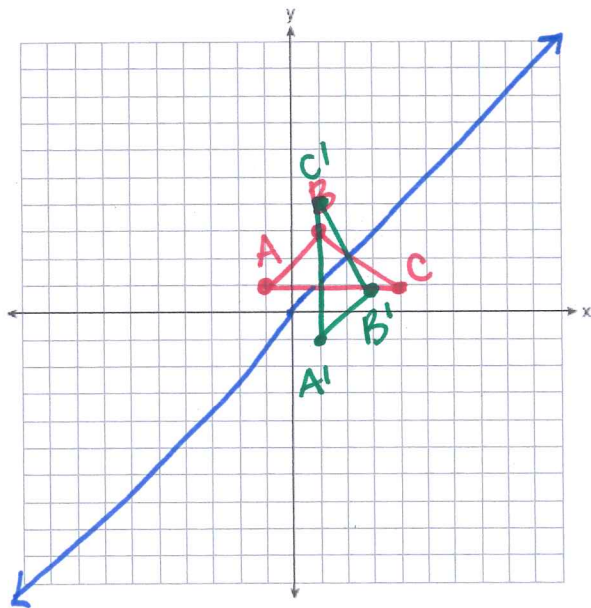


3. Triangle  $XYZ$ , shown in the diagram below, is reflected over the line  $x = 2$ . State the coordinates of  $\triangle X'Y'Z'$ , the image of  $\triangle XYZ$ .



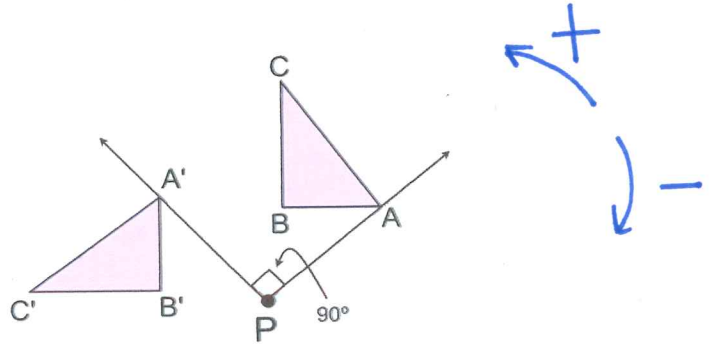
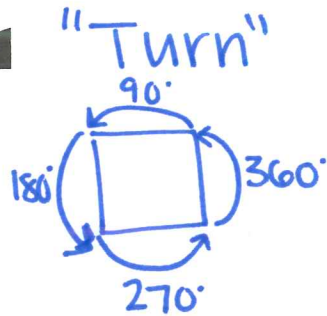
$X'(5, 1)$   
 $Y'(4, 4)$   
 $Z'(7, 4)$

4. Triangle  $ABC$  has vertices  $A(-1, 1)$ ,  $B(1, 3)$ , and  $C(4, 1)$ . The image of  $\triangle ABC$  after the transformation  $r_{y=-x}$  is  $\triangle A'B'C'$ . State and label the coordinates of  $\triangle A'B'C'$ . [The use of the set of axes below is optional.]



$A'(1, -1)$   
 $B'(3, 1)$   
 $C'(1, 4)$

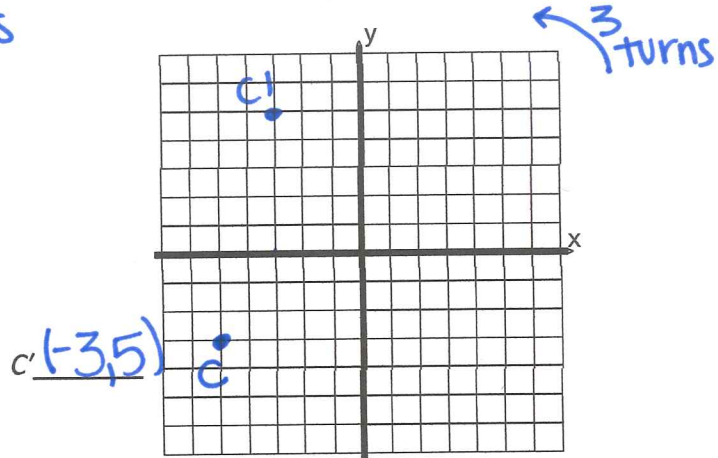
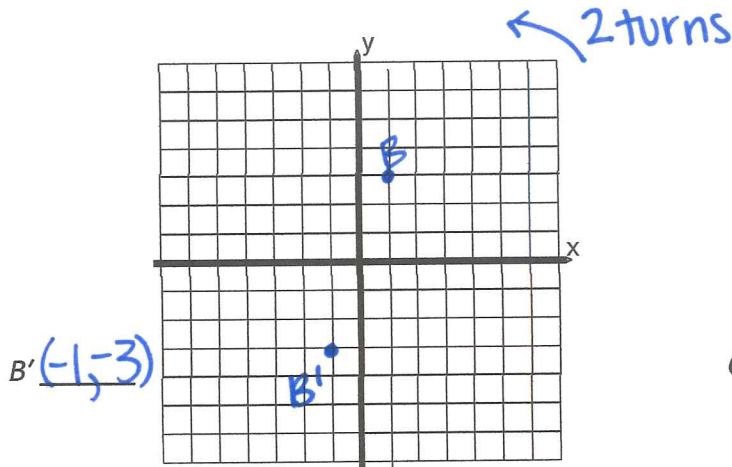
# Rotation



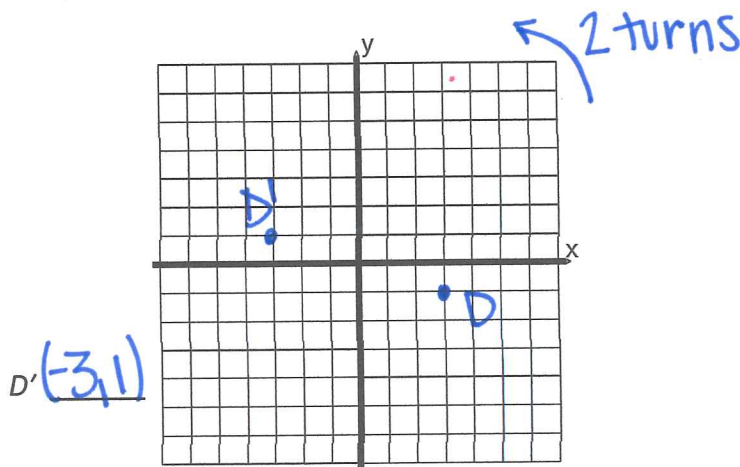
# Practice

Directions: Graph each pre-image, and new image, and state the appropriate coordinates.

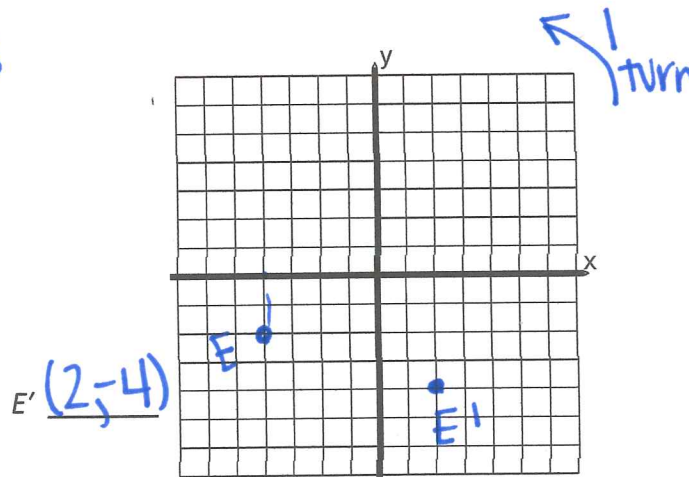
1. What are the coordinates of  $B(1, 3)$  after a rotation of  $180^\circ$  about the origin?
2. What are the coordinates of  $C(-5, -3)$  after a rotation of  $270^\circ$  about the origin?



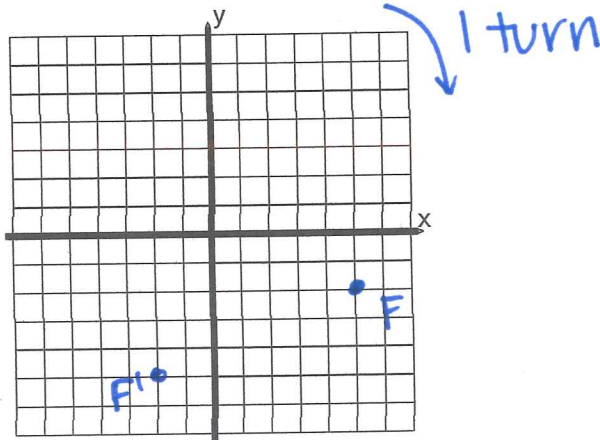
3. What are the coordinates of  $D(3, -1)$  after a rotation of  $180^\circ$  about the origin?



4. What are the coordinates of  $E(-4, -2)$  after  $R_{O, 90^\circ}$ ?

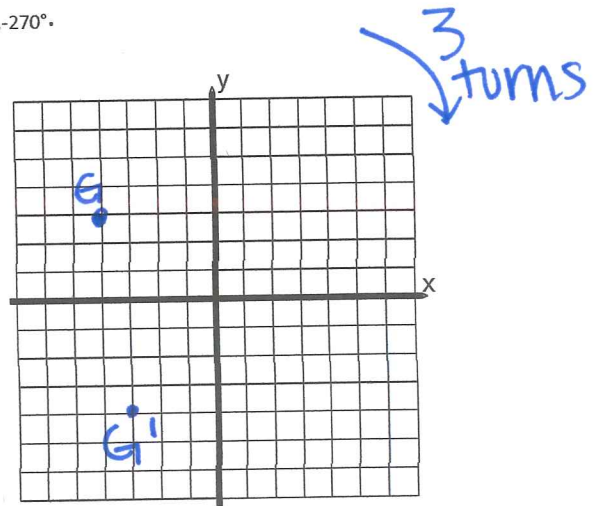


5. What are the coordinates of  $F'$  if  $F(5,-2)$  is rotated  $-90^\circ$  about the origin?



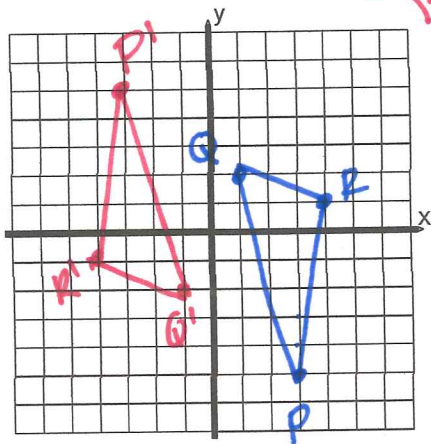
$F'(-2, -5)$

6. Find  $G'$ , the image of  $G(-4, 3)$  after  $R_{O, -270^\circ}$ .



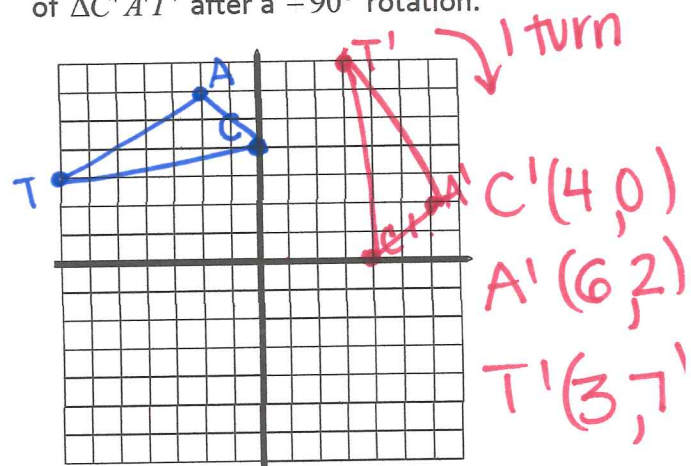
$G'(-3, -4)$

7.  $\triangle PQR$  has vertices  $P(3,-5)$ ,  $Q(1,2)$  and  $R(4,1)$ . State the coordinates of  $\triangle P'Q'R'$  after a  $180^\circ$  rotation.



$P'(-3, 5)$   
 $Q'(-1, 2)$   
 $R'(-4, 1)$

8.  $\triangle CAT$  has vertices  $C(0,4)$ ,  $A(-2,6)$  and  $T(-7,3)$ . State the coordinates of  $\triangle C'A'T'$  after a  $-90^\circ$  rotation.



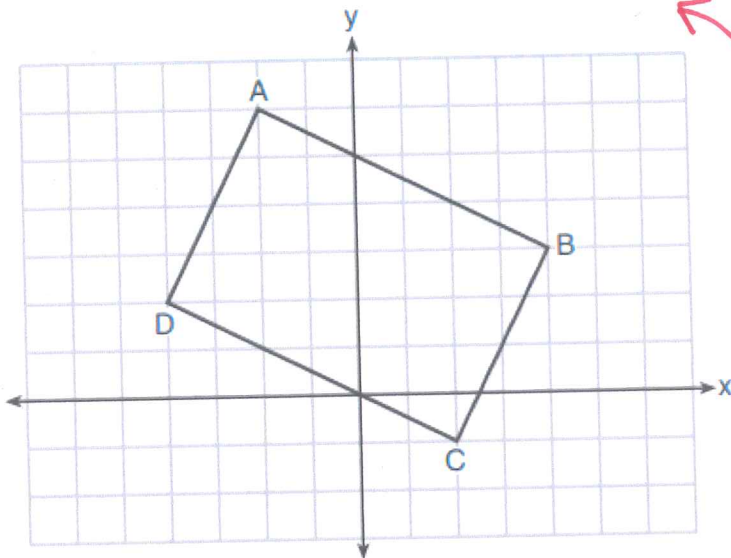
$C'(4, 0)$   
 $A'(6, 2)$   
 $T'(3, 7)$

Is a rotation RIGID MOTION?

YES! Size and shape are preserved!

# Practice

1. Quadrilateral  $ABCD$  is graphed on the set of axes below.



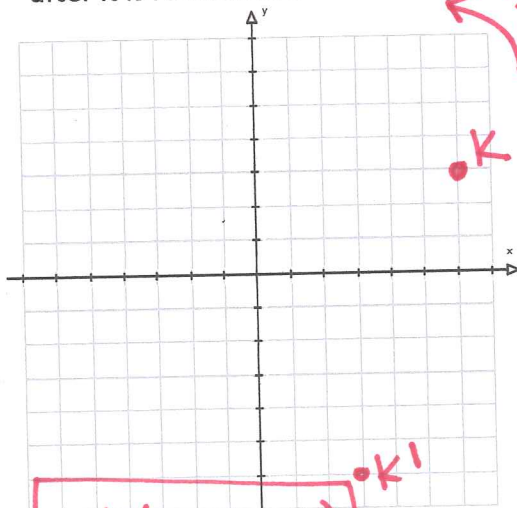
1 turn

$A'(6, -2)$   
 $B'(-3, 4)$   
 $C'(1, 2)$   
 $D'(-2, -4)$

When  $ABCD$  is rotated  $90^\circ$  in a counterclockwise direction about the origin, its image is quadrilateral  $A'B'C'D'$ . Is distance preserved under this rotation, and which coordinates are correct for the given vertex?

- ~~(1) no and  $C'(1, 2)$~~       ~~(3) yes and  $A'(6, 2)$~~   
~~(2) no and  $D'(2, 4)$~~       (4) yes and  $B'(-3, 4)$

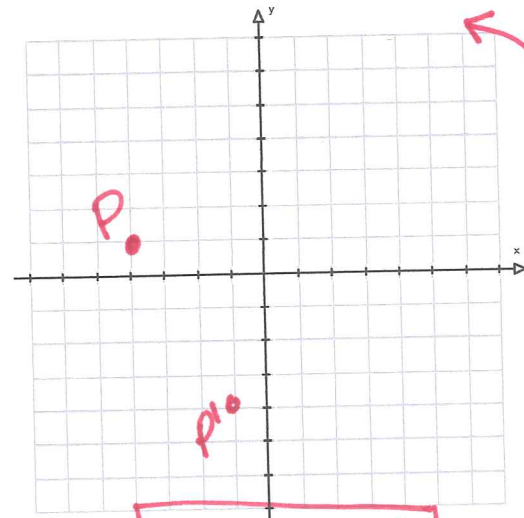
2. Find the image of the point  $K(6, 3)$  after it is rotated  $270^\circ$



3 turns

$K'(3, -6)$

3. Find the image of the point  $P(-4, 1)$  after it is rotated  $90^\circ$



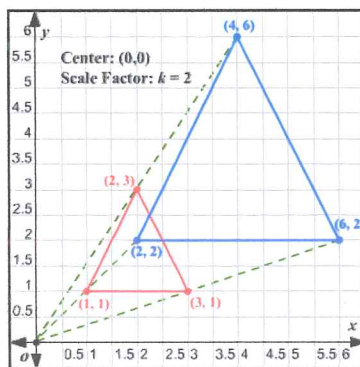
1 turn

$P'(-1, -4)$

# Dilation

"Enlargement" or a "shrink"

Rule:  $D_2$



\* if  $\# > 1 \rightarrow$  enlarge  
if  $\# < 1 \rightarrow$  shrink

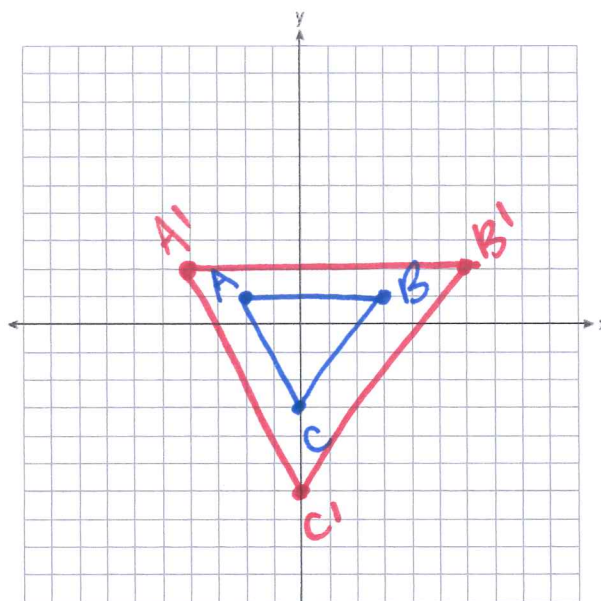
## Practice

- Triangle  $ABC$  has coordinates  $A(-2,1)$ ,  $B(3,1)$  and  $C(0,-3)$ . On the set of axes below, graph and label  $\triangle A'B'C'$ , the image of  $\triangle ABC$  after a dilation of 2 centered at the origin.

$$A(-2,1) \xrightarrow{D_2} A'(-4,2)$$

$$B(3,1) \rightarrow B'(6,2)$$

$$C(0,-3) \rightarrow C'(0,-6)$$



- The coordinates of the endpoints of  $\overline{AB}$  are  $A(2,3)$  and  $B(5,-1)$ . Determine the length of  $\overline{A'B'}$ , the image of  $\overline{AB}$  after a dilation of  $\frac{1}{2}$  centered at the origin.

$$A(2,3) \rightarrow A'(1, \frac{3}{2})$$

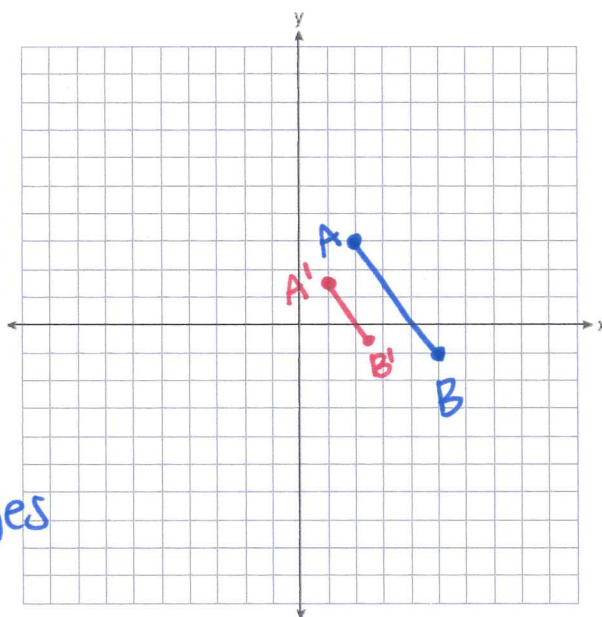
$$B(5,-1) \rightarrow B'(\frac{5}{2}, -\frac{1}{2})$$

Is a Dilation RIGID

MOTION?

NO!  
size changes

Dilations make similar figures!

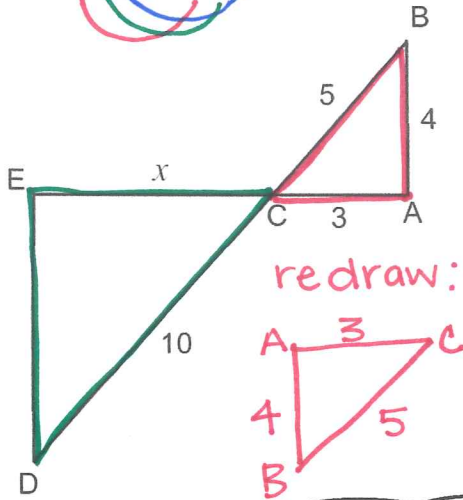


# Similar Triangles

- angle size is preserved
- sides are proportional
- $(\frac{a}{b} = \frac{c}{d} \rightarrow ad = bc)$

Each of the following are examples of similar triangles. For each example, solve for the given variable.  $\sim$  similar  $\cong$  congruent

1.  $\triangle ABC \sim \triangle EDC$

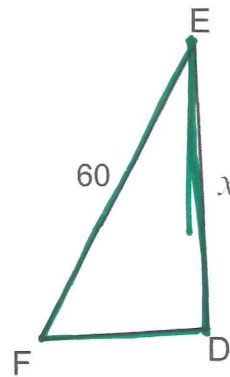


$$\frac{x}{3} = \frac{10}{5}$$

$$\frac{5x}{5} = \frac{30}{5}$$

$$x = 6$$

2.  $\triangle DEF \sim \triangle LMN$

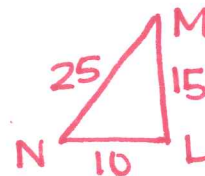


$$15 \frac{x}{25} = \frac{60}{25}$$

$$\frac{25x}{25} = \frac{900}{25}$$

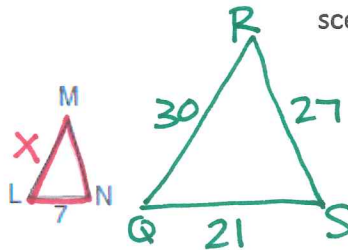
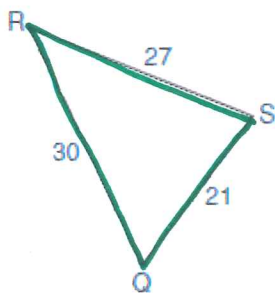
$$x = 36$$

re-draw



Re-draw, it helps.

3. In the accompanying diagram,  $\triangle QRS$  is similar to  $\triangle LMN$ . What is the length of  $\overline{ML}$ ?

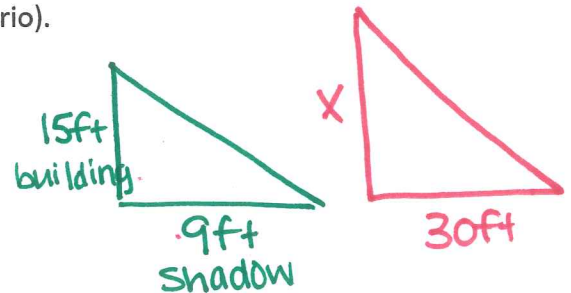


$$\frac{x}{30} = \frac{7}{21}$$

$$\frac{21x}{21} = \frac{210}{21}$$

$$x = 10$$

4. A 15 foot building casts a 9 foot shadow. How tall is the building that casts a 30 ft shadow at the same time? (Draw a picture of this scenario).



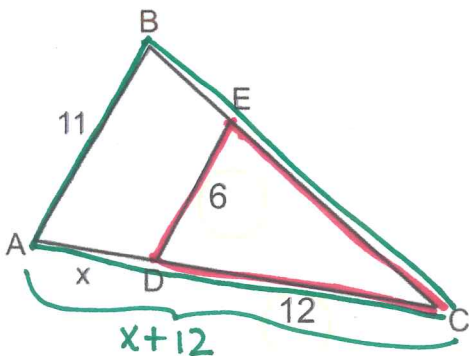
$$\frac{x}{15} = \frac{30}{9}$$

$$9x = 450$$

$$x = 50$$

# Triangles within triangles

1.  $\overline{AB} \parallel \overline{DE}$ . Find the value of  $x$ .



$$\frac{6}{11} = \frac{12}{x+12}$$

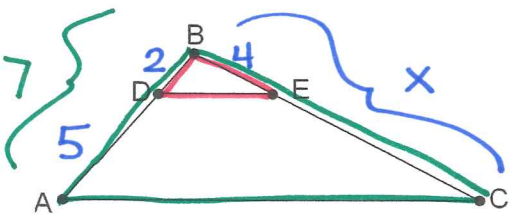
$$(11)(12) = 6(x+12)$$

$$132 = 6x + 72$$

$$\begin{array}{r} 132 \\ -72 \\ \hline 60 \end{array} = \frac{6x}{6}$$

$$60 = 6x \quad \boxed{x=10}$$

3. In the accompanying diagram of  $\triangle ABC$ ,  $\overline{DE} \parallel \overline{AC}$ ,  $BD = 2$ ,  $BE = 4$ , and  $DA = 5$ . Find the length of  $\overline{BC}$ .



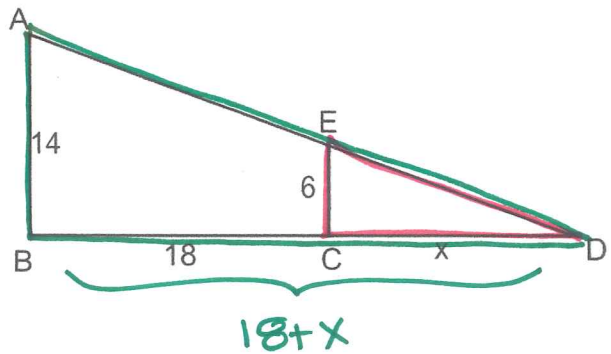
$$\frac{2}{7} = \frac{4}{x}$$

$$2x = 28$$

$$\frac{2x}{2} = \frac{28}{2}$$

$$\boxed{x=14}$$

2.  $\overline{AB} \parallel \overline{EC}$ . Solve for  $x$ .



$$\frac{6}{14} = \frac{x}{18+x}$$

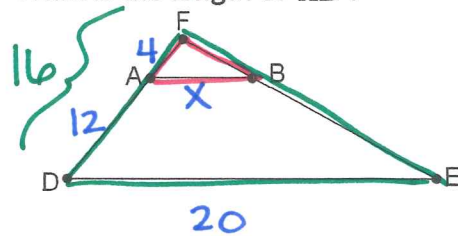
$$14x = 6(18+x)$$

$$14x = 108 + 6x$$

$$\begin{array}{r} 14x \\ -6x \\ \hline 8x \end{array} = \frac{108}{8}$$

$$8x = 108 \quad \boxed{x=13.5}$$

4. In the accompanying diagram of  $\triangle DEF$ ,  $\overline{AB} \parallel \overline{DE}$ ,  $AF = 4$ ,  $DA = 12$ , and  $DE = 20$ . What is the length of  $\overline{AB}$ ?



$$\frac{4}{16} = \frac{x}{20}$$

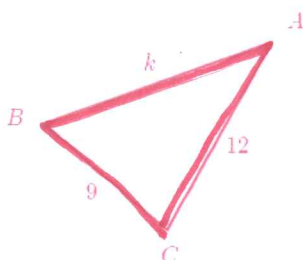
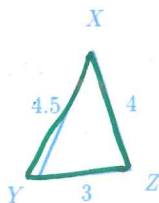
$$16x = 80$$

$$\frac{16x}{16} = \frac{80}{16}$$

$$\boxed{x=5}$$

## Practice

1. Find the value of  $k$  if  $\triangle XYZ \sim \triangle ABC$

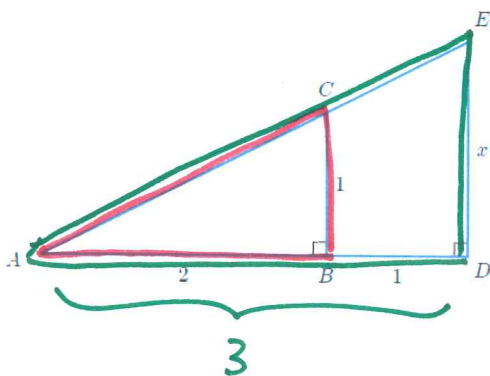


$$\frac{k}{4.5} = \frac{12}{4}$$

$$4k = 54$$

$$k = 13.5$$

2. Solve for  $x$ .



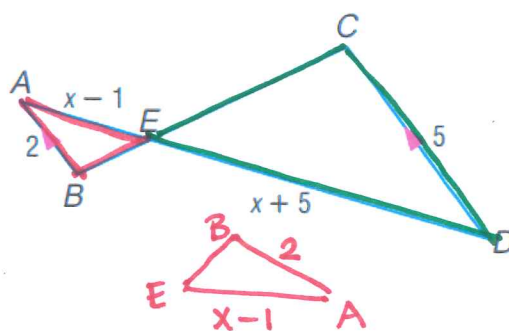
$$\frac{2}{3} = \frac{1}{x}$$

$$2x = 3$$

$$x = 1.5$$

3. Find the length of DE

$$DE = 5 + 5 = 10$$



$$\frac{2}{5} = \frac{x-1}{x+5}$$

$$2(x+5) = 5(x-1)$$

$$2x + 10 = 5x - 5$$

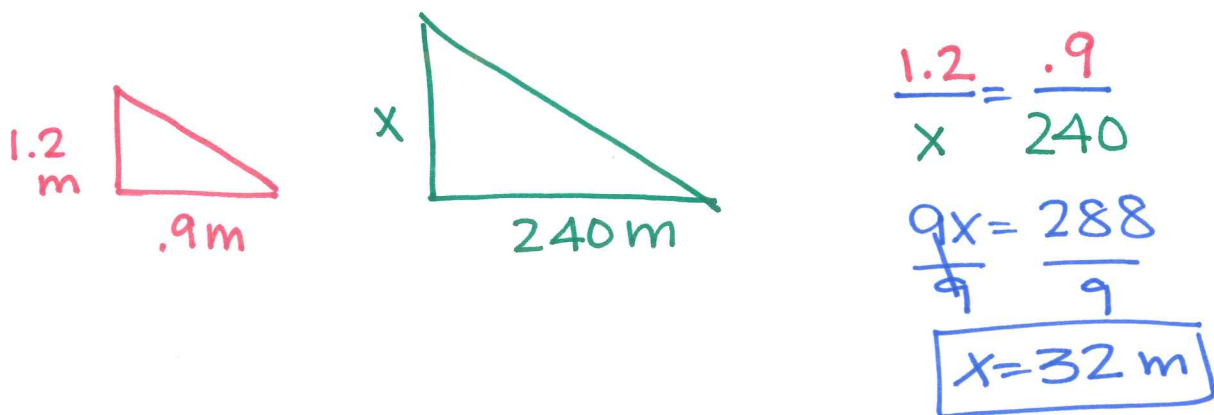
$$10 = 3x - 5$$

$$15 = 3x$$

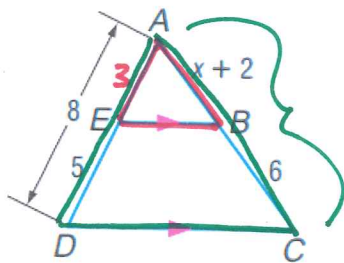
$$x = 5$$



4. Nina was curious about the height of the Eiffel Tower. She used a 1.2 meter model of the tower and measured its shadow at 2 p.m. The length of the shadow was 0.9 meter. She measured the Eiffel Tower's shadow, and it was 240 meters. What is the height of the Eiffel Tower?



5. Find the length of AC.



$$\frac{3}{8} = \frac{x+2}{x+2+6}$$



$$\frac{3}{8} = \frac{x+2}{x+8}$$

$$AC = (1.6 + 2) + 6 = \boxed{9.6}$$

$$8(x+2) = 3(x+8)$$

$$8x + 16 = 3x + 24$$

$$\begin{array}{r} -3x \\ \hline 5x + 16 = 24 \end{array}$$

$$\begin{array}{r} 5x + 16 = 24 \\ -16 \quad -16 \\ \hline 5x = 8 \end{array}$$

$$\begin{array}{r} 5x = 8 \\ \hline x = 1.6 \end{array}$$