

Kindergarten Math: Instructional Focus and Fluency

Transitioning to the NYS Next Generation Math Learning Standards for Grades K-8, Effective September 2022

Instructional Focus	Developmental Focus	Instructional Considerations (via Standards)
Develop a sound sense of numbers by representing and comparing numbers, initially using sets of objects.	<ul style="list-style-type: none"> ▶ Counting sets/using numbers to represent quantities, comparing sets ▶ Model simple addition and subtraction equations using “taking apart and putting together 	<p>NY-K.CC.2 Counting to 100 by ones beginning from any given number (instead of beginning at 1).</p> <p>NY-K.CC.4 Understand the relationship between numbers and quantities up to and including 20.</p> <p>NY-K.CC.6 Added language of “more than”, “fewer than” and “the same as”, which should be used when comparing objects without a quantity. “Greater than”, “less than”, and “equal to” should also be used when comparing numbers.</p>
Recognizing and describing shapes and using spatial relations.	<ul style="list-style-type: none"> ▶ Two and three dimensional shapes should include: <ul style="list-style-type: none"> ➢ squares, triangles, circles, rectangles, and hexagons ➢ cubes, cones, cylinders, and spheres 	<p>NY-K.G.3 Understand the difference between two-dimensional and three-dimensional shapes.</p> <p>NY-K.G.4 Students are analyzing, comparing and sorting two and three-dimensional shapes.</p> <p>NY-K.G.5 Students should be taught to model objects by building and/or drawing shapes.</p>

Fluency	Fluency development	Fluency Standard
Fluently add and subtract within 5.	▶ Students can find sums and differences within 5 reasonably quickly, and say or write it.	NY-K.OA.5 Fluently add and subtract within 5.

Note: Fluency involves a mixture of just knowing some answers (automaticity), knowing some answers from patterns, and knowing some answers from the use of strategies. Some students may still need to use fingers or make drawings (considered a developmental strategy).

Grade 1 Math: Instructional Focus and Fluency

Transitioning to the NYS Next Generation Math Learning Standards for Grades K-8, Effective September 2022

Instructional Focus	Developmental Focus	Instructional Consideration (via Standards)
Develop an understanding of addition, subtraction, and strategies for addition and subtraction within 20.	<ul style="list-style-type: none"> ▶ Model using unit cubes, adding/counting on, put together/take apart, making tens ▶ Understand the relationship between addition and subtraction <ul style="list-style-type: none"> ▶ knowing that $8 + 4 = 12$, one knows $12 - 8 = 4$ ▶ adding $6 + 7$ by creating the known equivalent $6 + 6 + 1 = 12 + 1 = 13$ (doubles plus 1) ▶ subtract $10 - 8$ by finding the number that makes 10 when added to 8 	<p>NY-1.OA.3 Apply properties of operations as strategies to add and subtract.</p> <p>NY-1.OA.4 Understand subtraction as an unknown-addend problem within 20.</p> <p>NY-1.OA.5 Relate counting to addition and subtraction.</p> <p>NY-1.OA.6a Add and subtract within 20. Use strategies such as: counting on, making ten, decomposing a number leading to a ten, using the relationship between addition and subtraction, creating equivalent but easier or known sums.</p>
Develop understanding of whole number relationships and place value, including grouping in tens and ones.	<ul style="list-style-type: none"> ▶ Comparing whole numbers (to 100) ▶ Strategies to add within 100 and subtract multiples of 10 ▶ Place value - tens and ones 	<p>NY-1.NBT.3 Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols $>$, $=$, and $<$.</p> <p>NY-1.NBT.4 Add within 100, including</p> <ul style="list-style-type: none"> • a two-digit number and a one-digit number • a two-digit number and a multiple of 10. <p>NY-1.NBT.6 Subtract multiples of 10 from multiples of 10 in the range 10-90 using</p> <ul style="list-style-type: none"> • concrete models or drawings • strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. <p>Relate the strategy used to a written representation and explain the reasoning.</p>

Grade 1 Math: Instructional Focus and Fluency

Instructional Focus	Developmental Focus	Instructional Consideration (via Standards)
Develop understanding of linear measurement and measuring lengths as iterating length units.	<ul style="list-style-type: none"> ▶ Measure an object using unit cubes/compare object lengths 	<p>NY-1.MD.1 Order three objects by length; compare the lengths of two objects indirectly by using a third object.</p> <p>NY-1.MD.2 Order three objects by length; compare the lengths of two objects indirectly by using a third object.</p>
Describe and analyze polygons based on the number of sides and vertices.	<ul style="list-style-type: none"> ▶ Classify polygons by examining sides and vertices ▶ Relate fraction work to geometry (i.e. area of part of a shape as a unit fraction of the whole) 	<p>NY-3.G.1 Recognize and classify polygons based on the number of sides and vertices (triangles, quadrilaterals, pentagons, and hexagons). Identify shapes that do not belong to one of the given subcategories.</p> <p>NY-3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.</p>

Fluency	Fluency development	Fluency Standard
Fluently add and subtract within 10.	<ul style="list-style-type: none"> ▶ Students can find sums and differences within 10 reasonably quickly, and say or write it. This involves a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies. 	<p>NY-1.OA.6b Fluently add and subtract within 10.</p>

Note: Fluency involves a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies. Reaching fluency and knowing from memory (automaticity) of all single digit multiplication will take much of the year for many students. Fluently (procedural) adding and subtracting within 1,000 uses strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Grade 2 Math: Instructional Focus and Fluency

Transitioning to the NYS Next Generation Math Learning Standards for Grades K-8, Effective September 2022

Instructional Focus	Developmental Focus	Instructional Consideration (via Standards)
Extend understanding of base-ten notation.	<ul style="list-style-type: none"> ▶ Counting in fives, tens, and multiples of hundreds, tens, and ones ▶ Place value understanding up to 1000 <i>use base-ten numerals, number names, and expanded form</i> 	<p>NY-2.NBT.2 Count within 1000; skip-count by 5s, 10s, and 100s</p> <p>NY-2.NBT.1 Understand that the digits of a three-digit number represent amounts of hundreds, tens, and ones.</p> <p>a. Understand 100 can be thought of as a bundle of ten tens, called a “hundred.”</p> <p>b. Understand the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).</p>
Build fluency with addition and subtraction.	<ul style="list-style-type: none"> ▶ Add and subtract within 100 ▶ Solve problems within 1000 using models (e.g. place value chart) 	<p>NY-2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.</p> <p>NY-2.NBT.6 Add up to four two-digit numbers using strategies based on place value and properties of operations.</p> <p>NY-2.NBT.7a Add and subtract within 1000, using</p> <ul style="list-style-type: none"> • concrete models or drawings • strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. <p>Relate the strategy to a written representation.</p>

Instructional Focus	Developmental Focus	Instructional Consideration (via Standards)
Use standard units of measure.	<ul style="list-style-type: none"> ▶ Use rulers in cm and inches for linear measurement 	<p>NY-2.MD.1 Measure the length of an object to the nearest whole by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.</p> <p>NY-2.MD.4 Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard “length unit”.</p>
Analyze and classify two-dimensional shapes as polygons or non-polygons.	<ul style="list-style-type: none"> ▶ Classify polygons(non) ▶ Decompose/compose shapes, partition shapes(basis for fraction understanding) 	<p>NY-2.G.1 Classify two-dimensional figures as polygons or non-polygons.</p> <p>NY-2.G.2 Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.</p> <p>NY-2.G.1 Partition circles and rectangles into two, three, or four equal shares. Describe the shares using the words halves, thirds, half of, a third of, etc. Describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.</p>

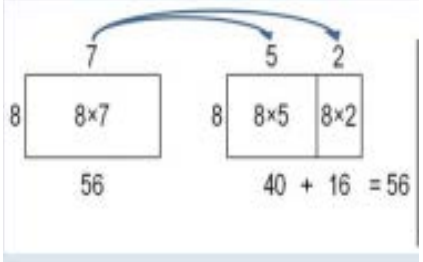
Fluency	Fluency development	Fluency Standard
Fluently add and subtract within 20 using mental strategies.	<ul style="list-style-type: none"> ▶ Students can find sums and differences within 20 reasonably quickly, and say or write it. This involves a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies. 	<p>NY-2.OA.2a Fluently add and subtract within 20 using mental strategies. Strategies could include: counting on, making ten, decomposing a number leading to a ten, using the relationship between addition and subtraction, creating equivalent but easier or known sums.</p>

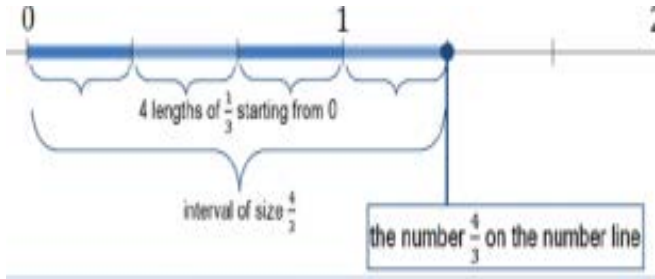
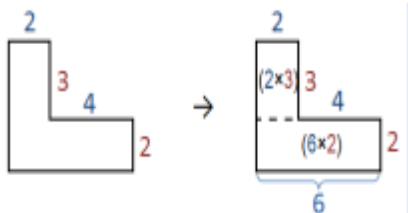
Fluency	Fluency development	Fluency Standard
Know from memory all sums within 20 of two one-digit numbers.	<ul style="list-style-type: none"> ► Fluency from memory means students are fast, accurate, flexible, and have understanding. They use strategies efficiently. 	<p>NY-2.OA.2b Know from memory all sums within 20 of two one-digit numbers.</p>
Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.	<ul style="list-style-type: none"> ► Students should be taught to use strategies based on place value, properties of operations, and the relationship between addition and subtraction; however, when solving any problem, students can choose any strategy. 	<p>NY-2.NBT.5 Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.</p>

Note: Reaching fluency with standards NY-2.OA.2a and NY-2.OA.2b will take students much of the year.

Grade 3 Math: Instructional Focus and Fluency

Transitioning to the NYS Next Generation Math Learning Standards for Grades K-8, Effective September 2022

Instructional Focus	Developmental Focus	Instructional Consideration (via Standards)
Develop understanding of multiplication and division and strategies for multiplication and division within 100 (whole numbers).	<p>► Equal-sized groups, arrays, and area models multiplication is finding an unknown product, and division is finding an unknown factor</p> <p>► If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication)</p> 	<p>NY-3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities.</p> <p>NY-3.OA.5 Apply properties of operations as strategies to multiply and divide.</p> <p>NY-3.OA.8 Solve two-step word problems posed with whole numbers and having whole-number answers using the four operations.</p>
Develop understanding of fractions, especially unit fractions (fractions with numerator 1). <i>Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.</i>	<p>► Use fractions along with visual fraction models to represent parts of a whole</p> <p>► Represent numbers equal to, less than, and greater than one</p>	<p>NY-3.NF.1 Understand a unit fraction, $\frac{1}{b}$, is the quantity formed by 1 part when a whole is partitioned into b equal parts. Understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.</p>

<p><i>Develop understanding of fractions continued...</i></p>	<p>➤ Compare fractions-visual fraction models/strategies based on equal numerators or denominators</p>	<p>NY-3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line.</p>  <p>NY-3.NF.3 Explain equivalence of fractions and compare fractions by reasoning about their size.</p>
<p>Develop understanding of the structure of rectangular arrays and of area.</p>	<p>➤ Find the total number of same-size units of area required to cover the shape</p> <p>➤ Decompose rectangles into rectangular arrays of squares, connecting area to multiplication</p>	<p>NY-3.MD.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.</p> <p>NY-3.MD.6 Measure areas by counting unit squares.</p> <p>NY-3.MD.7 Relate area to the operations of multiplication and addition.</p> 

<p>Describe and analyze polygons based on the number of sides and vertices.</p>	<p>➤ <i>(POST-TEST)</i> Classify polygons by examining sides and vertices</p> <p>➤ Relate fraction work to geometry (i.e. area of part of a shape as a unit fraction of the whole)</p>	<p>NY-3.G.1 <i>(POST-TEST)</i> Recognize and classify polygons based on the number of sides and vertices (triangles, quadrilaterals, pentagons, and hexagons). Identify shapes that do not belong to one of the given subcategories.</p> <p>NY-3.G.2 Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole.</p>
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Fluency	Fluency development	Fluency Standard
<p>Fluently solve single-digit multiplication and related divisions.</p>	<p>➤ Students use strategies such as the relationship between multiplication and division or properties of operations. <i>(procedural)</i></p> <p>➤ Example: use knowledge of $6 \times 7 = 42$ to understand $42 \div 6 = 7$</p>	<p>NY-3.OA.7a Fluently solve single-digit multiplication and related divisions, using strategies such as the relationship between multiplication and division or properties of operations.</p>
<p>Know from memory all products of two one-digit numbers.</p>	<p>➤ By the end of grade 3, students have sufficient experience with these strategies to know from memory (automaticity) all single-digit products.</p>	<p>NY-3.OA.7b Know from memory all products of two one-digit numbers.</p>
<p>Fluently <i>(procedural)</i> add and subtract within 1,000.</p>	<p>➤ Students use strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. A range of algorithms may be used. <i>(procedural)</i></p>	<p>NY-3.NBT.2 Fluently add and subtract within 1,000.</p>

Note: Fluency involves a mixture of just knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies. Reaching fluency and knowing from memory (automaticity) of all single digit multiplication will take much of the year for many students. Fluently (procedural) adding and subtracting within 1,000 uses strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

(POST-TEST)* refers to standards content that is taught **after the NYS grade 3-8 assessment. This time typically occurs late April - June.

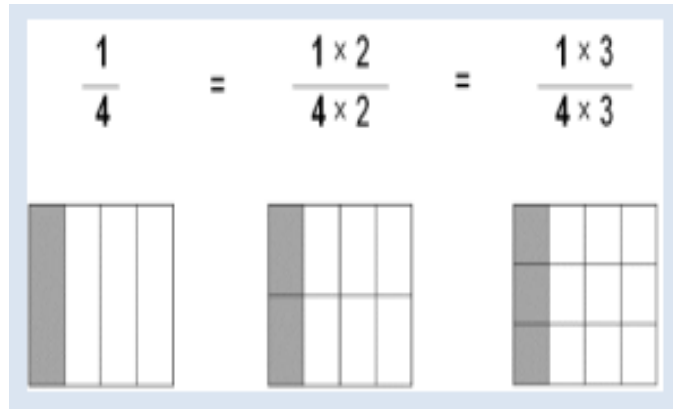
Grade 4 Math: Instructional Focus and Fluency

Transitioning to the NYS Next Generation Math Learning Standards for Grades K-8, Effective September 2022

Instructional Focus	Developmental Focus	Instructional Consideration (via Standards)
Develop <u>understanding and fluency</u> with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends.	<ul style="list-style-type: none"> ▶ Develop fluency with efficient procedures for multiplying whole numbers in problem solving ▶ Generalize methods to compute products of multi-digit whole numbers using models (equal-sized groups, arrays, area models, place value, and properties of operations) ▶ Generalize procedures to find quotients involving multi-digit dividends (place value, properties of operations, the relationship of division to multiplication) <i>Students should be taught to use equations, rectangular arrays, and area models; however, when illustrating and explaining any calculation, students can choose any strategy.</i> 	<p>NY-4.NBT.1 Interpret a multiplication equation as a comparison. Represent verbal statements of multiplicative comparisons as multiplication equations.</p> <p>NY-4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p> <p>NY-4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p>

Develop an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers. (Grade 4 expectations are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100).

► Develop methods for generating and recognizing equivalent fractions



► Compose fractions from unit fractions, decompose fractions into unit fractions

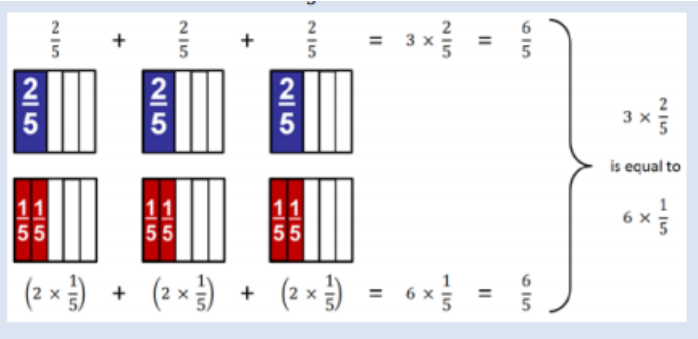
$$\frac{3}{8} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$$

$$\frac{3}{8} = \frac{1}{8} + \frac{2}{8}$$

$$2\frac{1}{8} = 1 + 1 + \frac{1}{8} = \frac{8}{8} + \frac{8}{8} + \frac{1}{8}$$

NY-4.NF.1 Explain why a fraction $\frac{a}{b}$ is equivalent to a fraction $\frac{a \times n}{b \times n}$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

NY-4.NF.3b Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions.

<p>Develop an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers continued...</p>	<p>► Multiply a fraction by a whole number n groups of a fraction (where n is a whole number), e.g., 4 groups of $1/3$; which lends itself to being thought about as repeated addition</p> 	<p>NY-4.NF.4 Apply and extend previous understandings of multiplication to multiply a whole number by a fraction.</p>
<p>Understand that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.</p>	<p>► Know properties of two-dimensional shapes (angles, parallelism, and symmetry)</p> <p><i>Students will need to focus on verbal, written and pictorial representations of vocabulary.</i></p> <p>http://p1232.nysed.gov/assessment/nysaa/2011-12/manual-12/framework/math-glossary.pdf</p> <p>https://www.graniteschools.org/mathvocabulary/vocabulary-cards/ (available in English, Spanish, Chinese, and French)</p>	<p>NY-4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.</p> <p>NY-4.MD.2a (POST-TEST)* Identify and name triangles based on angle size (right, obtuse, acute).</p> <p>NY-4.MD.2b (POST-TEST)* Identify and name all quadrilaterals with 2 pairs of parallel sides as parallelograms.</p> <p>NY-4.MD.2c (POST-TEST)* Identify and name all quadrilaterals with four right angles as rectangles.</p>

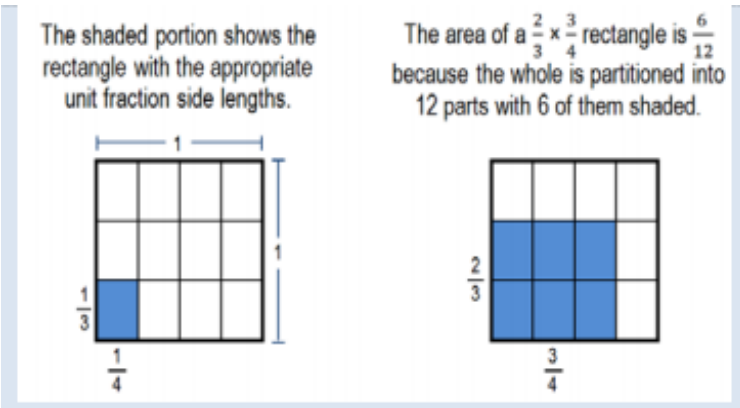
<p><i>Understand that geometric figures can be analyzed and classified based on their properties continued...</i></p>		<p>NY-4.MD.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.</p>
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Fluency	Fluency development	Fluency Standard
<p>Fluently add and subtract multi-digit whole numbers using a standard algorithm (<i>procedural fluency</i>).</p>	<p>► Developing fluency requires understanding why and how a procedure works. Understanding makes learning procedures easier, less susceptible to common errors, less prone to forgetting, and easier to apply in new situations.</p>	<p>NY-4.NBT.4 Fluently add and subtract multi-digit whole numbers using a standard algorithm</p>

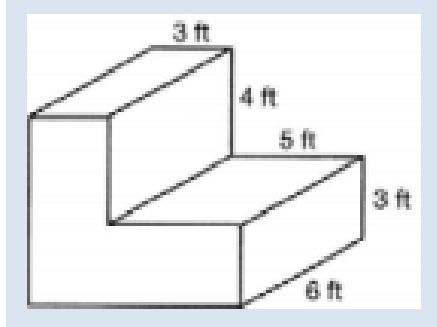
(POST-TEST) refers to standards content that is taught **after the NYS grade 3-8 assessment. This time typically occurs late April - June.*

Grade 5 Math: Instructional Focus and Fluency

Transitioning to the NYS Next Generation Math Learning Standards for Grades K-8, Effective September 2022

Instructional Focus	Developmental Focus	Instructional Consideration (via Standards)
<p>Develop <i>fluency</i> (<i>procedural</i>) with addition and subtraction of fractions; develop understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions).</p>	<ul style="list-style-type: none"> ▶ Represent addition and subtraction of fractions w/unlike denominators as equivalent calculations with like denominators <ul style="list-style-type: none"> ➢ include word problems ▶ Understand and explain why the procedures for multiplying and dividing fractions make sense <div style="text-align: center;">  </div> <p>Use the relationship between multiplication and division to explain that $\frac{1}{3} \div 4 = \frac{1}{12}$ because $\frac{1}{12} \times 4 = \frac{1}{3}$ and $4 \div \frac{1}{5} = 20$ because $20 \times \frac{1}{5} = 4$.</p> <ul style="list-style-type: none"> ➢ include word problems 	<p>NY-5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators.</p> <p>NY-5.NF.5 Interpret a fraction as division of the numerator by the denominator (includes word problems).</p> <p>NY-5.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number or a fraction.</p> <p>NY-5.NF.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Division of a fraction by a fraction is not a requirement until grade 6 (NY-6.NS.1)).</p>

<p>Extend division to 2-digit divisors, integrating decimals into the place value system and develop understanding of operations with decimals to hundredths, develop <i>fluency (procedural)</i> with whole number and decimal operations.</p>	<ul style="list-style-type: none"> ▶ Develop <i>fluency</i> with decimal computations to hundredths <ul style="list-style-type: none"> ➢ when solving any problem, students can choose to use a concrete model or a drawing. Their strategy must be based on place value, properties of operations, or the relationship between operations (this includes word problems) ➢ include word problems ▶ Understand and explain why the procedures for multiplying and dividing finite decimals make sense <ul style="list-style-type: none"> ➢ division problems are limited to those that allow for the use of concrete models or drawings, strategies based on properties of operations, and/or the relationship between operations (e.g., $0.25 \div 0.05$); problems should not be so complex as to require the use of an algorithm (e.g., $0.37 \div 0.05$) 	<p>NY-5.NF.7 Using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between operations:</p> <ul style="list-style-type: none"> • add and subtract decimals to hundredths; • multiply and divide decimals to hundredths. <p>Relate the strategy to a written method and explain the reasoning used.</p>
<p>Develop understanding of volume.</p>	<ul style="list-style-type: none"> ▶ Recognize volume as an attribute of three-dimensional space ▶ Understand that volume can be measured by finding the total number of same-size units of volume required to fill the space ▶ Know a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume 	<p>NY-5.MD.3 Recognize volume as an attribute of solid figures and understand concepts of volume measurement.</p> <p>NY-5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in., cubic ft., and improvised units.</p>

<p><i>Develop understanding of volume continued...</i></p>	<p>► Find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes in problem solving</p> <div data-bbox="688 354 1121 678" style="text-align: center;">  </div> <p>► apply the formulas $V = l \times w \times h$ and $V = B \times h$ in the context of solving real world and mathematical problems.</p>	<p>NY-5.MD.5 Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.</p> <p>NY-5.MD.2c Identify and name all quadrilaterals with four right angles as rectangles.</p> <p>NY-5.MD.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.</p>
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Fluency	Fluency development	Fluency Standard
<p>Fluently (<i>procedural</i>) multiply multi-digit whole numbers using a standard algorithm.</p>	<p>► Understand how and why an algorithm works</p> <p>► Students will need practice on selected problems to establish procedural fluency.</p>	<p>NY-5.NBT.5 Fluently multiply multi-digit whole numbers using a standard algorithm.</p>

Grade 6 Math: Instructional Focus and Fluency

Transitioning to the NYS Next Generation Math Learning Standards for Grades K-8, Effective September 2022

Instructional Focus	Developmental Focus	Instructional Consideration (via Standards)
Connect ratio and rate to whole number multiplication and division and use concepts of ratio and rate to solve problems.	<ul style="list-style-type: none"> ▶ Understand equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table ▶ Analyze simple drawings indicating the relative size of quantities ▶ Solve real and mathematical problems including: <ul style="list-style-type: none"> ➢ unit rate problems ➢ percent of a quantity as a rate per 100 ➢ finding the whole given a part and the percent ➢ finding a part of a whole given the percent ➢ convert of units within a given measurement system 	<p>NY-6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.</p> <p>NY-6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems.</p>
Complete understanding of division of fractions and extend the notion of number to the system of rational numbers, which includes negative numbers.	<ul style="list-style-type: none"> ▶ Explain why the procedures for dividing fractions make sense <ul style="list-style-type: none"> ➢ Include word problems ▶ Extend previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers/negative integers 	<p>NY-6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions.</p> <p>NY-6.NS.5 Understand that positive and negative numbers are used together to describe quantities having opposite directions or values. Use positive and negative numbers to represent quantities in real world contexts, explaining the meaning of 0 in each situation.</p>

Instructional Focus	Developmental Focus	Instructional Consideration (via Standards)
<p><i>Complete understanding of division of fractions and extend the notion of number to the system of rational numbers continued...</i></p>	<ul style="list-style-type: none"> ▶ Write, interpret, explain order of rational numbers in real world contexts <ul style="list-style-type: none"> ➤ use inequalities to determine position of rational numbers ➤ understand absolute value as the distance from zero ▶ Identify/plot the location of points on a coordinate plane 	<p>NY-6.NS.7 Understand ordering and absolute value of rational numbers.</p> <p>NY-6.NS.6 Understand a rational number as a point on the number line. Use number lines and coordinate axes to represent points on a number line and in the coordinate plane with negative number coordinates.</p>
<p>Write, interpret, and use expressions and equations.</p>	<ul style="list-style-type: none"> ▶ Use variables to represent an unknown <ul style="list-style-type: none"> ➤ identify the parts of an expression: coefficient, sum, difference, product, factor, quotient ▶ Use the properties of operations to rewrite and evaluate expressions in equivalent forms 	<p>NY-6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.</p> <p>NY-6.EE.3 Apply the properties of operations to generate equivalent expressions.</p> <p>NY-6.EE.4 Identify when two expressions are equivalent.</p>
<p><i>Write, interpret, and use expressions and equations continued...</i></p>	<ul style="list-style-type: none"> ▶ Solve simple one-step equations using properties of equality <div data-bbox="562 959 1157 1414" style="border: 1px solid #ccc; padding: 10px; margin: 10px 0;"> <p style="text-align: center;">Analogous arithmetical and algebraic solutions</p> <p>J. bought three packs of balloons. He opened them and counted 12 balloons. How many balloons are in a pack?</p> <p><i>Arithmetical solution</i> If three packs have twelve balloons, then one pack has $12 \div 3 = 4$ balloons.</p> <p><i>Algebraic solution</i> Defining the variable: Let b be the number of balloons in a pack. Writing the equation: $3b = 12$ Solving (mirrors the reasoning of the numerical solution): $3b = 12 \rightarrow \frac{3b}{3} = \frac{12}{3}$ $b = 4.$</p> </div>	<p>NY-6.EE.6 Use variables to represent numbers and write expressions when solving a real-world or mathematical problem. Understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.</p> <p>NY-6.EE.7 Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$; $x - p = q$; $px = q$; and $\frac{x}{p} = q$ for cases in which p, q, and x are all nonnegative rational numbers.</p>

Instructional Focus	Developmental Focus	Instructional Consideration (via Standards)
<p>Deepen understanding of area, surface area and volume.</p>	<ul style="list-style-type: none"> ▶ Use fractional side lengths for the volume of a right rectangular prism <ul style="list-style-type: none"> ➢ use area models to understand perfect squares and volume models to understand perfect cubes ▶ Find areas of polygons, surface areas of prisms 	<p>NY-6.G.2 Find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.</p> <p>NY-6.G.1 Find the area of triangles, trapezoids, and other polygons by composing into rectangles or decomposing into triangles and quadrilaterals. Apply these techniques in the context of solving real-world and mathematical problems.</p>
<p>Develop understanding of simple probabilities and statistical thinking.</p>	<ul style="list-style-type: none"> ▶ Describe and summarize numerical data sets - identifying clusters, peaks, gaps, and symmetry <ul style="list-style-type: none"> ➢ measures of center are mean, median, and mode. The measure of variation is the range. ▶ <i>(POST-TEST)*</i> Understand the probability of a chance event and develop probability models for simple events <i>Example: The probability of rolling a six-sided fair number cube and landing on a 2 is $\frac{1}{6}$. The probability of landing on an even number is $\frac{3}{6}$.</i> 	<p>NY-6.SP.2 Understand that a set of quantitative data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.</p> <p>NY-6.SP.3 Recognize that a measure of center for a quantitative data set summarizes all of its values with a single number while a measure of variation describes how its values vary with a single number.</p> <p>NY-6.SP.6 <i>(POST-TEST)*</i> Understand that the probability of a chance event is a number between 0 and 1 inclusive, that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $\frac{1}{2}$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.</p>

Fluency	Fluency development	Fluency Standard
Fluently (<i>procedural</i>) divide multi-digit numbers using a standard algorithm.	<ul style="list-style-type: none"> ▶ Understand how and why an algorithm works ▶ Students will need practice on selected problems to establish procedural fluency. 	NY-6.NS.2 Fluently divide multi-digit whole numbers using a standard algorithm.
Fluently (<i>procedural</i>) add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.	<ul style="list-style-type: none"> ▶ Students are fluent in the steps involved in the algorithm and why the algorithm works. 	NY-6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using a standard algorithm for each operation.

(POST-TEST) refers to standards content that is taught **after the NYS grade 3-8 assessment. This time typically occurs late April - June.*

Grade 7 Math: Instructional Focus and Fluency

Transitioning to the NYS Next Generation Math Learning Standards for Grades K-8, Effective September 2022

Instructional Focus	Developmental Focus	Instructional Consideration (via Standards)
<p>Develop understanding of and applying proportional relationships develop understanding of proportionality to solve single- and multi-step problems.</p>	<ul style="list-style-type: none"> ▶ Use ratios and proportionality to solve a wide variety of percent problems <ul style="list-style-type: none"> ➢ simple interest ➢ tax ➢ markups and markdowns ➢ gratuities and commissions ➢ percent increase/decrease, percent error ▶ Solve problems about scale drawings (corresponding lengths between the objects/relationships of lengths within an object are preserved in similar objects) <ul style="list-style-type: none"> ➢ ratios of lengths, areas ➢ ratio of quantities measured in like/different units (including across measurement systems). ▶ Graph proportional relationships, understand unit rate informally as a measure of the steepness of the related line <ul style="list-style-type: none"> ➢ identify the constant of proportionality in tables, graphs, equations, diagrams, and verbal descriptions ▶ Differentiate between proportional and nonproportional relationships <ul style="list-style-type: none"> ➢ test for equivalence through tables and graphs 	<p>NY-7.RP.1 Compute unit rates associated with ratios of fraction.</p> <p>NY-7.RP.2 Recognize and represent proportional relationships between quantities.</p> <p>NY-7.RP.3 Use proportional relationships to solve multistep ratio and percent problems.</p>

Instructional Focus	Developmental Focus	Instructional Consideration (via Standards)
<p>Develop understanding of operations with rational numbers and working with expressions and linear equations.</p>	<ul style="list-style-type: none"> ▶ Recognize fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers ▶ Extend addition, subtraction, multiplication, and division to all rational numbers <ul style="list-style-type: none"> ➢ show that a number and its opposite have a sum of 0 (are additive inverses) ➢ interpret sums of rational numbers by describing real-world contexts. ➢ understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. ▶ Explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers <ul style="list-style-type: none"> ➢ apply properties of operations (particularly the distributive property) as strategies to add, subtract, multiply and divide rational numbers. ➢ If p and q are integers, then $-\left(\frac{p}{q}\right) = \frac{-p}{q} = \frac{p}{-q}.$ ➢ interpret answers by describing real-world contexts. 	<p>NY-7.NS.1 Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers. Represent addition and subtraction on a horizontal or vertical number line.</p> <p>NY-7.NS.2 Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.</p> <p>NY-7.NS.2d Convert a fraction to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.</p> <p>NY-7.EE.1 Add, subtract, factor, and expand linear expressions with rational coefficients by applying the properties of operations.</p>

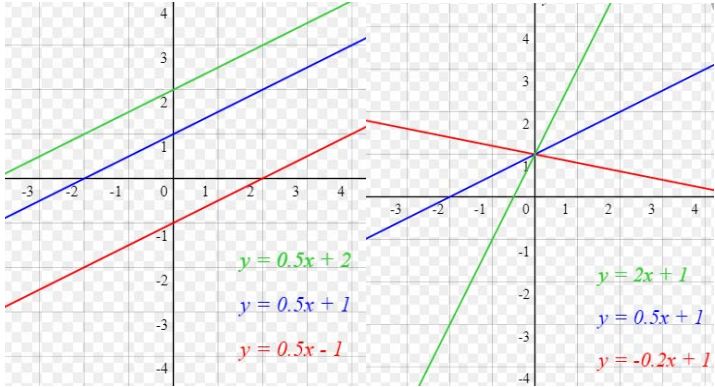
Instructional Focus	Developmental Focus	Instructional Consideration (via Standards)
<p><i>Develop understanding of operations with rational numbers and working with expressions and linear equations continued...</i></p>	<ul style="list-style-type: none"> ▶ Using rational numbers, formulate expressions and equations in one variable and use these equations to solve problems <ul style="list-style-type: none"> ➢ <i>If a woman making \$25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or \$2.50, for a new salary of \$27.50.</i> ➢ <i>If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar <u>about</u> 9 inches from each edge; this estimate can be used as a check on the exact computation."</i> 	<p>NY-7.EE.3 Solve multi-step real-world and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate. Assess the reasonableness of answers using mental computation and estimation strategies</p>
<p>Draw inferences about populations based on samples.</p>	<ul style="list-style-type: none"> ▶ Compare two data distributions and address questions about differences between populations <ul style="list-style-type: none"> ➢ examine degree of visual overlap ▶ Work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences <ul style="list-style-type: none"> ➢ interpret (not construct) box-plots that may contain outliers ▶ Calculate probabilities of compound events <ul style="list-style-type: none"> ➢ know that the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs ➢ use organized lists, tables, tree diagrams, and simulations to represent sample spaces for compound events 	<p>NY-7.SP.1 Construct and interpret box-plots, find the interquartile range, and determine if a data point is an outlier.</p> <p>NY-7.SP.4 Use measures of center and measures of variability for quantitative data from random samples or populations to draw informal comparative inferences about the populations.</p> <p>NY-7.SP.8 Find probabilities of compound events using organized lists, sample space tables, tree diagrams, and simulation.</p>

Grade 7 Math: Instructional Focus and Fluency

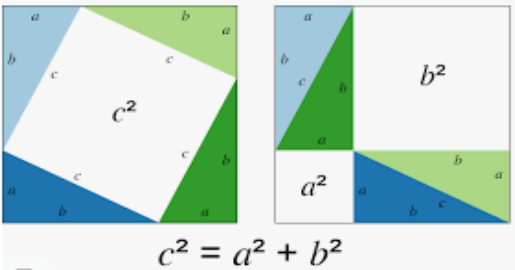
Fluency	Fluency development	Fluency Standard
Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are rational numbers. Solve equations of these forms fluently (<i>procedural</i>).	<ul style="list-style-type: none"> ▶ Students may need to simplify and/or combine like terms on the same side of an equation. ▶ Students will need practice constructing equations from word problems. 	NY-7.EE.4a Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

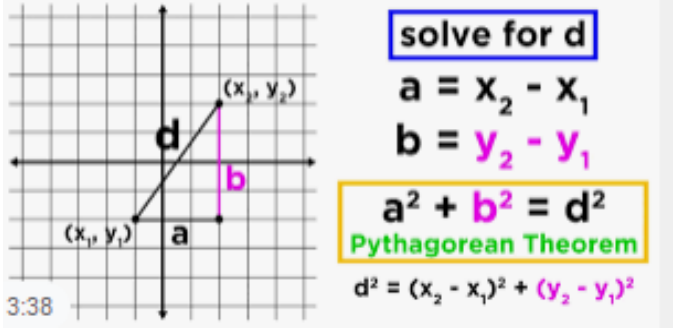
Grade 8 Math: Instructional Focus and Fluency

Transitioning to the NYS Next Generation Math Learning Standards for Grades K-8, Effective September 2022

Instructional Focus	Developmental Focus	Instructional Consideration (via Standards)
<p>Formulate and reason about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations.</p>	<ul style="list-style-type: none"> ▶ Recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$) <ul style="list-style-type: none"> ➢ understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin ➢ know the parts of the equation (x,y)represents the point, m represents the slope, and b represents the y intercept ▶ Understand that the slope (m) of a line is a constant rate of change ▶ Express a linear relationship between two quantities and interpret components of the relationship (such as slope and y-intercept) in terms of the situation <div style="text-align: center;">  </div> <ul style="list-style-type: none"> ➢ e.g., Compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. 	<p>NY-8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.</p> <p>NY-8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.</p> <p>NY-8.EE.7 Solve linear equations in one variable.</p>

Instructional Focus	Developmental Focus	Instructional Consideration (via Standards)
<p><i>Formulate and reason about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations continued...</i></p>	<ul style="list-style-type: none"> ▶ <i>(POST-TEST)*</i> Solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane(intersect, parallel, or are the same line) ▶ <i>(systems are POST-TEST)*</i> Apply linear and systems of linear equations to solve application problems <ul style="list-style-type: none"> ➢ one solution, no solutions, infinite solutions (may need to simplify) ➢ using rational number coefficients ➢ involving combining like terms, and using the distributive property 	<p>NY-8.EE.8 <i>(POST-TEST)*</i> Analyze and solve pairs of simultaneous linear equations.</p>
<p>Grasp the concept of a function and use functions to describe quantitative relationships a function as a rule that assigns to each input exactly one output.</p>	<ul style="list-style-type: none"> ▶ Understand that functions describe situations where one quantity determines another <ul style="list-style-type: none"> ➢ e.g., where the function is increasing or decreasing or when the function is linear or non-linear ▶ Translate among representations and partial representations of functions and describe how aspects of the function are reflected in the different representations <ul style="list-style-type: none"> ➢ function notation is NOT required in grade 8 ➢ include description, table, and graph representations ➢ include rate of change and initial value ➢ include linear and non-linear 	<p>NY-8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</p> <p>NY-8.F.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph. Sketch a graph that exhibits the qualitative features of a function that has been described in a real-world context.</p>

Instructional Focus	Developmental Focus	Instructional Consideration (via Standards)
<p>Analyze two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.</p>	<ul style="list-style-type: none"> ▶ Use translations, rotations, reflections, and dilations to describe, analyze, and solve problems involving two dimensional figures <ul style="list-style-type: none"> ▶ lines/line segments mapped to lines/line segments ▶ angles mapped to angles ▶ parallel lines mapped to parallel lines ▶ establishing congruence/similarity using properties of transformations ▶ lines of reflection are limited to both axes and lines of the form $y = k$ and $x = k$, where k is a constant when graphing ▶ rotations are limited to 90 and 180 degrees about the origin (unless otherwise specified, rotations are assumed to be counterclockwise) ▶ Show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines ▶ Understand the statement of the Pythagorean Theorem and its converse, and why the Pythagorean Theorem holds <div data-bbox="638 1141 1150 1409" style="text-align: center;">  <p>$c^2 = a^2 + b^2$</p> </div>	<p>NY-8.G.1 Verify experimentally the properties of rotations, reflections, and translations.</p> <p>NY-8.G.2 Know that a two-dimensional figure is congruent to another if the corresponding angles are congruent and the corresponding sides are congruent. Equivalently, two two-dimensional figures are congruent if one is the image of the other after a sequence of rotations, reflections, and translations. Given two congruent figures, describe a sequence that maps the congruence between them on the coordinate plane.</p> <p>NY-8.G.3 Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.</p> <p>NY-8.G.4 Know that a two-dimensional figure is similar to another if the corresponding angles are congruent and the corresponding sides are in proportion. Equivalently, two two-dimensional figures are similar if one is the image of the other after a sequence of rotations, reflections, translations, and dilations. Given two similar two-dimensional figures, describe a sequence that maps the similarity between them on the coordinate plane.</p>

Instructional Focus	Developmental Focus	Instructional Consideration (via Standards)
<p>Analyze two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem continued...</p>	<p>► Apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons.</p> <div data-bbox="558 418 1226 821" style="border: 1px solid gray; padding: 5px; margin: 10px 0;"> <p style="text-align: center;">Deriving and Using the Distance Formula</p>  </div> <p>https://youtu.be/wzQstiqxbuo</p>	<p>NY-8.G.5 Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.</p> <p>NY-8.G.6 Understand a proof of the Pythagorean Theorem and its converse.</p> <p>NY-8.G.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real world and mathematical problems in two and three dimensions.</p> <p>NY-8.G.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</p>

Fluency	Fluency development	Fluency Standard
<p>Solve systems of linear equations in two variables graphically, algebraically, and using a table. (<i>Procedural</i>).</p>	<p>► (<i>POST-TEST</i>)* Solving systems algebraically will be limited to at least one equation containing at least one variable whose coefficient is 1. Algebraic solution methods include elimination and substitution.</p>	<p>NY-8.EE.8b (<i>POST-TEST</i>)* Solve systems of two linear equations in two variables with integer coefficients: graphically, numerically using a table, and algebraically. Solve simple cases by inspection.</p>

**(POST-TEST) refers to standards content that is taught after the NYS grade 3-8 assessment. This time typically occurs late April - June.*